Effect of Stress-Path on the Stress-Strain-Volume Change Relationships of a River Sand

A. VARADARAJAN
Assistant Professor, Civil Engineering Department, Indian Institute of Technology, Delhi
S. S. MISHRA
Research Scholar, Civil Engineering Department, Indian Institute of Technology, Delhi
G. L. WADHWA
Formerly Graduate Student, Civil Engineering Department, Indian Institute of Technology, Delhi

SUMMARY The effect of stress-path on the stress-strain-volume change relationships of a river snad is presented. It is shown that these relationships are very much a function of stress-paths. Modulus values are evaluated for different stress-paths and relationships between stress-paths and modulus values are established. A procedure for using these relationships in the analyses of field problems using finite element method is indicated.

1 INTRODUCTION

The current integrated analyses of movements and failure in soil masses using finite element method incorporate in them stress-strain volume change relationships of soil. The results obtained from such analyses are as realistic as the soil parameters used in the analyses. The stress-strain behaviour of soils is a complex function of many factors such as soil type, structure, confining pressure, stress-history, drainage condition, stress-path etc. (Lambe and Whitman, 1969; Duncan and Chang, 1970; Yudhbir and Varadarajan, 1975; Lade and Duncan, 1975, 1976). Considerable research work is now directed towards developing analytical models for describing the soil behaviour (Desai, 1977). Research work is in progress at IIT Delhi in developing an analytical model for granular soils. This paper deals with one of the major factors influencing the stress-strain volume change behaviour of soils, viz. stress-path. Experimental results for a local river sand are presented and discussed. Evaluation of parameters based on theory of elasticity for different stress paths is indicated. A procedure for using these parameters in the analyses of field problems is also dealt with in the paper.

2 TESTING PROGRAMME

Two test series were carried out using standard triaxial testing equipment. In the first series the samples were consolidated isotropically and sheared to failure using standard triaxial compression tests. Three consolidation stresses σ_c, 1.05, 2.11 and 3.16 kg/cm^2 were used.

In the second series, the samples were anisotropically consolidated with a stress ratio K=0.42 to approximate the insitu state of stress condition. The samples were then sheared using six stress-paths (Figure 1), (i) vertical stress, σ_1 constant and lateral stress, σ_3 increasing (ii) σ_1 and σ_3 increasing such that $\sigma_3/\sigma_1 = 0.42$ (iii) σ_3 constant and σ_1 increasing (iv) σ_3 decreasing σ_1 constant (v) σ_1 and σ_3 decreasing such that $\sigma_3/\sigma_1 = 0.42$ and (vi) σ_3 constant and σ_1 decreasing. The stress-levels $p_c = (\sigma_1 + \sigma_3)/2$, 1.97, 3.94 and 5.91 kg/cm^2 were adopted.

2.1 Soil Used

Jamuna river sand was used for experimental investigations. River Jamuna is located near Delhi and is a major tributory to the famous river Ganga in

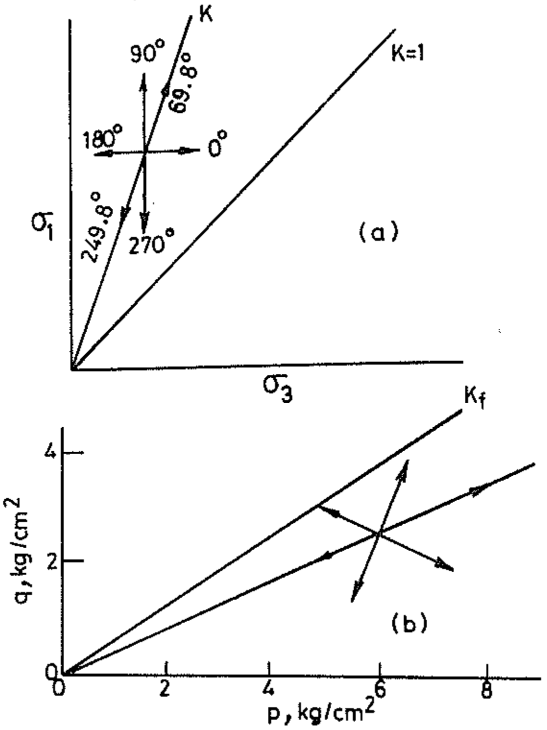

Figure 1 Stress-paths used for the tests in σ_3- σ_1 and p-q plots

the northern region of Indian subcontinent. The sand was of uniform size with ninety per cent of the soil having grain sizes between 0.1 mm and 0.4 mm. The sand contained mica flakes and was grey in colour. The specific gravity of sand material was 2.66. A single batch of sand was used for all the tests.

2.2 Experimental Procedure

Standard triaxial compression testing equipment was used for all the tests. 3.81 cm diameter and 7.62cm long cylindrical samples were used. The samples were prepared under saturated condition (Bishop and Henkel, 1957). A uniform procedure of tamping the mould was used to achieve the same density of the samples. Cell pressure was applied by the self compensating mercury pot system and the vertical loading was applied through the loading frame-proving ring system. For tests with σ_3 constant, strain controlled loading was applied. For other stress-path tests cell pressure and the vertical loading were applied by manual control in small increments. The loading to be applied was estimated at every increment after using appropriate area correction for the sample. For speedy calculations during experiment a standard programme was used with the aid of a 200 step programmable calculator. Vertical displacement and volume change readings were noted after steady condition was reached for each increment of loading. Smaller increments of loading were used as failure was reached. The vertical displacements and volume changes were measured to the accuracy of 0.000254 cm and 0.05 cc respectively.

3 EXPERIMENTAL RESULTS

3.1 Consolidation and Rebound

Figure 2 shows consolidation and rebound curves for isotropic and anisotropic stress conditions. Both the curves show similar behaviour. Very small amount of particle crushing was observed at higher pressure levels and this appears to have also contributed for the curvature of loading curves at those pressure levels. The rebound curves for the two stress conditions show that the irrecoverable volumetric strain is more in the case of isotropic unloading condition.

3.2 Stress-Strain-Volume Change Relationships

3.2.1 Isotropically consolidated samples

Figure 3 shows stress-strain-volume-change relationship of σ_3 constant stress-path test. All the

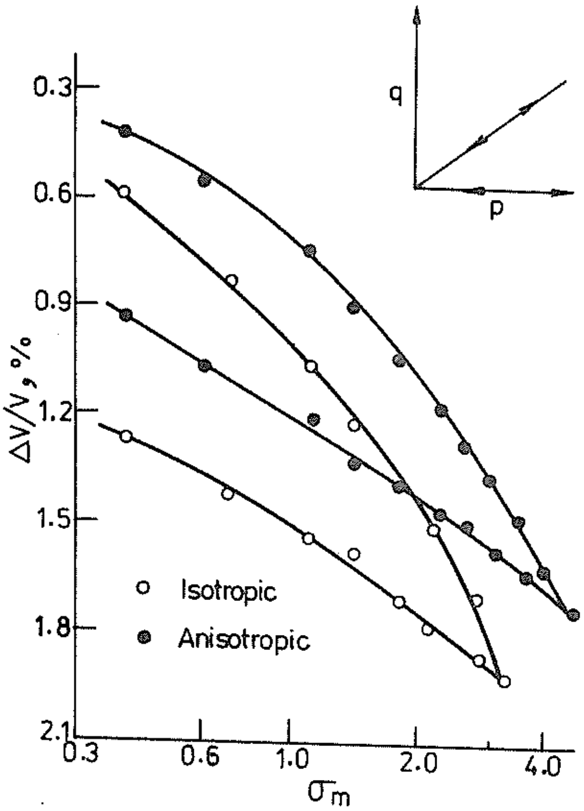

Figure 2 Isotropic and anisotropic loading and rebound curves

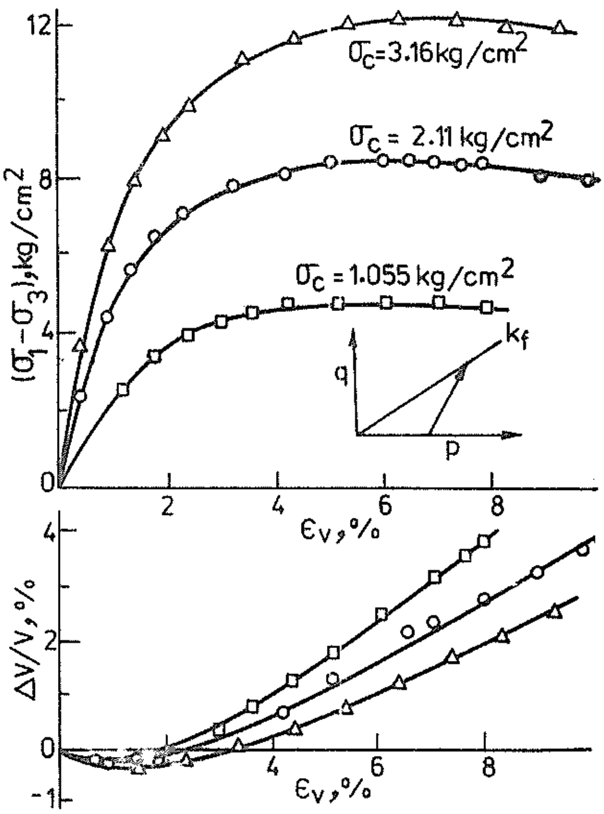

Figure 3 Stress-strain-volume change relationships of standard triaxial compression test (σ_3 constant).

tests indicate increase in vertical failure strain with higher consolidation pressures. Volume expansion is noted for all the tests near failure state.

3.2.2 Anisotropically consolidated samples

Figure 1a shows the stress-paths used in σ_3 - σ_1 stress-space. The stress-paths are designated as ASP0, ASP69.8, ASP90, ASP180, ASP249.8 and ASP270, the numbers indicating the angles with respect to horizontal in the counter clockwise direction. ASP indicates anisotropically consolidated samples.

In Figures 4 and 5 are shown the stress-strain volume-change relationships for ASP90 and ASP180 tests. These tests were carried out till failure was reached. In both the cases higher vertical failure strains are observed for higher pre-shear stress level

$$p_c \left(= \frac{\sigma_1 + \sigma_3}{2}\right)$$

Volume compression is observed only in the initial range of loading in ASP90 test and in the higher loading range volume expansion is noted whereas in ASP180 test volume expansion is noted from the very beginning of the test. Figures 6 and 7 show stress strain-volume-change relationships for ASP69.8 and ASP249.8 tests. These are loading and unloading tests along the K-line. The relationships are almost linear. As expected ASP69.8 shows compres-

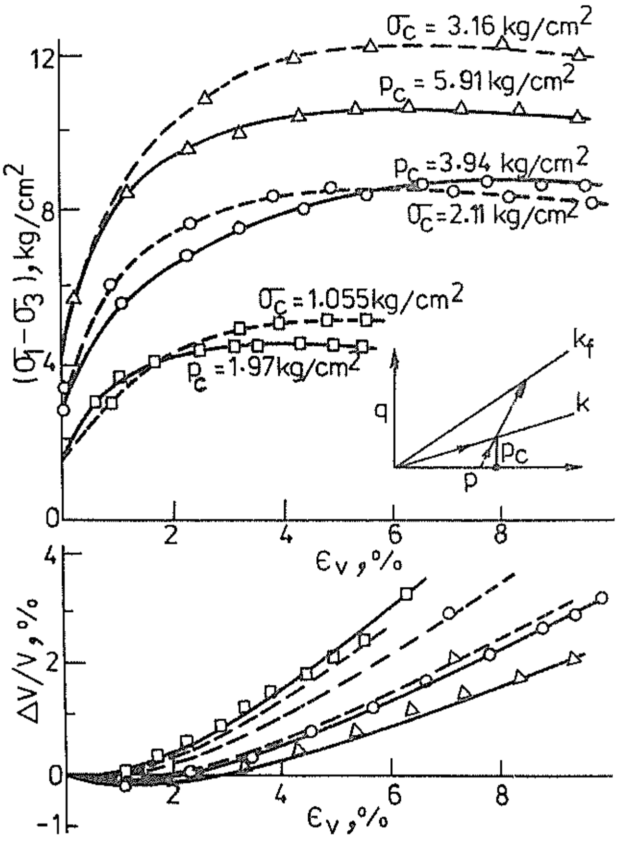

Figure 4 Stress-strain-volume change relationships of ASP90 and ISP90 tests

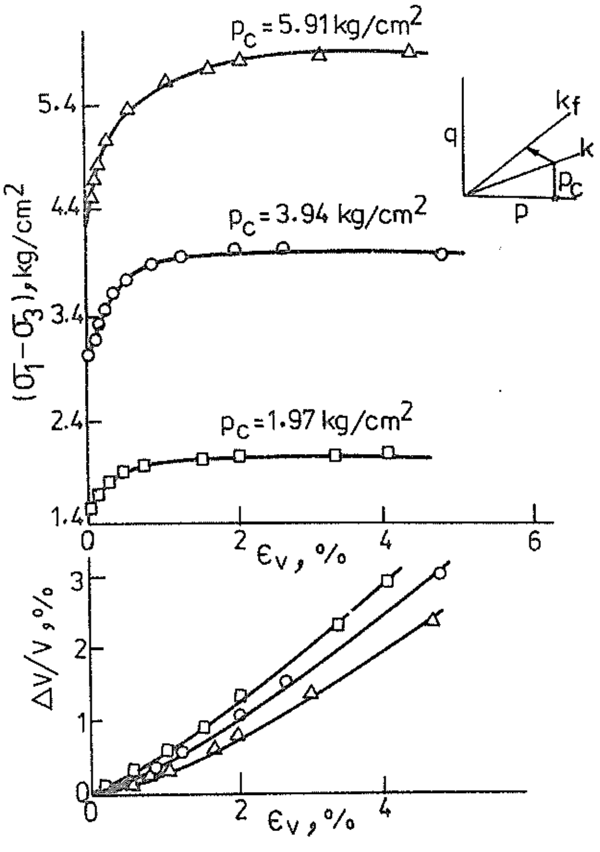

Figure 5 Stress-strain-volume change relationships of ASP180 tests

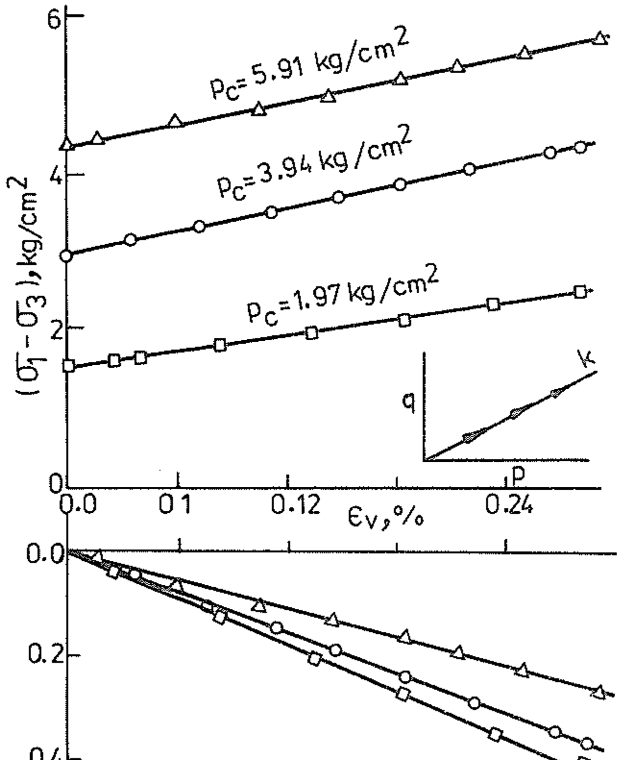

Figure 6 Stress-strain-volume change relationships of ASP69.8 tests

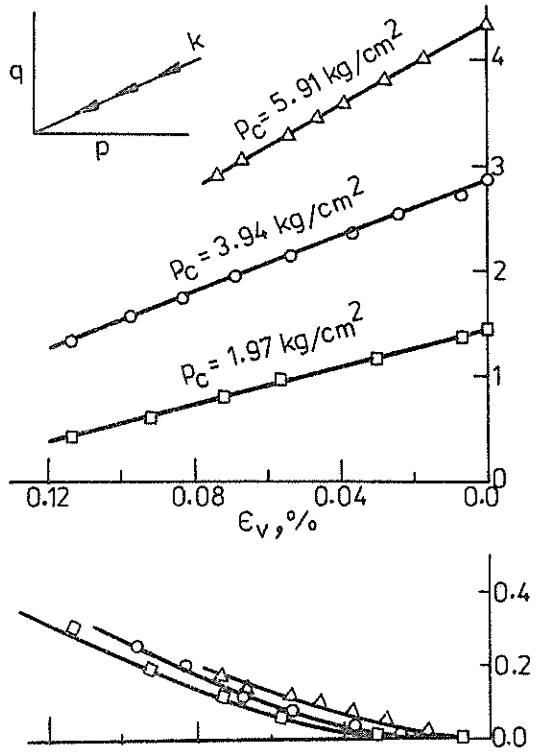

Figure 7 Stress-strain-volume change relationships of ASP249.8 tests

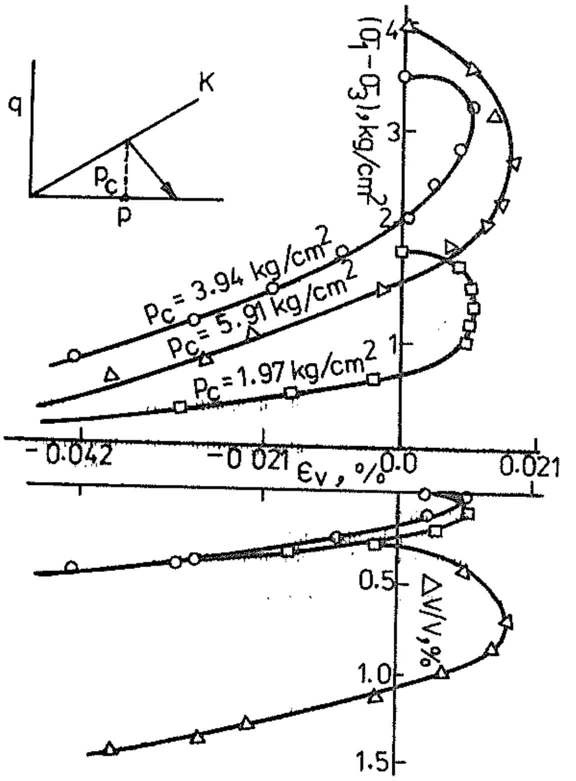

Figure 8 Stress-Strain-Volume change relationships of ASPO tests

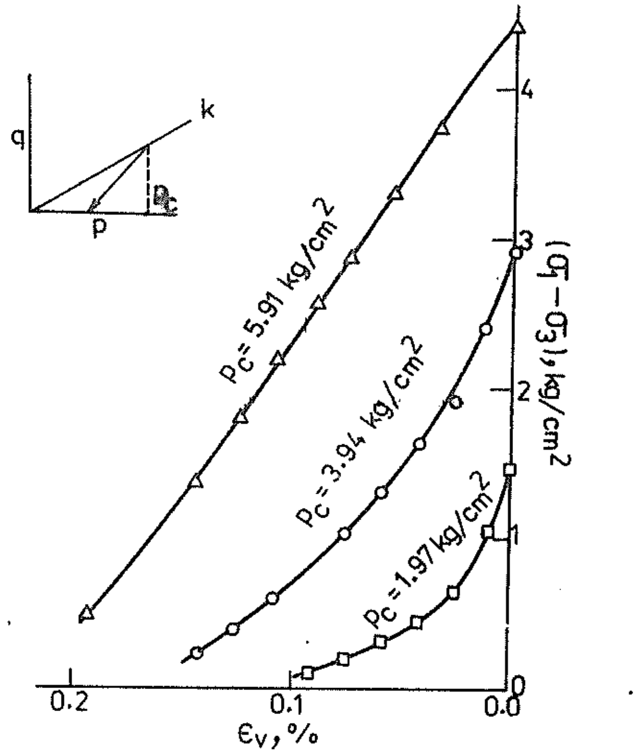

Figure 9 Stress-Strain relationships of ASP270 tests.

sion and ASP249.8 indicates expansion. Stress-strain-volume change relationships for ASPO are shown in Figure 8. In these tests shear stress was decreased up to zero value only (due to limitations of standard triaxial cell). In the initial stages of loading small increase in vertical strains and for subsequent loading reduction in vertical strains are observed. During all stages of loading tests show volumetric compression.

Figure 9 shows stress-strain relationships for ASP 270. Negative vertical strain is indicated all through the test. For all the stress ranges negligible volume expansion is observed (not shown in Figure).

A comparative stress-strain-volume change relationships for all the stress-paths are presented in Figure 10 at p_c = 3.94 kg/cm^2. For ASPO, ASP270 and ASP249.8 tests, the shear stress is decreased and for ASP180, ASP90 and ASP69.8 the shear stress is increased. Whereas for the stress-paths ASPO, ASP270 and ASP249.8 vertical strains are negative, for the stress-paths ASP180, ASP270 the vertical strains are positive.

The ASPO and ASP69.8 volume contraction is noted whereas for ASP80 and ASP249.8 volume expansion is observed. For ASP90 volume contraction is noted at initial loading range and volume expansion is indicated at higher loading range. Negligible volume change is observed for ASP270.

In Figure 4 are also shown the portions of stress-strain-volume change relationships of standard triaxial compression tests in addition to those of anisotropically consolidated samples. Though the stress conditions before shearing (with σ_3 constant) are same for both cases the relationships are

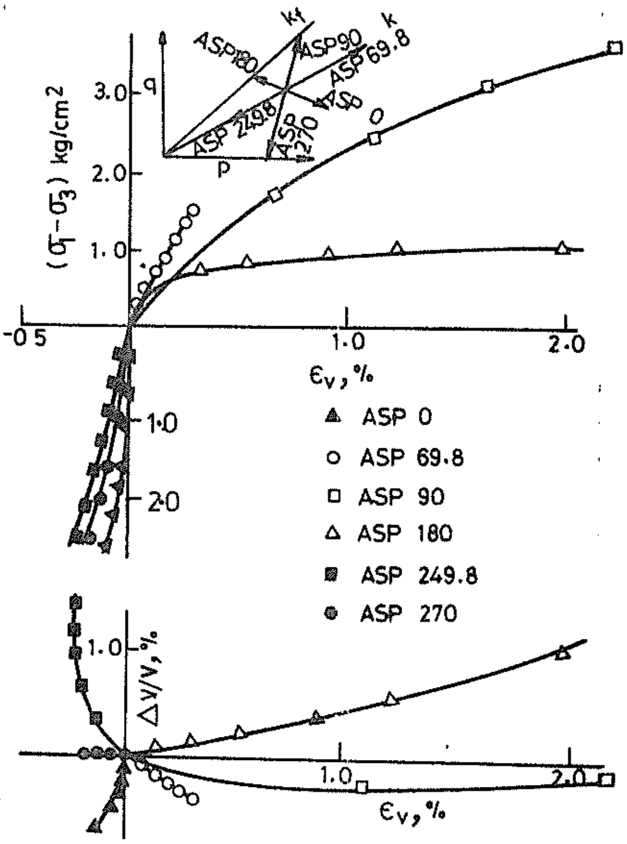

Figure 10 Comparison of stress-strain volume change relationships of different stress-path tests

markedly different.

4 STRENGTH ENVELOPE

The Mohr-Coulomb envelope obtained from the test results show slight curvature for high stress levels. The angle of shearing resistance at lower stress range is 42° and it is 39° at higher stress range.

5 ELASTIC STRESS-STRAIN PARAMETERS

To study the effect stress-path on the stress-strain-volume change relationships the Young's moduli, E and Poisson's ratios, ν, were calculated using the stress-probe level (n = 0.2) following Lewin and Burland(1970) as

$$\sigma_n = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} \tag{1}$$

$$\Delta\sigma_n = 0.2\ \sigma_n = \sqrt{\Delta\sigma_1^2 + \Delta\sigma_2^2 + \Delta\sigma_3^2} \tag{2}$$

$\Delta\sigma_1$ and $\Delta\sigma_3$ values were calculated for this stress-level and corresponding values of strains $\Delta\epsilon_1$ and $\Delta\epsilon_3$ values for each stress-path were obtained from the experimental results and these E and ν values were obtained using theory of elasticity (Lambe and Whitman, 1969) as follows

$$E = \frac{(\Delta\sigma_1 + 2\Delta\sigma_3)(\Delta\sigma_1 - \Delta\sigma_3)}{\Delta\sigma_3(\Delta\epsilon_1 - 2\Delta\epsilon_3)+\Delta\sigma_1\Delta\epsilon_1} \tag{3}$$

and $$\nu = \frac{\Delta\sigma_3\ \Delta\epsilon_1 - \Delta\epsilon_3\ \Delta\sigma_1}{\Delta\sigma_3(\Delta\epsilon_1 - 2\Delta\epsilon_3)+\Delta\sigma_1\Delta\epsilon_1} \tag{4}$$

where,

$\Delta\sigma_1$ and $\Delta\sigma_3$ = change in σ_1 and σ_3

ϵ_1 and ϵ_3 = corresponding change in vertical and lateral strains.

The effect of consolidation pressure on E values was expressed using Janbu's (1963) relationships

$$E_i = K\ p_a \left(\frac{\sigma_m}{p_a}\right)^n \tag{5}$$

where,
k = modulus number
n = exponent determining rate of variation of E with σ_m
p_a = atmospheric pressure expressed in the same unit as E and σ_m

$$\sigma_m = \frac{\sigma_1 + 2\sigma_3}{3}$$

E_i = initial tangent modulus

Within the stress ranges considered, it was observed that K, n vs σ_m relationships were bilinear. K and n relationships for the two stress ranges are presented in Table 1. It may be observed that the K value which is also equal to E value at σ_m=1 kg/cm² is very much a function of stress path. Between the minimum and maximum value of K the ratio is in the order of about 10. The Poisson's ratio shows a wide variation for different stress-paths (not shown in the Table). Thus, the E and values are a function of stress level and stress-path. In analysing field problems it is, therefore, necessary to use appropriate modulus values.

Table 1

Stress-path	σ_m 1.54-3.08 kg/cm² K	n	3.08-4.62 kg/cm² K	n
ASP0	681	0.987	1554	-0.315
ASP69.8	542	0.64	1167	-0.61
ASP90	292	0	87.2	1.62
ASP180	84	0.92	154.3	-0.21
ASP249.8	1156	0.65	2677	-0.33
ASP270	2023	1.05	8437	-1.21

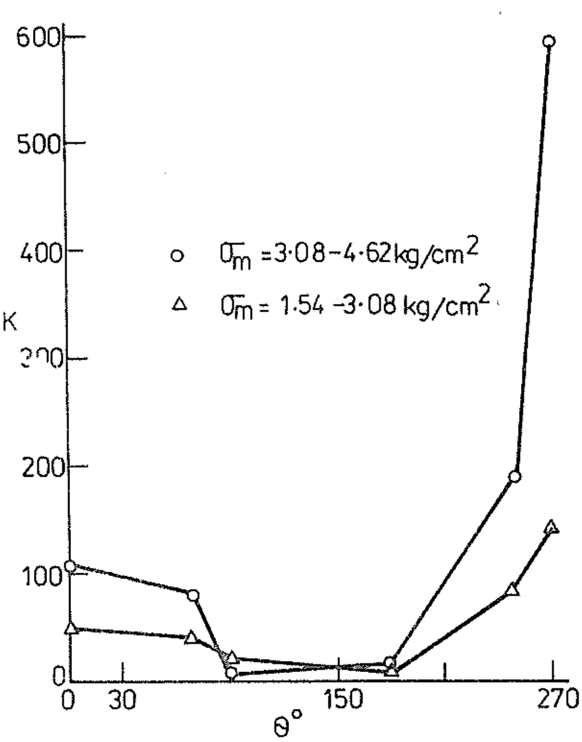

Figure 11 K - θ relationship

6 USE OF ELASTIC PARAMETERS

K and n vs stress-paths expressed quantitatively as θ in $\sigma_1 - \sigma_3$ stress space are shown in Figures 11 and 12. From these figures, equations can be developed between K and n and θ values. Values of K and n can be conveniently obtained from the figures or equations.

For realistic analysis of field problems with finite element method at higher factors of safety, the modulus values evaluated by this procedure may be advantageously used. To start with the analyses of any field problem may be carried out using K and n values of a stress-path which is representative for the field problem (for example, for the footing problem σ_3 constant stress path is the representative one). The stress paths for each element may be evaluated in terms of θ from the stresses

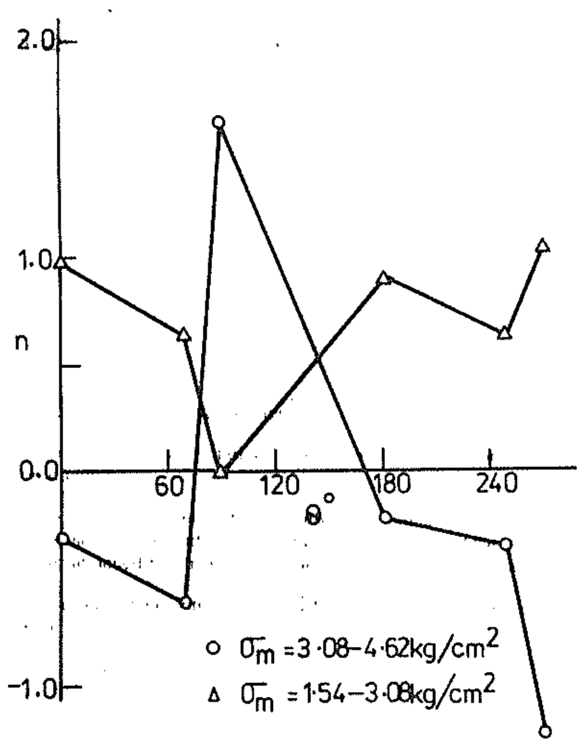

Figure 12 n – θ relationship

obtained. Knowing the θ values, the analysis may be repeated using appropriate values of K and n in each element. Two or three such iterations may be necessary before the modulus values used are consistent with the stress-paths. Either an average value of Poisson's ratio or appropriate values of Poisson's ratio obtained from similar relationships as for E may be used.

7 CONCLUSIONS

On the basis of the limited experimental investigations carried out on Jamuna sand, it is observed that the stress-strain-volume change relations are very much a function of stress-path. Stress-paths can be quantified and the relationships so established. This relationships may be used for the realistic analyses of field problems using finite element method.

8 REFERENCES

BISHOP, A.W. and HANKEL, D.J. (1957). The measurement of soil properties in triaxial test. Edward Arnold (Publishers) Ltd., London.

DESAI, C.S.(1977). Constitutive law for geologic media, In C.S. Desai and J.T. Christian (ed.), Numerical Methods in Geotechnical Engineering, McGraw-Hill Book Company, New York.

DUNCAN, J.M. and CHANG, C.Y.(197). Nonlinear analysis of Stresses and strain in soil, Jnl.of soil mechanics and foundation division, ASCE,Vol.96, pp.1629-1653.

JANBU, N.(1963). Soil compressibility as determined by Oedometer and triaxial tests, European Conference on soil mechanics and foundation engineering, Germany, Vol.1, pp.19-25).

LADE, P.V. and DUNCAN, J.M.(1975). Elastoplastic stress-strain theory for cohesionless soil. Jnl. of the geotech. engng. divn., ASCE, 101, pp.1037-1054.

LADE, P.V. and DUNCAN, J.M.(1976). Stress-path dependent behaviour of cohesionless soil, Jnl. of the geotech. engng. divn., ASCE, 102,pp.51-68.

LAMBE, T.W. and WHITMAN, R.V.(1969). Soil mechanics, John Wiley and Sons, Inc., New York.

LEWIN, P.I. and BURLAND, J.B.(1970). Stress probe experiments on saturated normally consolidated clay, Geotechnique, Vol.20, pp.38-56.

YUDHBIR and VARADARAJAN, A.(1975). Stress-path dependent of deformation moduli of clay, Jnl. of the geotechnical engng. divn., Proc. ASCE,Vol.101, pp.315-327.