Stability of Cut Slopes in a Pumice Soil Deposit with Particular Reference to Tensile Failure

T. YAMANOUCHI
Professor of Civil Engineering, Kyushu University, Japan
K. GOTOH
Associate Professor of Civil Engineering, Kyushu University, Japan
H. MURATA
Associate Professor of Civil Engineering, Kagoshima Technical College, Japan

SUMMARY The properties of a pumice soil called *Shirasu* distributed especially in southern Kyushu of Japan are varied from those of granular soil to those of weak rock in its undisturbed state. In dealing with its cut-off slopes, the tensile failure is unavoidable in the traditional vertical slopes. This paper reports with regard to the problem of tensile failure causing in the cut-off *Shirasu* slopes, dividing into three parts; (1) the identification and classification of the undisturbed *Shirasu* applying a practical measure called soil hardness, (2) strength and elastic constants of the undisturbed specimens in the triaxial compression tests, and (3) stability analyses of the cut-off *Shirasu* slopes of a few kinds of slope angles not only at the ordinary time but also at the time of earthquake, using the finite element method and the seismic coefficient method jointly. The third part indicates where the tensile failure arises in the slopes, and the result proved the superiority of Kyushu Transverse Expressway in which the gentle cut-off slope of 45 degrees was for the first time adopted in the *Shirasu*-distributed area. The gulley erosion is there prevented by means of scrupulous drainage arrangements.

1 INTRODUCTION

The properties of a pumice soil called *Shirasu* distributed in certain areas in Japan are varied from those of granular soil to those of weak rock in its undisturbed state, and there are known no cases of rotational shear-slide failure occurring to cut slopes in usual soil deposits. *Shirasu* may be classified as a soil similar to the yellow-brown pumice soil distributed in North Island of New Zealand. In dealing with these cut-off slopes, two kinds of problems have to be faced; one is the gulley erosion resulted from heavy rainfalls and the other is tensile failure that is unavoidable in the traditional steep cut-off slopes. The gulley erosion is, however, prevented now by means of careful drainage works, but there is much to be made clear about the problem of tensile failure.

The authors made an analytical study on the cut-off slopes of *Shirasu* using the finite element method and from the viewpoint of tensile failure, exclusively on the statical condition (Yamanouchi et al, 1975). They have also published a study on the failure mechanism of the undisturbed samples of *Shirasu* where the rock mechanics approach was applied (Yamanouchi and Murata, 1979).

This paper consists of three parts and the first part deals with the identification and classification of the original *Shirasu* ground according to the soil hardness, giving the result of statistical analysis with the analysis of principal components as its main point. In the second part, strength constants of the undisturbed *Shirasu* specimen are considered in relation with the tensile strength and the hardness. In the third part, the stress condition of slope, either at the ordinary time or at the earthquake time, is analysed to compare the value of slope angle with the occurrence and the magnitude of tensile stress, and at the same time, the significance of the result thereby in the actual damage is discussed. Incidentally, in carrying out the analysis the finite element method and the seismic coefficient method are jointly used by regarding the original *Shirasu* ground bearing a slope as an isotropic and linearly elastic body.

2 IDENTIFICATION AND CLASSIFICATION OF UNDISTURBED *SHIRASU*

2.1 Object of Identification and Classification, and Definition of Hardness

Since engineering properties of the original *Shirasu* ground are not homogeneous, being considerably varied according to its state, careful consideration should be paid to the above-mentioned difference in the properties in the project of slope angle, the slope protection work and the drainage arrangements of cut-off slopes. For that purpose, the identification and classification were carried out depending upon the hardness by means of Yamanaka's soil penetrometer (Yamanaka and Matsuo, 1962) which renders a measurement comparatively easy in detecting the mechanical properties of *Shirasu* at the fields.

According to Yamanaka (1965), the hardness is defined as the force required for the material of greater hardness to penetrate into the inner part of soil against the welded bonds among particles. In this case, the authors will define the value measured by Yamanaka's soil hardness-meter, where a spring that contracts 40 mm precisely against the load of 78.45 N is used as the hardness of *Shirasu*. This hardness of soil is obtained by reading the resistance required for driving a part of the cone, 40 mm long, 18 mm in bottom diameter and 12°40' in vertical angle, into the soil by checking the contraction length of spring.

2.2 Determination of Boundary Hardness by Statistical Analysis

To identify and classify the original *Shirasu* ground, the authors have to determine first the boundary hardnesses which are to be the indication of classification. The principal component analysis was carried out at first, based on the data obtained from the field investigation, followed by the identification and classification based on the hardness from the result. Then the frequency distribution was examined by χ^2-test to see whether it would assume a normal distribution, the significance of

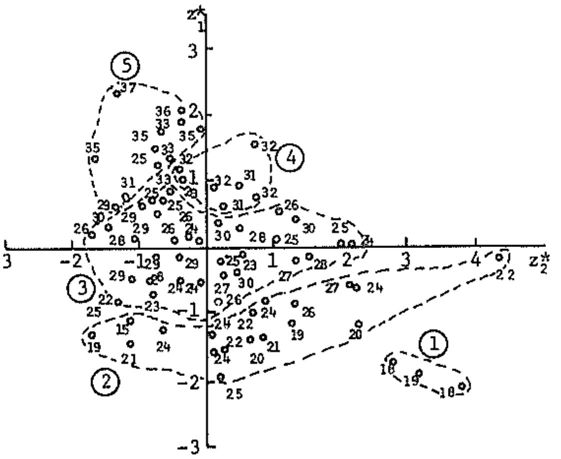

Figure 1 Group classification based on principal component analysis

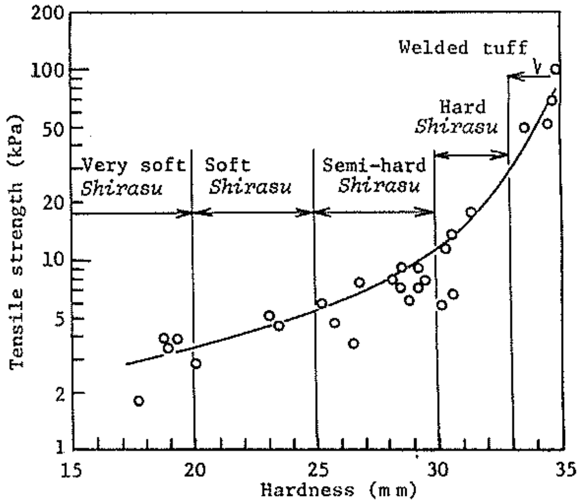

Figure 2 Relation between tensile strength and hardness of *Shirasu*

group classification is confirmed by undergoing F-test and t-test. Boundary hardness will be determined later on by the average and the standard deviation of hardness of each group already classified respectively. The data of investigation used here have been collected from 84 sites in the *Shirasu*-distributed area of Kagoshima Prefecture by Kyushu Engineering Office, Kyushu Construction Bureau, Ministry of Public Works.

Figure 1 shows the result obtained through the principal component analysis by using 5 variables of moisture content ratio, specific gravity of particle, void ratio, coefficient of uniformity and hardness. The ordinate and the abscissa in the figure represent the first principal component and the secondary one respectively, both of which are the values already normarized. The numerical values fixed to each point signify the hardness (mm), and this hardness makes it possible to devide each point in the figure into 5 groups, as shown by a broken line. In order to check whether or not the frequency distribution of the hardness belonging to each group is normal one, the authors evaluated theoretical frequency distribution based on the average of measured values and the standard deviation and carried out the test. As the result, the hardness distribution of each group was found out to be in a good accordance with that of theoretical frequency. When the significance of individual group classification was examined by F-test and t-test, therefore, it was confirmed that 5 groups fall under other population respectively. Accordingly, it is possible, if we take (average hardness ± standard deviation) to distinguish *Shirasu* thus classified into 5 categories according to the hardness, to consider that the first group lies within the range of 17.8 to 18.8, the second group 19.5 to 25.1, the third group 25.0 to 29.6, the fourth group 29.9 to 32.1 and the fifth group 32.5 to 35.9 and that the boundary hardness of each group is 20, 25, 30 and 33.

From the result of analysis made so far *Shirasu* may be classified into 5 groups of one with the hardness below 20 mm, one between 20 to 25 mm, one between 25 to 30 mm, one between 30 to 33 mm and one above 33 mm, and each group will be assumed as equivalent to its conventional name respectively, namely, very soft *Shirasu*, soft *Shirasu*, semi-hard *Shirasu*, hard *Shirasu* and welded tuff.

3 STRENGTH CONSTANTS OF UNDISTURBED *SHIRASU* SPECIMEN

3.1 Relation between Tensile Strength and Hardness

Undisturbed samples taken from the fields by using a specially devised cutter are characterized by the fact that it bears tensile strength resulted from the geological welding effect. This tensile strength can be measured simply by means of the splitting tensile test from the following equation (Akazawa, 1943)

$$\sigma_t = \frac{2P}{\pi d l} \qquad (1)$$

where, σ_t: splitting tensile strength, P: compression load, d: diameter of sample, l: thickness of sample. Incidentally, the fact that the splitting tensile strength of undisturbed *Shirasu* becomes nearly equivalent to the uniaxial tensile strength has already been confirmed (Murata and Yamanouchi, 1977). Figure 2 shows the relation between the splitting tensile strength of undisturbed *Shirasu* and the soil hardness measured at the same field as the place where the undisturbed specimen is sampled. As expected from the figure, it is obvious that the higher hardness index number rises, the higher tensile strength goes up. Also, the higher the hardness is, the larger unit weight grows, and therefore, the hard *Shirasu* is estimated to possess properties most resembled to soft rock. In southern Kyushu, therefore, there was a customary practice to build the cut-off *Shirasu* slope almost perpendicular as the countermeasure to protect the cut surface from water flow in that rainy district, though no measure were taken yet about the mechanical stability. The tensile strength of very soft *Shirasu* is extremely small, to a degree almost negligible, it seems, but when compared with disturbed samples, the effect from a welding effect is not to be overlooked.

3.2 Relation between Tensile Strength and Constants

Elastic constants and shear strength constants can be evaluated by carrying out the triaxial compressive test, using the undisturbed samples taken from the fields by using the specially devised cutter. The characteristics of the sress-strain curve obtained from the test are; i) the relation of stress with volumetric strain decreases linearly in the initial stress stage, followed by the occurrence of rapid dilatancy, ii) the difference between the maximum axial stress and the residual deviator stress is large. Since the Griffith failure criterion is applicable to the stress limit of the former, *Shirasu* may be assumed as an elastic body in the above stress zone. At this time of elastic limit, the relation between Young's modulus E_e in the case

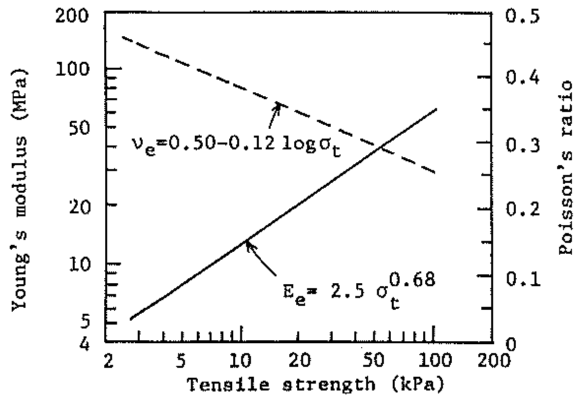

Figure 3 Relation between Young's modulus, Poisson's ratio and tensile strength

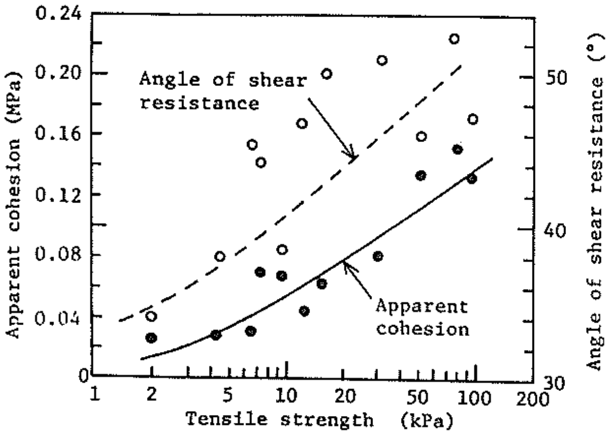

Figure 4 Relation between apparent cohesion, angle of shear resistance and tensile strength

when the confining pressure is zero and Poisson's ratio ν_e and tensile strength may be expressed by the following equation, as seen by Figure 3 (Yamanouchi and Murata, 1979).

$$E_e = 2.5\,\sigma_t^{0.68} \tag{2}$$

$$\nu_e = 0.50 - 0.12 \log \sigma_t \tag{3}$$

As stated above, undisturbed samples have an elastic domain in their initial deformation zone, but ultimately they cause shear slide failure in the same way as the disturbed *Shirasu*, resulting in shear failure. That point of time falls on that of the maximum axial stress, and under this stress condition, the modified Griffith failure criterion is applicable.

In Figure 4 are given the relation between apparent cohesion c_f and angle of shear resistance ϕ_f, which have been evaluated from the failure envelop of Mohr-Coulomb under the maximum deviator stress condition with tensile strength. The Mohr-Coulomb failure envelop of the undisturbed *Shirasu* changes to a curve with the cohesion, and the value has stopped below 0.2 MPa. Since the apparent cohesion and the angle of shear resistance, evaluated from the Mohr-Coulomb failure envelop under the condition of residual deviator stress, are almost constant independently of the tensile strength and nearly agree with the values of them when the very soft *Shirasu* gives the maximum axial stress, a rapid growth of c_f and ϕ_f with the increase of tensile strength is understood as mainly resulted from the welding effect of undisturbed *Shirasu*.

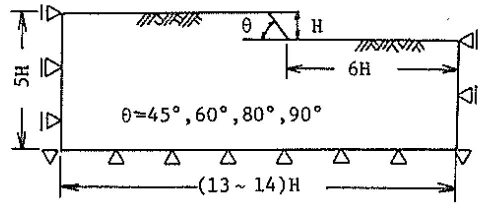

Figure 5 Analysis domain and boundary conditions for pseudo-dynamic F.E.M. analysis of cut-off *Shirasu* slope

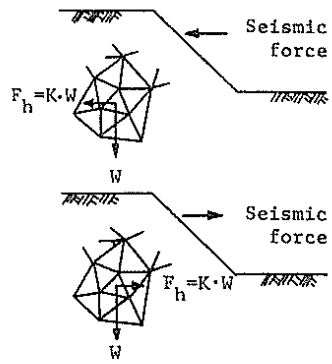

Figure 6 Key figure for seismic coefficient method

4 STABILITY ANALYSES OF CUT-OFF *SHIRASU* SLOPES

4.1 Computing Method

The analysis domain for the finite element method is given, as shown in Figure 5. As the object of the present analysis is laid in grasping of the slope behavior, it is planned in a way that the effect from sides as well as lower boundary may be prevented as much as possible. As for boundary conditions, the horizontal displacement is restricted against lateral boundary and for the lower boundary the condition of perfect fix is given.

Concerning the elastic constant, here is used the value of static condition as it is, even for the pseudo-seismic analysis, because there is hardly any dynamic value to be obtained, excepting the observed value by Omote et al (1973), nor is the result of observation made by Omote et al satisfactory enough. The tensile strength of each *Shirasu* material, elastic constant and shear strength constants to be used for analysis are given in Table I.

The key figure of the seismic coefficient method is shown in Figure 6. As seen in the figure, each

TABLE I STRENGTH CONSTANTS USED FOR ANALYSES

Kinds of *Shirasu*	Soft *Shirasu*	Semi-hard *Shirasu*	Hard *Shirasu*
Tensile strength σ_t (kPa)	5	8	22
Apparent unit weight γ_t (kN/m^3)	12.7	13.7	15.0
Young's modulus E_e (MPa)	8	11	22
Poisson's ratio ν_e	0.41	0.38	0.33
Apparent cohesion c_f (kPa)	30	42	80
Angle of shear resistance ϕ_f (°)	38	41	45

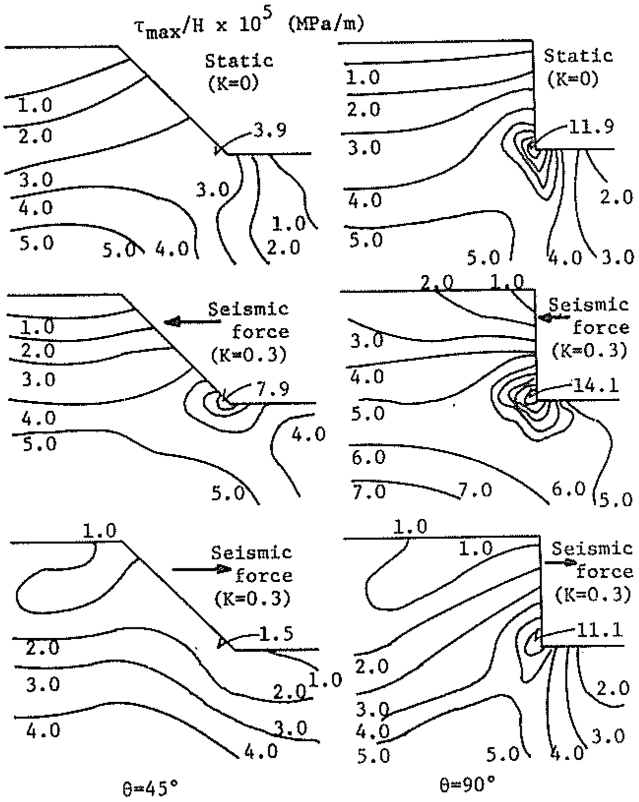

Figure 7 Maximum shear stress contours in case of semi-hard *Shirasu*

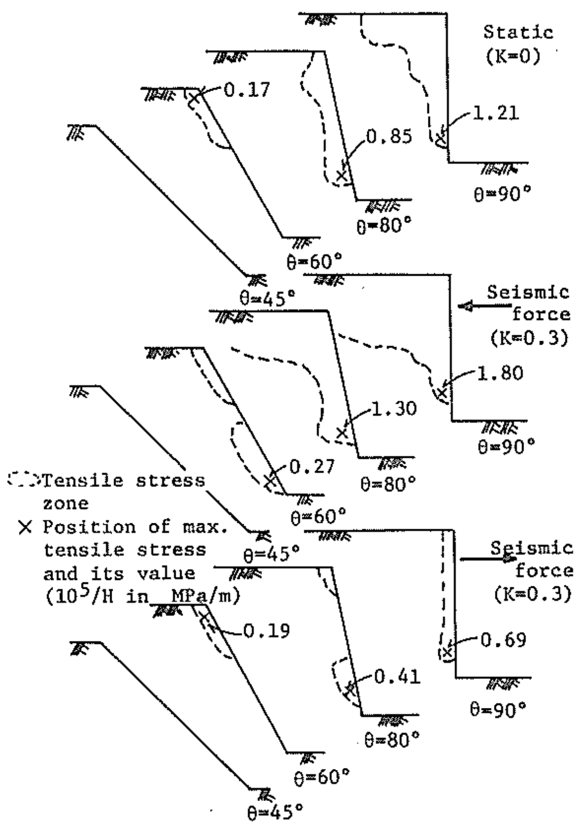

Figure 8 Relation between magnitude of tensile stress zone and slope angle in case of semi-hard *Shirasu*

element is subject to horizontal force, which is obtained by multiplying by the horizontal seismic coefficient K besides its own weight. In the case the finite element method is used, the horizontal seismic force can be taken optionally for it, but it was assumed here for simplicity that all elements are constant, and the value K = 0.3 which was estimated at the time of 1968 Ebino earthquake (Research Committee on *Shirasu*, Japanese Society of SM and FE, 1968) was adopted. Furthermore, as shown in the figure, there are two ways of input method in the seismic force acting upon the slope. One is the case when the seismic force goes toward the inside of the slope and the other is the case when it goes out of the slope, and the direction of the horizontal force acting upon each element agrees with that of this seismic force.

4.2 Distribution of Maximum Shear Stress

In Figure 7, the maximum shear stress is traced after equivalent points on the slope where the slope angle of semi-hard *Shirasu* is either 45° or 90° respectively. As plain from the figure, the maximum shear stress at the part near the toe is smaller than its static condition when the earthquake force acts in the outward direction of the slope, but it gets larger when the force acts in the incoming direction. And on the gentle slope of θ = 45°, there appears conspicuously such a concentration of the maximum shear stress on the toe as that seldom seen under a static condition. The magnitude of it is, however, far smaller than the steep slope of θ = 90°, and not only under static condition but also at the time of earthquake the danger of shear failure in the neighborhood of the slope toe is presumably higher on the slope of a steep slope.

4.3 Tensile Stress Zone and Its Magnitude

According to the result of analysis, while the major principal stress of each element is on the side of compression, the minor principal stress turns to the direction vertical to the slope, inclining to the compression side in case of gentle slope, but showing the tensile side in case of steep slope. Figure 8 shows tensile stress zone and its maximum value concerning semi-hard *Shirasu*. What is to be pointed out as a remarkable characteristics in the figure is that, on a gentle slope of θ = 45°, there occurs no tensile stress even at the time of earthquake, just as in case of static condition. In case of the slope of θ = 60°, there is merely the occurrence of tensile stress only on the slope shoulder, similarly as in case of static condition, with respect to the earthquake force going out of the slope, but for the earthquake force coming into the slope the domain of tensile stress is seen to spread almost entirely over the slope, showing the maximum value at the slope toe. As it comes to the steep slopes of θ = 80° and 90°, the tensile stress is seen to spread wider than that in case of static condition, not only the entire surface of the slope but also down to the depth of the slope, especially when earthquake force acts in the direction entering the slope.

Figure 9 shows the relation between the magnitude of tensile stress at the part of slope toe and slope shoulder and slope angle. There the value at the time of earthquake is recorded in the case when earthquake force enters the slope. As easily understood from the figure, the tendency of tensile stress at the time of earthquake is the same as that under the static condition, where the tensile stress at slope shoulder slightly increases with the increase of slope angle, whereas the tensile stress at the slope toe increases remarkably with the increase of the slope angle. And the value is then much larger than that under a static

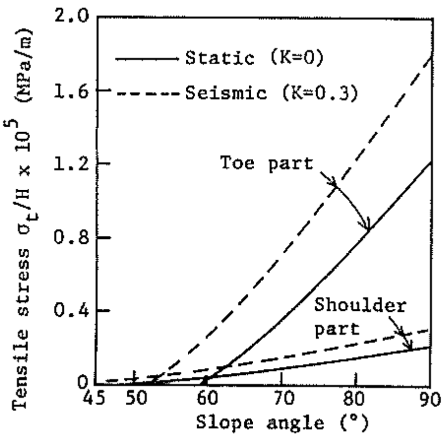

Figure 9 Relation between magnitude of tensile stress and slope angle in case of semi-hard *Shirasu*

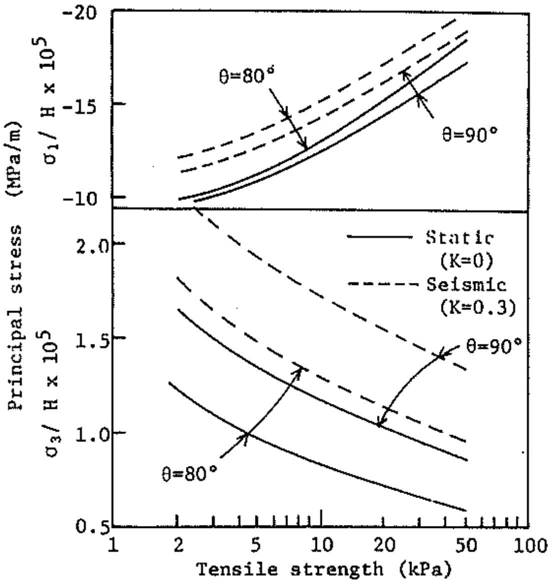

Figure 10 Relation between magnitudes of principal stresses at the toe part and tensile strength

condition. Tensile failure of the slope at the time of earthquake is, therefore, hardly possible to occur at a gentle slope, but in case of a steep slope the said danger is presumed to be greater at the toe part, under a static condition.

Figure 10 shows the relation of major principal stress and minor principal stress (tensile stress) at the slope toe in the steep slope with the tensile strength borne by *Shirasu*. It is obvious that the minor principal stress (tensile stress) which acts in the direction almost normal to the slope is larger when the tensile strength is smaller. As clearly seen from this, the major principal stress (compressive stress) gives a fairly large value and acts upon the slope in almost parallel direction, the circumstances readily lead to cause tensile failure in case of steep slope.

4.4 Relation between Local Tensile Failure and Local Shear Failure

When the height of a slope, where the maximum tensile stress occurring in the slope becomes equal to the tensile strength borne by *Shirasu*, is defined as non-cracking critical height H_{ct}, H_{ct} may be

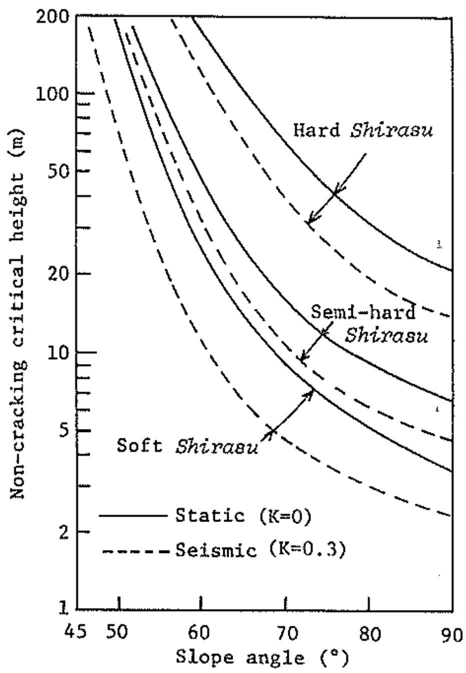

Figure 11 Relation between non-cracking critical height and slope angle

evaluated by the following equation.

$$H_{ct} = \sigma_t / \sigma_{td.max} \quad (4)$$

Where, σ_t: the tensile strength possessed by *Shirasu*, $\sigma_{td.max}$: maximum tensile stress occurring in the slope. Hence, so-called non-cracking critical height signifies that it is a certain element in the slope, representing slope height at the time when there occurs tensile failure locally, and it is important for investigating the stability of slope. By giving light to the relation between this non-cracking critical height and the slope angle of the slope with respect to three kinds of *Shirasu*, we can disclose the following facts, as shown in Figure 11.

First of all, let us pay attention to the kinds of *Shirasu*. While with hard *Shirasu* it is found in non-cracking state on the slope as high as some 20m, even when it is a steep slope above θ = 80°, with semi-hard and soft *Shirasu* of small tensile strength, non-cracking critical height considerably decreases. In case of θ = 60°, there is expected tensile failure on the slope shoulder, but the non-cracking critical height of hard *Shirasu* and semi-hard *Shirasu* arises above 50 m, and there is no danger of failure. At the time of earthquake, the non-cracking critical height of three kinds of *Shirasu* decreases further compared with that of static case. By the way, since the non-cracking critical height in the case when earthquake force acts in the direction going out of the slope arises higher than that of static case, it is left out of the figure.

With the elements in a slope, as it is sometimes expected that there might occur local shear failure, we have to investigate superfluity level S_L which is detected vertical to the Mohr-Coulomb failure envelop of shear stress of each element. S_L will be given by the following equation, referring to Figure 12.

$$S_L = \frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_3)/2 \cdot \sin\phi_f + c_f \cos\phi_f} \quad (5)$$

Where, σ_1, σ_3: major principal stress and minor

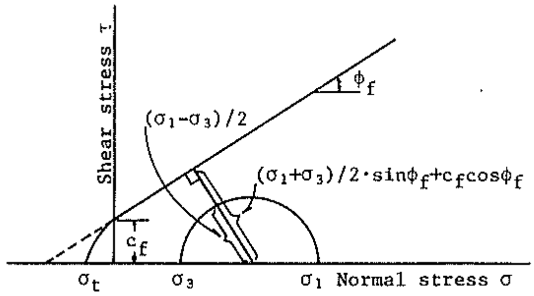

Figure 12 Illustrative figure for superfluity level S_L against shear failure

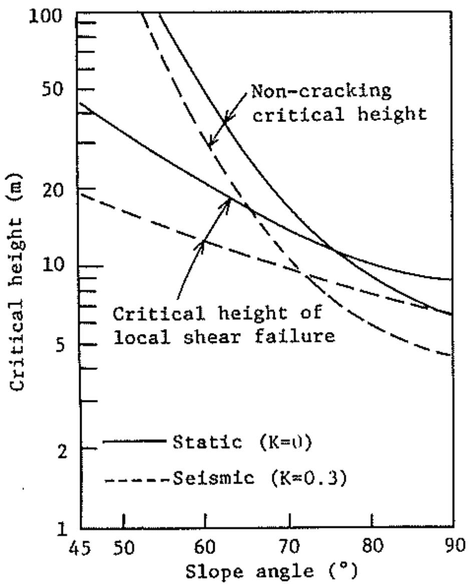

Figure 13 Relation between two kinds of critical heights and slope angle in case of semi-hard *Shirasu*

principal stress respectively, c_f, ϕ_f: the apparent cohesion and angle of shear resistance.

It is possible, therefore, to define the slope height where elements $\sigma_{td} < \sigma_t$ as well as $S_L > 1$ have appeared in the slope as the slope height H_{cs} at which local shear failure takes place. Figure 13 shows the relation between critical height H_{ct}, H_{cs} of each semi-hard *Shirasu* and slope angle of the slope. As seen from the figure, H_{ct} is larger than H_{cs} in case of $\theta \leq 75°$ to be inverse in case of $\theta \geq 75°$, whether it is under a static condition or at the time of earthquake. Such a tendency remains the same either with hard *Shirasu* or with soft *Shirasu*. It is therefore evident in case of a steep slope that the local tensile failure at the slope toe plays a dominant part in the failure of slope. Also, with the slope of $\theta = 60°$, it is presumed that tensile failure has endangered the stability of the slope before the occurrence of a complete shear failure. Moreover, on the slope of $\theta = 45°$, where there has taken place local shear failure with H = 45 m (static condition) and H = 20 m (earthquake time condition), the domain is confined only to the surface of the slope, nor there has occurred any local tensile failure, we can safely judge that it is mechanically stable.

Again as the countermeasure against tensile failure on the slope, the authors have made it known that cutting of shoulder part or some retaining works at the toe part are effective (Yamanouchi et al, 1975).

5 CONCLUSIONS

Considerations made so far are on the stability of the slope, which was worked out by the application of the engineering classification of undisturbed *Shirasu* and the finite element method as well as the seismic coefficient method to each kind of the original *Shirasu* ground bearing cut-off slopes. The result obtained will be summarized as follows.

(1) The original ground *Shirasu* may be classified into four categories according to its hardness.
(2) The maximum shear stress is conspicuously concentrated on the slope toe, more in case of steep slope than in the gentle slope, and its degree is higher at the time of earthquake and in case of soft *Shirasu*.
(3) While tensile stress at each part of the slope, except at the toe, increases only slightly when the slope angle is larger than about 60°, the tensile stress at the slope toe increases with the increase of slope angle, and its value is larger at the time of earthquake and in case of soft *Shirasu* than under the static condition.
(4) With the slope of the slope angle larger than 75°, the local tensile failure surpasses the local shear failure at the toe, and the tensile failure of the slope is predominant. On the other hand, non-cracking critical height decreases considerably at the time of earthquake.

6 ACKNOWLEDGEMENTS

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