The Application of a Critical State Soil Model to Cyclic Triaxial Tests

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SUMMARY Recently, several sophisticated constitutive models have been proposed to predict the behaviour of soils under cyclic loading. In this paper the concepts of the critical state soil mechanics have been used to develop a simple model which predicts many aspects of clays under repeated loading.

1 INTRODUCTION

An understanding of the behaviour of soils under cyclic loading is important in the fields of earthquake, offshore and highway engineering. The properties of sands under cyclic loading have been studied extensively (Seed and Lee, 1966; Seed, 1979) and engineering theories developed for particular classes of problems(Martin et al ,1970). More recently data pertinent to cyclic loading of clay have been collected (Taylor et al, 1965, 1969; Andersen, 1975, 1976; Van Eekelen and Potts, 1978). Although the conclusions of these tests are not unanimous, several facts emerge. The most important of these is that under undrained loading excess pore pressures are generated and if cyclic loading is continued for a sufficiently long time a failure or critical state condition may be reached.

There have been several attempts at modelling this behaviour mathematically (Mróz et al 1979; Prévost, 1977). These models are complex involving multiple yield surfaces and both kinematic and isotropic hardening and involve the specification of a number of parameters which may be difficult to determine in practice. A less complicated model, which is potentially applicable to cyclic loading, has been suggested by Pender (1977, 1978).

In this paper the concepts of critical state soil mechanics have been extended to provide a description of the response of clay under cyclic loading. Only one additional parameter, which can be determined from the number of cycles to failure in an undrained cyclic triaxial test, is required.

2 THEORETICAL DEVELOPMENT

2.1 Modified Cam Clay

In the interest of clarity the essential features of the modified Cam-clay model are described. Attention is restricted to triaxial conditions where it assumed that the state of effective stress may be completely described by the quantities $p' = \frac{1}{3}(\sigma_z' + 2\sigma_r')$ and $q = \sigma_z' - \sigma_r' = \sigma_z - \sigma_r$, where σ_z', σ_r', σ_z, σ_r are the axial and radial components of effective and total stress respectively. The symbol u will be used to represent excess pore pressure. The convenient measures of strain are the volume strain, $v = \varepsilon_1 + 2\varepsilon_3$ and a measure of the octahedral strain, $\varepsilon = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$; where ε_1, and ε_3 are major and minor principal strains respectively.

The modified Cam-clay model requires the specification of five parameters, values of which may be readily obtained from standard oedometer and triaxial compression tests. These parameters are :

- λ the gradient of the normal consolidation line in e-$\ell n\ p'$ space,
- κ the gradient of the swelling and recompression line in e-$\ell n\ p'$ space
- e_{cs} a value of voids ratio which locates the consolidation lines in e-$\ell n\ p'$ space, conveniently taken as the value of e at unit p' on the critical state line,
- M the value of the stress ratio q/p' at the critical state condition; M is related to ϕ', the angle of friction obtained in triaxial compression tests, by $M = 6 \sin \phi'/(3-\sin \phi')$
- G the elastic shear modulus.

For states of stress within the current yield surface the soil responds elastically and the incremental effective stress-strain law may be written as

$$\begin{pmatrix} dp' \\ dq \end{pmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \cdot \begin{pmatrix} dv \\ d\varepsilon \end{pmatrix} \qquad (1)$$

where $K = (1+e)p'/\kappa$ is the bulk modulus and the shear modulus G is constant.

Yielding of the material occurs whenever the stresses satisfy the following criterion

$$q^2 - M^2\{p'(p_c' - p')\} = 0 \qquad (2)$$

where p_c' is a hardening parameter - analogous to a preconsolidation pressure - which defines the nonzero intersection of the current elliptical yield locus and the p' axis in effective stress space - see Figure 1. Plastic flow is determined by an associated flow rule and the permanent volume strain dv^p is related to the change in the hardening parameter p_c' as follows

$$dv^p = \left(\frac{\lambda-\kappa}{1+e}\right) \frac{dp_c'}{p_c'} \qquad (3)$$

Types of loading can be categorised in terms of a variable p_y', defined as

$$p_y' = p' + q^2/(Mp') \qquad (4)$$

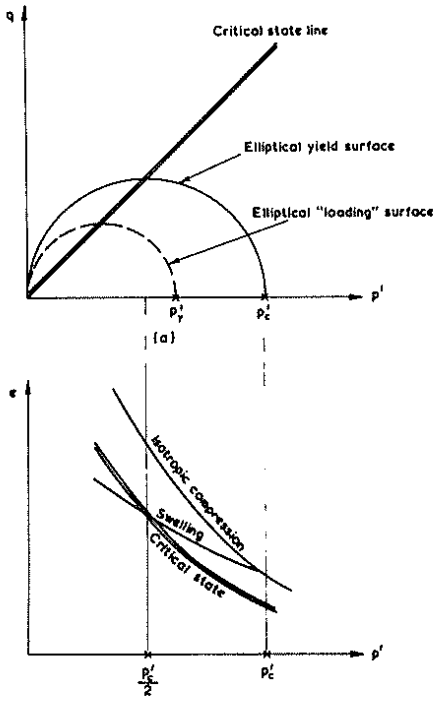

Figure 1 Some aspects of the modified Cam-clay model for triaxial conditions

Equation (4) is also the locus of an ellipse, in p'-q space, which passes through the current stress point and the origin, and is centred on the p' axis, i.e. it has the same shape as the yield locus - see Figure 2. This variable p_y' is the (non-zero) value of p' at which the ellipse cuts the p'-axis and is a convenient way of comparing the current stress state with the current yield locus represented by p_c'.

The material is elastic whenever $p_y' < p_c'$ and during the elastic deformation

$$dp_c'/p_c' = 0 \qquad (5)$$

The material behaves plastically whenever $p_y' = p_c'$ and three conditions can be identified. These are (a) the material hardens whenever $dp_y' = dp_c' > 0$, (b) the material softens whenever $dp_y' = dp_c' < 0$, and (c) 'neutral loading', when the yield locus does not change while plastic behaviour occurs, $dp_y' = dp_c' = 0$. Condition (a) requires $p' > p_c'/2$, i.e. the material is said to be 'wet' of critical, and (b) requires $p' < p_c'/2$, i.e. the material is said to 'dry' of critical.

During plastic behaviour the yield locus changes according to the law

$$dp_c'/p_c' = dp_y'/p_y' \qquad (6)$$

The incremental stress-strain relation during yielding may be shown to be

$$\begin{Bmatrix} dv \\ d\varepsilon \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \cdot \begin{Bmatrix} dp' \\ dq \end{Bmatrix} \qquad (7)$$

where the compliance coefficients are given by

$$C_{11} = \left(\frac{\lambda-\kappa}{1+e}\right)\frac{a}{p'} + \left(\frac{\kappa}{1+e}\right)\frac{1}{p'}$$

$$C_{12} = C_{21} = \left(\frac{\lambda-\kappa}{1+e}\right)\left(\frac{1-a}{p'}\right)$$

$$C_{22} = \left(\frac{\lambda-\kappa}{1+e}\right)\frac{b}{p'} + \frac{1}{3G}$$

and

$$a = (M^2-\eta^2)/(M^2+\eta^2),\quad b = 4\eta^2/(M^4-\eta^4),$$

$$\eta = \text{the stress ratio } q/p'$$

As would be expected, (7) breaks down when the soil reaches the critical state condition $\eta = M$.

2.2 A Model for Cyclic Loading

The modified Cam-clay model has been shown to match well the observed behaviour of insensitive clays subjected to monotonic loading for which the stress level increases. However, the predictions are not as satisfactory when the soil undergoes repeated loading.

When saturated clay is unloaded and then reloaded it is found that permanent strains occur earlier than predicted by the modified Cam-clay model. One way of interpreting this real behaviour is to assume that the position and perhaps the shape of the yield surface have been affected in some way by the elastic unloading.

For the sake of simplicity in developing a new model it is assumed that the form of the yield surface is unchanged but that its size has been reduced in an isotropic manner by the elastic unloading. This can only mean that the hardening parameter p_c' has been reduced by the unloading process. In order to specify how this reduction occurs a relation is proposed between the hardening parameter p_c' and the loading parameter p_y'. In view of (5) it seems reasonable to postulate that when the material is elastic ($p_y' < p_c'$) and when $dp_y' < 0$, the following relation holds

$$dp_c'/p_c' = \theta\, dp_y'/p_y' \qquad (8)$$

If θ takes a value of unity, then the yield surface would shrink back in such a way that the stress state always lay on it. It is to be expected that the yield surface will recede only a fraction of this amount and the values of θ will tend to be quite small. If, however, the material is elastic, but $dp_y' > 0$, it is postulated that the current yield surface is not changed, i.e.

$$dp_c'/p_c' = 0 \qquad (9)$$

The distinction between these types of behaviour is shown schematically in Figure 2.

It has been shown elsewhere that θ may be regarded as an OCR degradation parameter (Carter et al, 1980). A consequence of introducing this degradation parameter θ into the model is that repeated loading under fully drained conditions will result in a continued densification of the soil sample. In modified Cam-clay, for which θ = 0, no such densification will occur. Some predictions of the new model for undrained loading under triaxial test conditions are now discussed.

3 PREDICTION OF THE BEHAVIOUR OF NORMALLY CONSOLIDATED CLAY

In order to illustrate the behaviour predicted by this model one set of values for the conventional Cam-clay parameters has been selected. These are

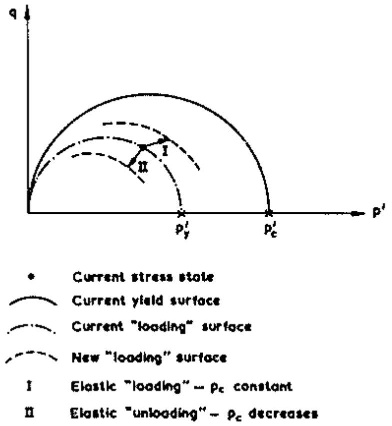

Figure 2 The yield surface and the "loading" surface in p'-q space

$\lambda = 0.25$, $\kappa = 0.05$, $M = 1.2$, $G = 200\ c_{uo}$, where c_{uo} is the initial value of undrained strength predicted by the modified Cam-clay model. For all calculations in which the soil is initially in a normally consolidated state, the initial voids ratio is taken as $e_o = 0.6$.

3.1 Undrained Stress Controlled Loading

Calculations have been performed for the case of cyclic axial load at constant cell pressure in the triaxial test. In each case loading is applied so that the deviator stress q is varied continuously between limits of 0 and q_c, i.e. one way compression loading where $\sigma_z \geq \sigma_r$ with σ_r constant.

Typical results for calculations with $\theta = 0.1$ and $q_c = 1.5c_{uo}$ are shown in Figure 3. The effective stress path, plotted in p'-q space, is shown in Figure 3(a). In the first half of the first cycle the yield surface expands, i.e. the material work hardens, and the stress path is identical to that predicted by modified Cam-clay. During the second half of the first cycle the soil is unloaded (q decreasing) and it responds elastically. As no drainage occurs there is no change in p', however, the value of p_c' will have decreased according to (8), i.e. the yield surface will have contracted slightly. On reloading in the second cycle the material behaves elastically until the stress point reaches the yield locus again*, therafter the material yields, the yield surface expands, further plastic deformations occur, the stress state migrates toward the critical state condition and additional excess pore pressure is generated. This sequence is repeated at each additional load cycle and ultimately, if this process is continued, a critical state condition is reached. In every cycle there is yielding and associated permanent strains and, in particular, during any cycle there is an increment of permanent volume strain. Because the deformation occurs at constant volume there must be a corresponding elastic volume increase and this implies a decrease in mean effective stress, i.e. an increase in pore pressure. The accumulation of excess pore pressure with each cycle is plotted against mean effective stress in Figure 3(c) and against shear strain in Figure 3(d). The relation between deviator stress and shear strain is also shown in Figure 3 (b).

For this material, which has $\theta = 0.1$, failure occurs on the loading portion (q increasing) of the 12th cycle. In general the number of cycles to failure N_f will be dependent not only on the value of θ but also on the cyclic load level q_c. Results are presented in Figure 4 for a number of values of θ and a range of different load levels. It can be seen that for a given material, i.e., a particular value of θ, the number of cycles to failure increases as the amplitude of loading is decreased. For a given amplitude of loading the number of cycles to failure decreases as θ increases. This is as expected since a larger value of θ implies a greater contraction of the yield surface with elastic "unloading". Consequently there are greater permanent volume strains and greater excess pore pressures generated per cycle and thus the material will reach critical state after fewer cycles.

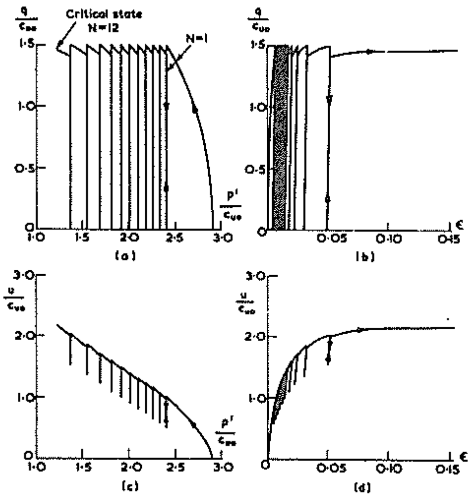

Figure 3 Predictions for a one-way, stress controlled, undrained triaxial test: OCR = 1, $\theta = 0.1$

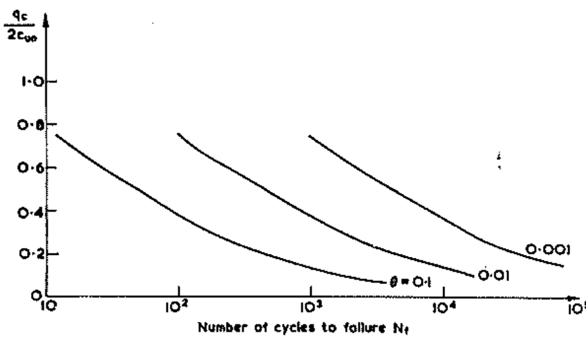

Figure 4 Variation of the number of cycles to failure with cyclic stress amplitude q_c, in a one-way, stress controlled, undrained, triaxial test: OCR = 1

Another important feature predicted by this model is indicated in Figure 5 where the ratio of the undrained shear strength c_u, measured immediately after the "Nth" cycle, to the original undrained strength c_{uo}, measured before cycling, is plotted against the ratio N/N_f. The results show a continual reduction in the undrained shear strength for soils subjected to repeated increments of

* In modified Cam-clay the yield surface will have remained fixed during the unloading and elastic behaviour would be predicted for all subsequent cycles and there would be no further increase in pore pressure.

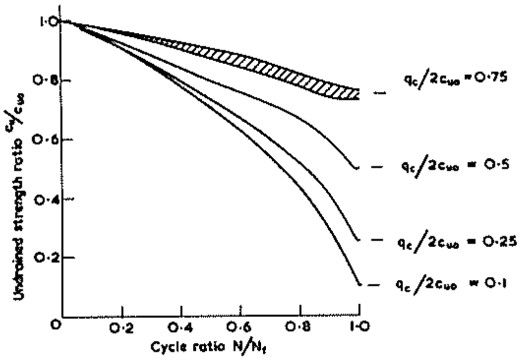

Figure 5 Effect of cyclic stress amplitude q_c on the change in undrained strength

deviator stress. Each of the curves of Figure 5 corresponds to a different amplitude of cyclic deviator stress and results for materials with θ in the range $0.001 \leq \theta \leq 0.1$ appear to lie on either a unique curve or in a narrow region as shown. When the soil reaches failure after N_f cycles the final undrained shear strength is equal to one half of the amplitude q_c of the cyclic deviator stress. This effect of a reduction in strength after cyclic loading with increasing number of cycles has been observed in tests on many clays (e.g. Taylor and Bacchus, 1969; Andersen, 1975, 1976).

4 PREDICTIONS OF THE BEHAVIOUR OF OVERCONSOLIDATED CLAY

The behaviour of an initially overconsolidated sample when subjected to repeated loading may be contrasted to that of an initially normally consolidated soil. In Figure 6 results are presented for a material with $\theta = 0.001$ which has been initially isotropically consolidated to an effective stress of $3.85c_{uo}$ and has then been allowed to swell to a mean effective stress equal to $0.961\ c_{uo}$, so that the conventional overconsolidation ratio is equal to 4. The soil has then been subjected to a continuous variation of deviator stress between the limits $0 \leq q \leq q_c$, where $q_c = 1.9c_{uo}$, under undrained conditions. All stress levels quoted here have been expressed as multiples of the undrained strength c_{uo}, which is the value after swelling to an OCR of 4 but prior to cyclic loading.

The initial swelling and the period when q decreases in each cycle constitute elastic "unloading" as defined above, i.e. p'_y decreasing. During each of these unloading events the yield surface contracts until eventually the stress point contacts the yield surface. Thereafter, there will be periods of plastic loading in each cycle. In this particular example the first plastic strains were observed in the 51st cycle. Thus during the first 50 cycles the material responds entirely elastically; there are no permanent strains and the excess pore pressure oscillates between 0 and $\frac{1}{3}\ q_c$. After 51 cycles, permanent strains occur and in this particular case the material dilates and plastically softens because the stress state is on the "dry" side of critical. Since the deformation is occurring at constant volume the increase in plastic volume strain must be compensated by a decrease in elastic volume strain, i.e. the stress state migrates towards critical state and the pore pressure decreases. In common with modified Cam-clay the cyclic model predicts a peak strength in a stress defined test under certain circumstances; hence failure may occur either by the stress state reaching critical state or by reaching this peak

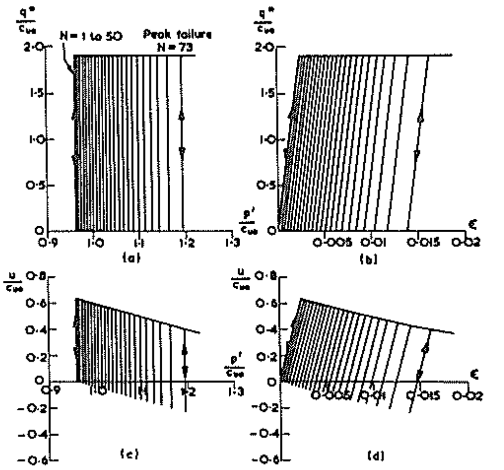

Figure 6 Predictions for a one-way, stress controlled, undrained, triaxial test: OCR = 4, $\theta = 0.001$, $G = 200\ c_{uo}$

undrained strength, whichever occurs first. In samples which are initially highly overconsolidated, such as the one considered here, peak failure is likely to occur. In contrast, soils which are slightly overconsolidated, i.e. on the "wet" side of critical, will, after sufficient cycles, behave in the manner of initially normally consolidated soils. It is also a feature of this model that all initially overconsolidated soils will eventually respond to cyclic loading in the same manner as an initially normally consolidated soil, as long as the deviator stress q is never greater than M times p'.

4.1 The Effect of Initial OCR on Cyclic Behaviour

Calculations have been performed for a number of ideal soils with different values of θ but all having the same conventional overconsolidation ratio of 4.

Figure 7 shows the prediction of the number of cycles to failure N_f in a one way stress controlled test plotted against the magnitude of the applied deviator stress q_c. Curves have been plotted for three different materials corresponding to θ = 0.001, 0.01 and 0.1. The trend is the same as that for normally consolidated soils, i.e. the number of cycles to failure increases as q_c decreases and as θ decreases. Broken curves have also been plotted in Figure 7 for soils with OCR = 1 and the same values of θ. A comparison of the three pairs of curves shows that the number of cycles to failure is also a function of the initial overconsolidation ratio of any soil. The model predicts that overconsolidated soils fail sooner, i.e. in fewer cycles, in repeated load tests than do normally consolidated samples of the same soil. This prediction is in agreement with the trends shown in laboratory tests on Drammen clay (e.g. Andersen, 1975).

5 COMPARISON OF PREDICTIONS WITH EXPERIMENTAL RESULTS

5.1 Tests of Taylor and Bacchus

Taylor and Bacchus (1969) reported the results of cyclic triaxial tests in which one hundred sinusoidal strain-controlled cycles were applied to artificially prepared saturated clay samples. The significant effect on normally consolidated clay was to reduce the mean effective stress p' by an amount which depended on the applied strain

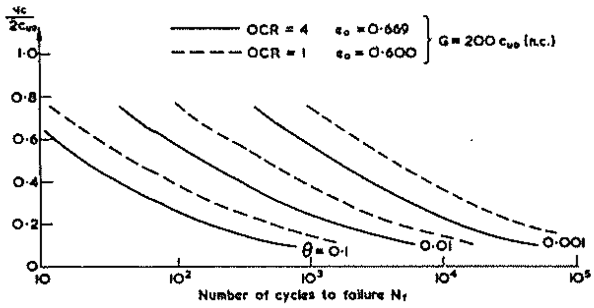

Figure 7 Effect of initial OCR on the number of cycles to failure in a one-way, stress controlled, undrained triaxial test

amplitude. The results of one of these tests in which the initial OCR = 1, e_o = 0.962 and p_o' = 64 lbf/in^2, are plotted in Figure 8 for the case where the strain was varied continuously in the range $-0.003 \leq \varepsilon \leq 0.003$. Also shown on this plot are some predictions made using the new model. Values selected for the model parameters are λ = 0.132, κ = 0.021, M = 1.5, G = 5000 and 2500 lbf/in^2 and θ = 0.03. It can be seen that the predictions in these strain-controlled tests are very dependent on the value selected for the elastic shear modulus G. In both predictions the rate of decrease in p' is overpredicted in the latter stages of both tests and possible reasons for this behaviour are discussed below. Nevertheless, the model predicts the correct trend in this type of cyclic test.

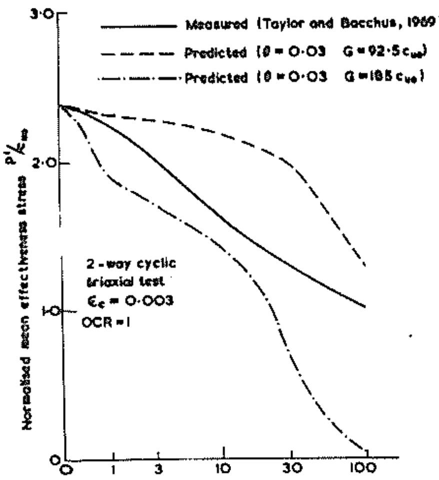

Figure 8 Comparison of model predictions with test results of Taylor and Bacchus (1969) for a two-way strain controlled, undrained, triaxial test

Test results reported by Tayler and Bacchus, and predictions made using the cyclic model are shown in Figure 9 for the case of a monotonic triaxial compression test under undrained conditions. It can be seen, that although the predictions for the ultimate deviator stress are very accurate, the predicted shear stress-strain responses are both too stiff prior to failure. These predictions for monotonic loading, which are the same as would be provided by the modified Cam-clay model, do not show enough plastic shear strain and in fact over-predict the plastic volume strain. As a result a given drop in p' (or increase in u) is predicted in a cyclic test in fewer cycles than is observed. Both static and cyclic tests suggest that the

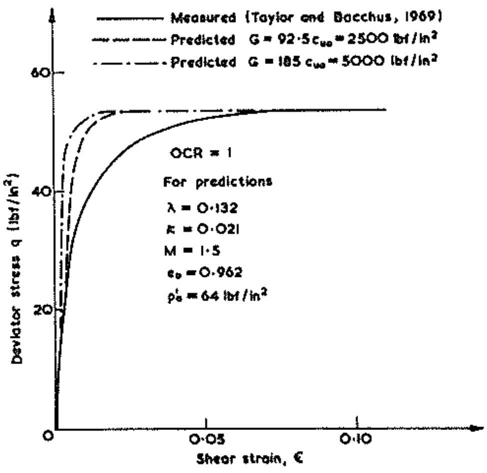

Figure 9 Comparison of model prediction with test results of Taylor and Bacchus (1969) for an undrained, monotonic, triaxial compression test.

elliptical yield locus used in the model (which is identical to the plastic potential because of an associated flow rule) is not an accurate representation of the actual behaviour. Better predictions might be obtained, for this particular material if some other shape is used for a yield locus; one in which plastic shear strains are greater at lower values of deviator stress q, than predicted by the ellipse. A shape like the original Cam-clay yield locus might be better as long as the singularity at the isotropic axis is removed.

6 CONCLUSIONS

A soil model, capable of predicting many of the observed features of the behaviour of clay when subjected to repeated loading, has been presented. The model possesses most of the characteristics of the former critical state models but with a simple, yet important modification. This involves a specified contraction of the yield surface as the soil sample is unloaded (with the definition of unloading as given above). With the introduction of this modification an additional parameter must also be defined. A value for this parameter may be determined, in a straight forward manner, from a laboratory triaxial test involving repeated, undrained loading. For example, if the number of cycles to failure N_f is measured and the cyclic deviator stress q_c is known, the parametric results, such as those presented in Figure 4, may be used to infer a value for θ.

It should be emphasised that the model described in this paper cannot be expected to reproduce accurately all features of the behaviour of a real clay under monotonic and cyclic loading. Indeed, it is believed that no mathematical model, that can be used sensibly and economically for design calculations, is likely to achieve this modest aim. The philosophy behind this work has been the need to develop as simple a family of models as possible that reproduce qualitatively the salient features of cyclic behaviour of soils, and that are expressed in terms of soil parameters that have physical meaning and which can be easily measured in conventional laboratory tests.

7 ACKNOWLEDGEMENTS

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