The Behaviour of Circular Tanks on Deep Elastic Foundations

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1 INTRODUCTION

Cylindrical tanks are widely used for storage purposes. Often they are sufficiently large that they exert considerable pressure on the soil and so induce significant settlement. While the behaviour of circular rafts has been investigated extensively, Borowicka (1963), Habel (1937), Holmberg (1946), Brown (1969), there has been surprisingly little research into the modifications of the raft behaviour induced by interaction with the tank walls. In this paper the behaviour of a cylindrical tank resting on a deep elastic soil is investigated, the important features of the problem are identified by examination of a realistic example. A study of the behaviour, of a uniform tank with equal wall and base thickness, is then presented, for a range of geometric and stiffness parameters.

2 THEORY

It might perhaps be thought that, with the existence of high speed computers and the extensive development of finite element methods, the analysis of cylindrical tanks would be a straightforward matter which could be accomplished using standard finite element codes Smith, (1970). Unfortunately this is not the case, experience with circular rafts, Brown (1969), indicate that considerable care is necessary to obtain reasonably accurate solutions. To illustrate this point a rigid raft has been analysed using a conventional finite element approach. The reaction pressure q_s obtained using the finite element method is compared with the analytic solution in Figure 1 and it is clear that the finite element results, calculated from the nodal forces are inaccurate and oscillate about the correct solution. To overcome this difficulty the semi-analytic technique presented below has been developed. It is essentially a substructure approach, the equations governing the behaviour of the soil, the circular base plate and the cylindrical walls are developed and then combined to obtain the complete response.

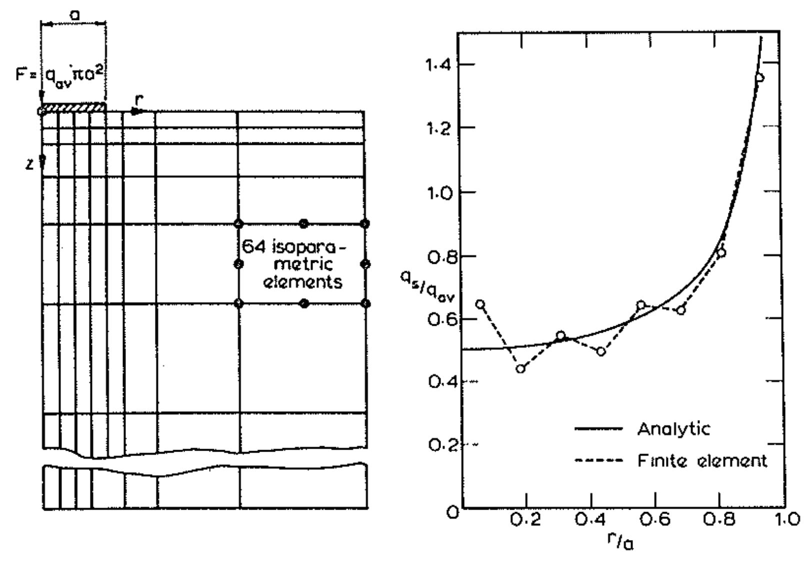

Figure 1 Comparison of finite element and analytic solutions for a rigid raft

2.1 Analysis of Soil

Suppose that a tank of radius, a, rests on a deep (semi-infinite) layer of a homogeneous soil with Young's Modulus E_s and Poisson's ratio ν_s, suppose also that the normal traction, q_s, exerted by the base plate on the soil can be expressed in the form:

$$q_s = \sum_{n=1}^{m} F_n \phi_n(r) \qquad (1)$$

where the coefficients F_n can be considered as generalised forces.

It is convenient to introduce the generalised deflections δ_n defined by

$$\delta_n = \int_0^a r\,\omega(r)\,\phi_n(r)dr \qquad (2)$$

where $\omega(r)$ is the deflection of the soil surface.

It may be shown using the theory of elasticity, that the relation between generalised forces and deflections may be expressed in the form

$$\underset{\sim}{\delta} = C\underset{\sim}{F}$$

where

$$\underset{\sim}{\delta} = (\delta_1, \ldots, \delta_m)^T$$

$$\underset{\sim}{F} = (F_1, \ldots, F_m)^T$$

and C is the generalised flexibility matrix with coefficients

$$C_{mn} = \frac{2(1-\nu_s^2)}{E_s} \int_0^\infty \Phi_m(\alpha)\Phi_n(\alpha)\,d\alpha \qquad (4)$$

where $$\Phi_m(\alpha) = \int_0^\infty r\,J_o(\alpha r)\,\phi_m(r)\,dr$$

Brown (1969) has shown that a suitable set of functions $\{\phi_n\}$ is $\{(a^2-r^2)^{\frac{1}{2}}, 1, (a^2-r^2), (a^2-r^2)^2 \ldots\}$. In this investigation it was only found necessary to use the first six terms of this sequence.

2.2 Analysis of Tank Base

The tank base is assumed to be a circular plate of radius a and rigidity D_p which is subjected to an applied normal traction q_A. The tank will be acted on by a pressure $q_p = q_A - q_s$ and a moment, M_e, induced by the presence of the tank walls (it will be assumed that radial deflections of the base plate may be neglected). It may be shown that the deflected shape the plate $\omega(r)$ is given by

$$\omega(r) = \omega_o + \int_0^a r_o G(r,r_o)\,q_p(r_o)/D_p\,dr_o + M_e r^2/(2D_p(1+\nu_p)) \qquad (5)$$

where ω_o is the central deflection and ν_p is Poisson's ratio of the plate and

$$G(r,r_o) = \frac{r_o^2}{4}\left(1+\log\frac{r}{r_o}\right) + \left(\frac{1-\nu_p}{1+\nu_p}\right)\frac{r^2 r_o^2}{8a^2} \quad r \le r_o$$

$$G(r,r_o) = \frac{r^2}{4}\left(1+\log\frac{r_o}{r}\right) + \left(\frac{1-\nu_p}{1+\nu_p}\right)\frac{r^2 r_o^2}{8a^2} \quad r_o \le r \qquad (6)$$

Equations (1,2,5) lead to the flexibility relation

$$\begin{aligned} \underset{\sim}{\delta} &= -H\underset{\sim}{F} - \underset{\sim}{\beta} M_e + \underset{\sim}{\alpha}\,\omega_o + \underset{\sim}{\gamma} \\ a\theta_e &= -\underset{\sim}{\beta}^T\underset{\sim}{F} - \frac{a^2}{D_p(1+\nu_p)}M_e + S \\ 0 &= \underset{\sim}{\alpha}^T\underset{\sim}{F} + \frac{P_A}{2\pi} \end{aligned} \qquad (7)$$

where θ_e is the edge rotation and

$$H_{mn} = \frac{1}{D_p}\int_0^a\int_0^a r r_o\,G(r,r_o)\,\phi_m(r_o)\,\phi_n(r)\,dr\,dr_o$$

$$\gamma_m = \frac{1}{D_p}\int_0^a\int_0^a r r_o\,G(r,r_o)\,\phi_m(r)\,q_A(r_o)\,dr\,dr_o$$

$$\left(\frac{P_A}{2\pi}, S\right) = \int_0^a \left(r_o, \frac{r_o^3}{2D_p(1+\nu_p)}\right) q_A(r_o)\,dr_o$$

$$(\alpha_m, \beta_m) = \int_0^a \left(r_o, \frac{r_o^3}{D_p(1+\nu_p)}\right)\phi_m(r_o)\,dr_o$$

2.3 Analysis of Tank Wall

The tank wall is assumed to be a cylindrical shell of height d, thickness t_w and radius a, with a Young's modulus E_w and rigidity D_w, which is subjected to a pressure which increases linearly from zero at the top of the tank to γd at the base. Integration of the governing differential equations (Timoshenko Woinowsky-Krieger (1959)) can then be used to establish that for shells of practical properties

$$\theta_e = \frac{Me}{2D_w\zeta} + \frac{\gamma d^2}{E_w t_w}(1-\zeta d) \qquad (8)$$

where $$\zeta^4 = \frac{t_w E_w}{4a^2 D_w}$$

2.4 Analysis of the Soil-Base-Wall System

Assuming that compression of the base plate can be neglected and that the wall-plate joint is rigid, the behaviour of the soil, base, and tank wall (equations (3,7,8)) may now be combined to obtain the following equations governing the combined system

$$\begin{bmatrix} A, & \underset{\sim}{\beta}, & -\underset{\sim}{\alpha} \\ \underset{\sim}{\beta}^T, & b, & 0 \\ -\underset{\sim}{\alpha}^T, & 0, & 0 \end{bmatrix} \begin{bmatrix} \underset{\sim}{F} \\ M_e \\ \omega_o \end{bmatrix} = \begin{bmatrix} \underset{\sim}{\gamma} \\ c \\ -\frac{P_A}{2\pi} \end{bmatrix} \qquad (9)$$

where $A = H + C$

$$b = \frac{a^2}{D_p(1+\nu_p)} + \frac{a}{2D_w\zeta}$$

$$c = S - \frac{\gamma a^3(1-\zeta d)}{E_w t_w}$$

EXAMPLES:

The first example is of a water storage tank constructed with walls of constant thickness, and founded on a deep uniform clay layer. The following properties, and dimensions were chosen for the tank-soil system:-

Unit weight of fluid	γ	=	9.81 kN/m^3
Depth of tank	d	=	7.5 m
Radius of tank	a	=	9.0 m
Thickness of walls	t	=	360 mm

Elastic modulus of tank E_p = 1.4×10^4 MPa
Elastic modulus of soil E_s = 20 MPa
Poisson's ratio of tank ν_p = 0.3
Poisson's ratio of soil ν_s = 0.4

Results of the analysis are plotted in Figures 2-5 which show the radial moment resultant M_R (Figure 2), the thrust resultant N_θ (Figure 3) the reaction distribution (Figure 4) and the base deflection w (Figure 5) for the tank walls and base. This analysis shows that the most important quantities are the radial moments which occur at the centre and edge of the tank base, these moments having opposite signs. Also of importance is the maximum tensile force resultant in the wall, N_θ, which occurs at some distance from the base of the wall.

In order to allow rapid determination of some of the important quantities required in the design of reinforced concrete water tanks, non-dimensional plots are presented for a range of geometric and stiffness parameters. In all cases it is assumed that $\nu_p = \nu_w = 0.3$ and that the top of the tank wall is pinned. It is found that for a tank with uniform wall and base (i.e. same thickness and elastic modulus) that the results depend upon three parameters, K, $\frac{d}{a}$, $\frac{a}{t}$ where

$$K = \frac{E_p}{E_s} (1 - \nu_s^2) \left(\frac{t}{a}\right)^3$$

(E_p, E_s, t, a, ν_s have been defined previously).

Figure 6 shows the edge moment resultant, M_e (i.e. moment/unit length) for various radius to thickness ratios a/t, and values of K. The edge moment is not very sensitive to the d/a ratio, values of d/a = 1, 2 giving coincident values in this figure. A similar plot may be made for the central moment resultant M_c (see Figure 7). Differential deflections in the base of the tank are shown in Figures 8-10, for various values of K. Ratios of d/a again have little influence on results for differential deflections. The central or maximum deflection of the tank base ω_0 is given in Figure 11 for various a/t ratios, and values of K.

Finally values of the maximum tensile force resultant N_θ in the walls is presented in Figure 12 for various value of K, and a/t ratio. Curves are presented for d/a values of 1, 2.

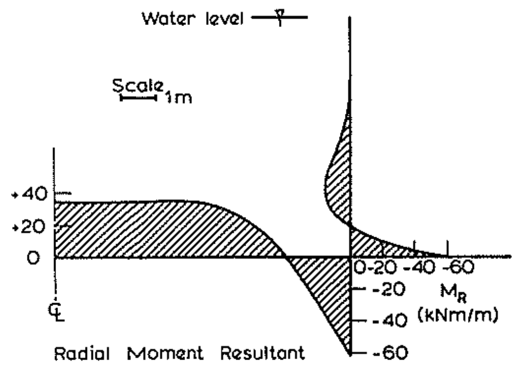

Figure 2 Radial moment resultant

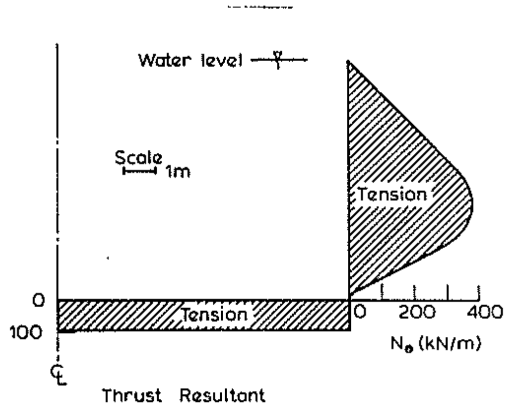

Figure 3 Thrust resultant

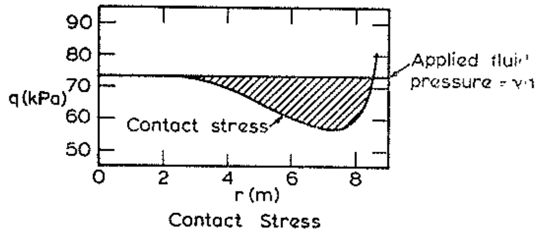

Figure 4 Contact stress

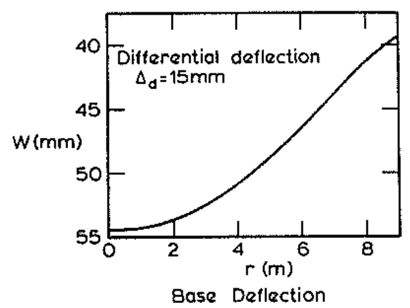

Figure 5 Base deflection

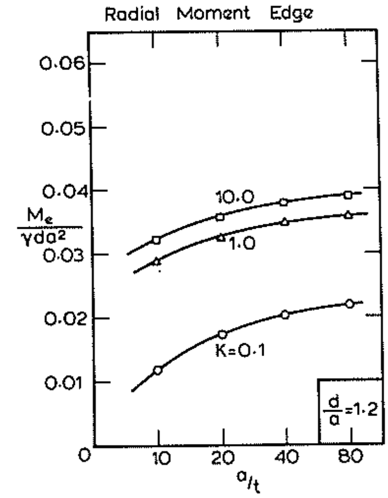

Figure 6 Radial moment - edge

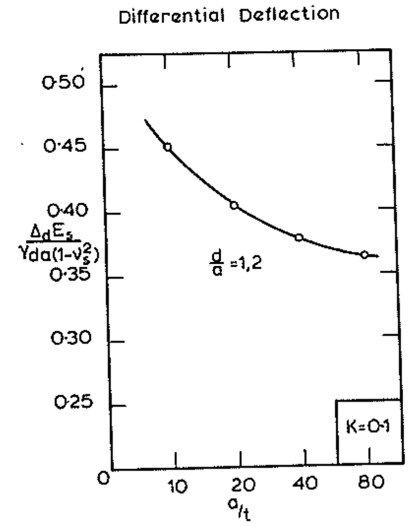

Figure 8 Differential deflection K=0.1

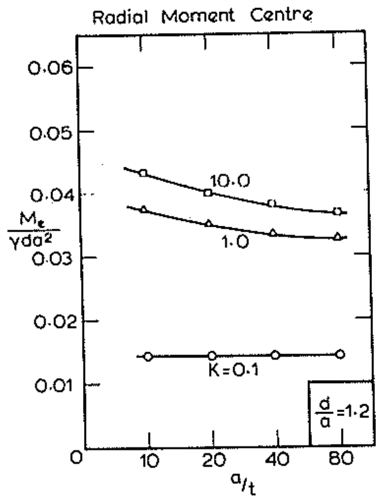

Figure 7 Radial moment - centre

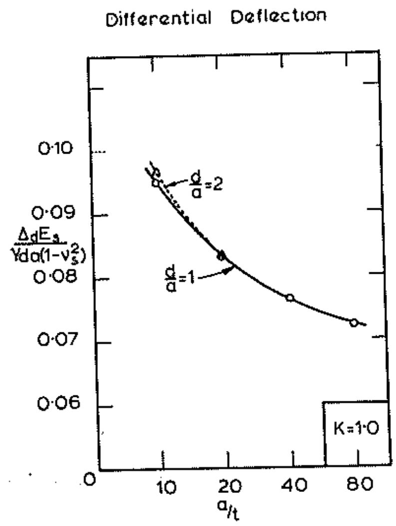

Figure 9 Differential deflection K=1.0

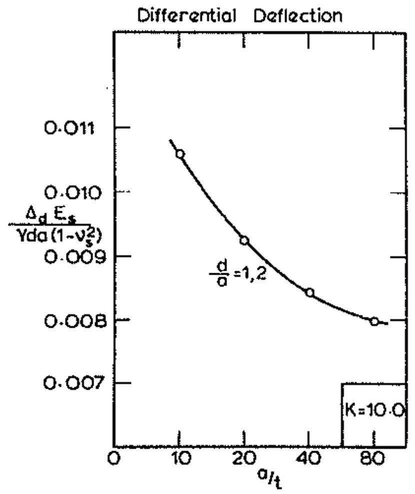

Figure 10 Differential deflection K=10.0

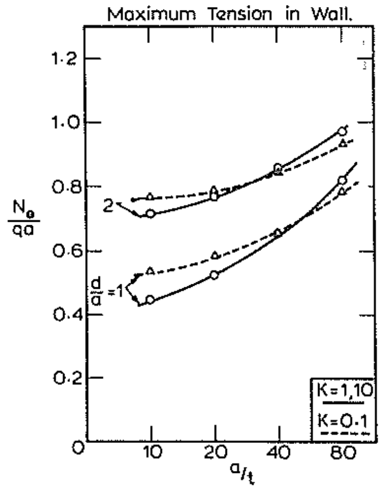

Figure 12 Maximum wall tension

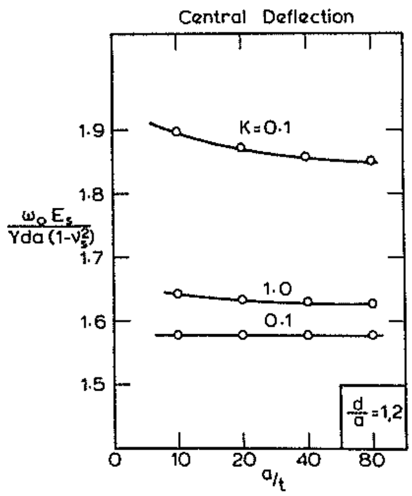

Figure 11 Central deflection

4 CONCLUSIONS

A semi-analytic technique for the analysis of the behaviour of a cylindrical tank resting on a deep clay layer has been presented. The method has been used to analyse a realistic problem and to perform a parametric study of a homogeneous tank having base and walls having equal thickness.

The method can easily be extended to include situations in which the tank wall is tapered or stepped and the effects of only partial filling of the tank.

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