Prediction of Structure-Foundation Interaction Behaviour

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SUMMARY. An examination of the influence of interaction between a three dimensional frame structure and a raft foundation. The effect on column loads, raft differential settlement and maximum positive and negative bending moments is considered for 3 bay and 5 bay multistorey structures. The supporting soil is considered to be an isotropic perfectly elastic continuum with either a constant modulus or a modulus which increases linearly with depth.

Results of the analyses are presented in terms of two relative stiffness parameters in such a way that predictions of interaction behaviour for a wide range of structure and raft conditions can be made. The graphs show under what conditions interaction can reasonably be ignored or when a full interaction analysis will be required.

1. INTRODUCTION

The designer has two basic problems to consider in the design of a raft foundation for a framed structure. Firstly the total and differential settlements of the foundation must be predicted and compared to the allowable settlements that the structure can withstand. Secondly the distribution of bending moment within the raft must be predicted so that the detailed structural design can be completed.

Traditional methods (Teng, 1962) for calculating settlements and bending moments of raft foundations ignore the influence of the structure thus implying that the forces transmitted to the raft are independent of the differential settlements of the system. Recent studies have shown that for certain situations this is not the case and structure-foundation interaction should be considered (Lee and Brown, 1972) and (Hain and Lee, 1974). The extent to which interaction causes a redistribution of forces will depend on the stiffness characteristics of the frame structure, the raft and the supporting soil. Thus it is essential that the designer have at his disposal a means of readily assessing the stiffness characteristics of these three components and thus predicting the interaction behaviour.

Meyerhof (1953) suggested a means of evaluating the combined stiffness of the structure and the raft which he then compared to the stiffness of the supporting soil to give one relative stiffness parameter. The influence of structure foundation interaction could then be assessed using behaviour of a uniformly loaded raft of stiffness equal to the combined structure and raft. This approach was subsequently adopted by the A.C.I. Committee 436 (1966) for their recommendations regarding raft foundation design and the influence of structure-foundation interaction. However, this simplified approach can be shown to lead to an overprediction of differential settlement and raft bending moments.

Brown (1975) examined the behaviour of a multibay plane frame on a strip foundation in terms of three relative stiffness parameters. The results enable an assessment of the likely influence of interaction for a two dimensional situation; however they may not always be reliable for the three dimensional situation involving a raft foundation. Hain (1977) has shown that a two dimensional analysis which neglects twisting moments in the raft and the redistribution of load that occurs between frames can lead to significantly different results to a three dimensional analysis which considers these aspects.

The present paper presents the results of a series of three dimensional analyses of a multibay multi-storey space frame supported on a raft foundation. The system is described by two relative stiffness parameters and the results are presented in such a way that the designer can readily follow the trends in behaviour and therefore predict the likely effects of structure foundation interaction.

2. NOTATION

a = thickness of shear wall
B_R = width of the raft foundation
E' = Young's modulus of the materials used in the structure
E_L = Young's modulus of the supporting soil layer at a depth of $L_R/2$
E_o = Young's modulus of the supporting soil layer at the surface
E_o^* = $E_o/(1 - \nu_s{}^2)$
E_s = Young's modulus of the supporting soil layer at depth
h = height of shear wall
h_ℓ = storey height of lower columns at storey j
h_u = storey height of upper collumns at storey
i = frame number
I_b = moment of inertia of the beam at storey j
I_b' = effective moment of inertia of the beam at storey j
I_ℓ = moment of inertia of lower columns at storey j
I_u = moment of inertia of upper columns at storey j
j = storey number

K_b = I_b/ℓ = stiffness of the beam at storey j

$(K_f)_i$ = effective stiffness of frame i

K_ℓ = I_ℓ/h_ℓ = stiffness of the lower columns of storey j

K_R = stiffness of the raft foundation per unit width compared to the stiffness of the supporting soil

K_S = stiffness of the structure per unit width compared to the stiffness of the supporting soil

K_T = $K_S + K_R$ = total stiffness of the structure and raft compared to the supporting soil

K_u = I_u/h_u = stiffness of the upper columns of storey j

ℓ = bay length of frame i

L_f = total length of frame i

L_R = length of the raft foundation

n_f = number of structural frames spaced across the foundation width B_R

n_s = number of storeys in the structure

t_R = thickness of the raft foundations

ν_s = Poisson's ratio of the supporting soil

3. INTERACTION ANALYSIS

The interaction analysis used in this analysis is based on the substructuring method and has been presented by Hain (1977). The components of the system have been modelled as follows:-

(i) the supporting soil is represented by an isotropic perfectly elastic continuum of infinite extent with either a constant modulus or a modulus which increases linearly with depth.

(ii) the raft foundation is represented by an assemblage of thin plate bending finite elements (Zienkiewicz, 1971).

(iii) the structure is represented by a three dimensional assemblage of beam elements according to traditional methods of structural analysis.

Figure 1 shows a simplified two dimension representation of the problem considered. Two multistorey structures, 3 bays and 5 bays in both directions, were considered for uniformly distributed floor loadings on every storey. Two supporting soil conditions, homogeneous and linearly increasing modulus with depth such that $E_L/E_o = 2$, were also considered. For these cases analyses covering the practical range of structure, raft and supporting soil stiffnesses were performed.

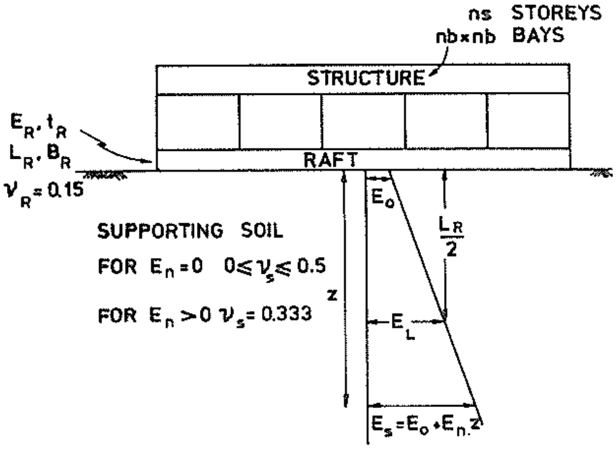

Figure 1. The problem analysed

4. RELATIVE STIFFNESS PARAMETERS

The problem can be described by three independent parameters - the structure stiffness, the raft stiffness and the supporting soil stiffness. From these two independent relative stiffness parameters were selected as follows:-

$$K_R = \frac{\text{stiffness of the raft foundation per unit width}}{\text{stiffness of the supporting soil}} \tag{1}$$

$$K_S = \frac{\text{stiffness of the structure per unit width}}{\text{stiffness of the supporting soil}} \tag{2}$$

A third parameter, the combined stiffness of the structure and raft compared to the supporting soil, was given by:-

$$K_T = K_S + K_R \tag{3}$$

The parameters K_R and K_S, can be calculated from:-

$$K_R = \frac{4\ E_R\ B_R\ t_R^3\ (1 - \nu_S^2)}{3\ .\ \pi\ .\ E_o\ .\ L_R^4} \tag{4}$$

$$K_S = \frac{\sum\limits_{i=1}^{i=nf} \frac{(K_f)_i}{B_R}}{E_o^*\ .\ L_R^3} \tag{5}$$

Meyerhof (1953) suggests the following approximate expression for the stiffness of a frame subjected to differential settlement of the columns.-

$$(K_f)_i = \sum_{j=1}^{j=ns} \left(E'I_b' + \frac{E'.ah}{12}\right)_j \tag{6}$$

$$\text{where } I_b' = I_b\ .\left[1 + \frac{K_\ell + K_u}{K_b + K_\ell + K_u}\ .\left(\frac{L_f}{\ell}\right)^2\right]$$

5. DISCUSSION OF RESULTS

The relative stiffness parameters defined in equations (1) and (2) allow the examination of the total system stiffness as well as the distribution of stiff-ness between the structure and the raft. Analyses were performed for K_T = 10.0, 1.0, 0.1, and 0.01 for K_S/K_R values covering the range 0.01 to 100.0.

5.1 Column Loads

Figures 2 and 3 show the results in terms of the column loads for various ratios of K_S/K_R when K_T = 1.0. Actual column loads have been normalized with respect to the average column load and curves are shown for the corner column, the average of all the edge columns and the average of all the interior columns. When K_S/K_R tends towards 0.01, the distribution of column loads approaches the rigid raft (zero differential settlement) conditions. As K_S/K_P increases, interaction leads to a transfer of load from interior columns to edge and corner columns and the perfectly flexible raft condition is approached. Most of the redistribution of column loads occurs within the region $0.1 < K_S/K_R < 10.0$ and thus this would appear to be the area where an interaction analysis may be necessary. For values of K_T within the range 0.1 to 10.0 the interaction results were found to be virtually identical to results shown in Figures 2 and 3. For K_T = 0.01 there was significantly less redistribution of column load as this situation represents a very stiff supporting soil. As could be expected the stiffer non-homogeneous supporting soil leads to a reduction in column load redistribution compared to the homogeneous case.

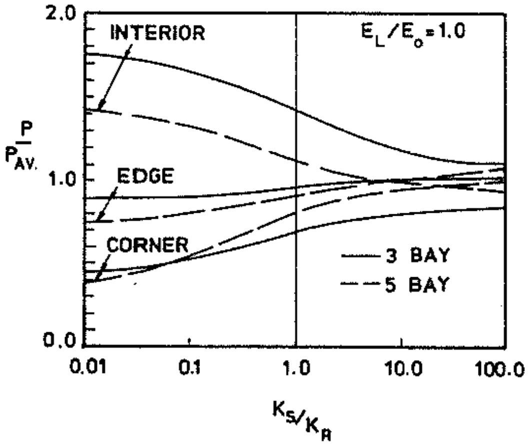

Figure 2. Column Loads for K_T = 1.0 when E_L/E_o = 1

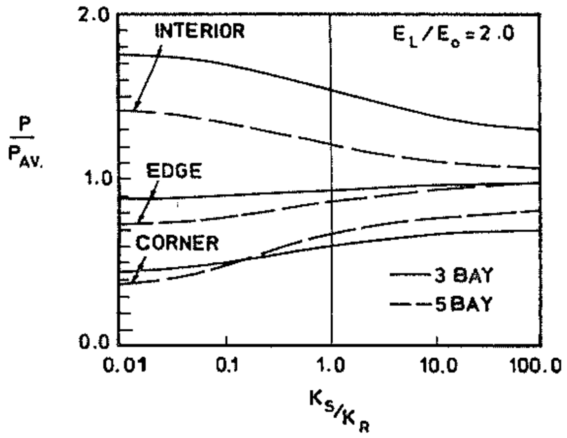

Figure 3. Column Loads for K_T = 1.0 when E_L/E_o = 2

5.2 Differential Settlement

The differential settlement presented is the maximum differential settlement which occurs between the corner column and the interior column closest to the centre of the raft. Figures 4 and 5 show the variation of raft differential settlement for K_T = 1.0 and various K_R values.

The maximum differential settlement obtained when only the raft is analysed is shown as the CCL (constant column load) result. The constant column loads applied to the raft were determined from a zero differential settlement analysis of the structure. The importance of the CCL result is that all interaction results will converge to this result as K_S tends to zero.

The results indicate that providing the structure is at least as stiff as the raft, i.e. $K_S > K_R$, then the maximum differential settlement is determined by the total system relative stiffness, K_T, independent of the raft relative stiffness, K_R. This differential settlement is always of the order of 50% of

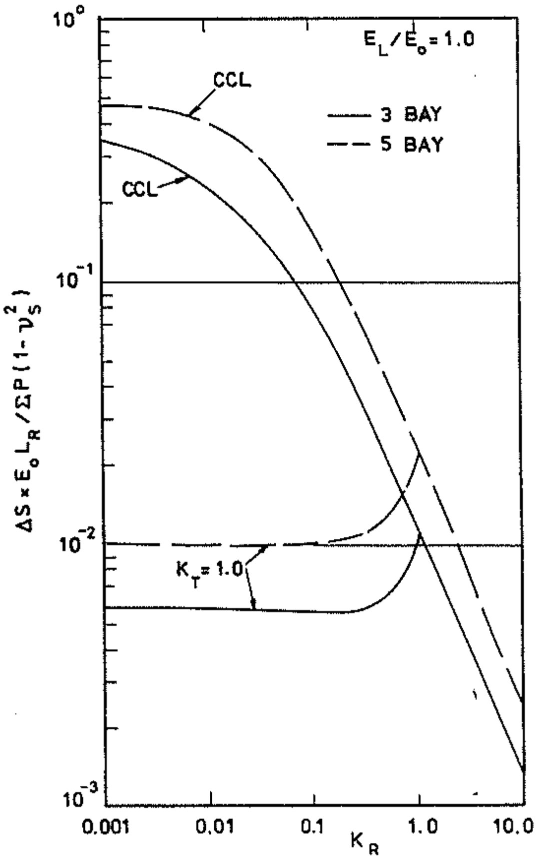

Figure 4. Raft differential settlement for K_T = 1.0 when E_L/E_o = 1

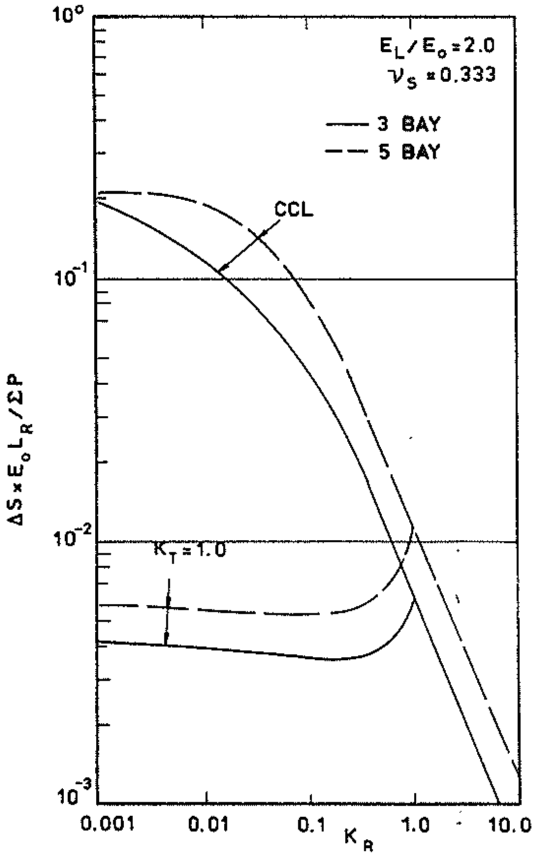

Figure 5. Raft differential settlement for K_T = 1.0 when E_L/E_o = 2

the maximum differential settlement of a raft without a structure of the same stiffness as the actual structure and raft. Indicating the effectiveness of the structure in reducing differential settlements because it reinforces the raft between those points where the maximum loads, and hence settlements, occur. If the raft is stiffer than the structure, i.e. $K_R > K_S$, then the differential settlement is very sensitive to the ratio K_R/K_T and there is considerable increase in values as this ratio tends towards one. The present value for $K_T = 1.0$ can be used to indicate results for values K_T in the range 0.1 to 10.0. Within this range K_T contours are practially identical in shape and will intersect the CCL curve at the point where $K_T = K_R$.

5.3 Raft Bending Moments

Figures 6 and 7 show the maximum positive raft bending moments for $K_T = 1.0$ as a function of K_R. For the frames considered the maximum positive bending moment occurs at the interior column closest to the centre of the raft. Interaction which redistributes some of this load to the peripheral columns produces a significant reduction in bending moment most of which occurs over the range $0.1 < K_R/K_T < 1.0$. Results for other values of K_T in the range 0.1 to 10.0 can be predicted using the present results. For $0.01 < K_R/K_T < 1.0$ all K_T contours have a similar shape while for $K_R/K_T < 0.01$ all contours asymptote to the same value of maximum bending moment for a given frame and E_L/E_o value. Again the CCL result is shown and it is noted that K_T contours converge onto this result as K_R/K_T tends towards 1. A better appreciation of the reduction in bending moment that occurs when interaction is considered can be obtained from a study of Table 1 which compares interaction bending moments with those obtained from the conventional CCL analysis.

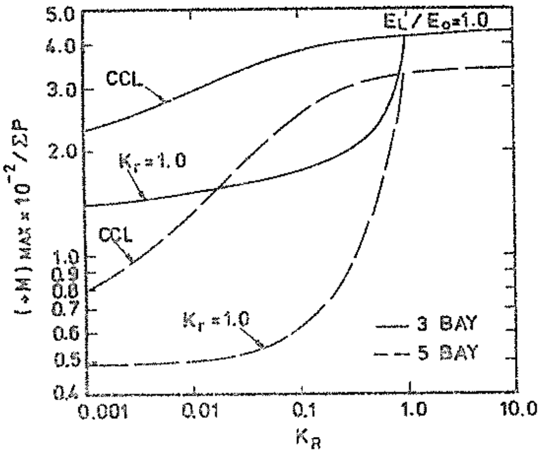

Figure 6. Maximum positive bending moments for $K_T = 1.0$ when $E_L/E_o = 1$

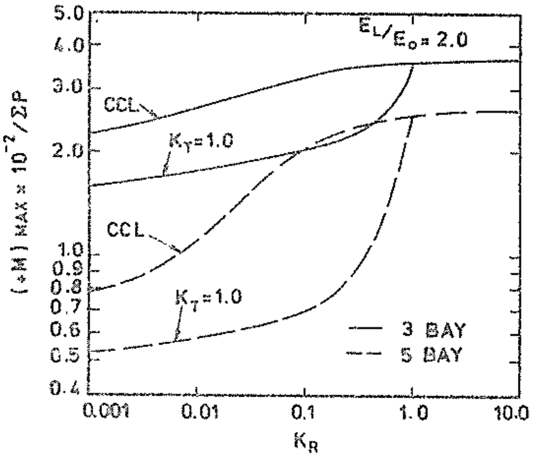

Figure 7. Maximum positive bending moments for $K_T = 1.0$ when $E_L/E_o = 2$

TABLE I

COMPARISON OF RAFT BENDING MOMENTS FOR $K_T = 1.0$ AND $K_R = 0.1$

	3 Bay Frame		5 Bay Frame	
	$\frac{E_L}{E_o} = 1$	$\frac{E_L}{E_o} = 2$	$\frac{E_L}{E_o} = 1$	$\frac{E_L}{E_o} = 2$
No interaction considered (CCL)	100	85	100	78
Interaction considered	46	52	24	27

Figures 8 and 9 show the variation of the maximum negative bending moments for $K_T = 1.0$ as a function of K_R. The maximum negative bending moment occurs along the line between an edge column and an interior column and is influenced by the size of the edge column load as well as the raft relative stiffness. Interaction between the structure and the raft redistributes load from interior columns to edge columns and hence negative bending moments will increase. For a given K_T value the maximum negative bending moment occurs when the structure and raft relative stiffnesses are equal (i.e. $K_R = K_S$). If K_S is then increased and K_R appropriately reduced, then little additional change in column loads occurs and hence negative bending moments reduce as K_R reduces.

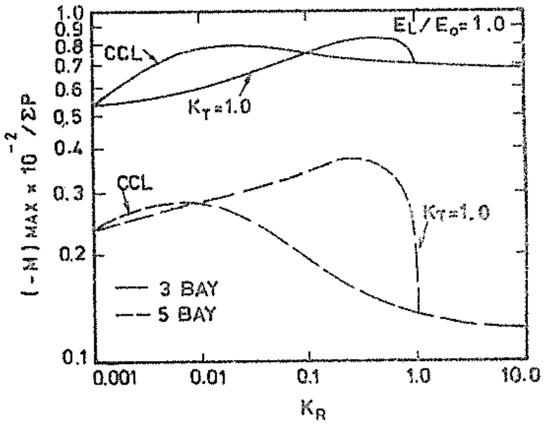

Figure 8. Maximum negative bending moments for $K_T = 1.0$ when $E_L/E_o = 1$

Comparing the curves for the 3 bay and the 5 bay structures in Figures 8 and 9 indicates that although the values are greater for the 3 bay structure, there is a greater range of values for the 5 bay structure. Comparison with Figures 6 and 7 shows that negative bending moments are generally of a similar size to the positive bending moments when interaction is considered.

Results for values of K_T in the range 0.1 to 10.0 can be estimated from the $K_T = 1.0$ curves shown in Figures 8 and 9 by observing the following characteristics. All K_T contours have a similar shape in the range $0.1 < K_R/K_T < 1.0$ with a maximum value when $K_R = K_S$.

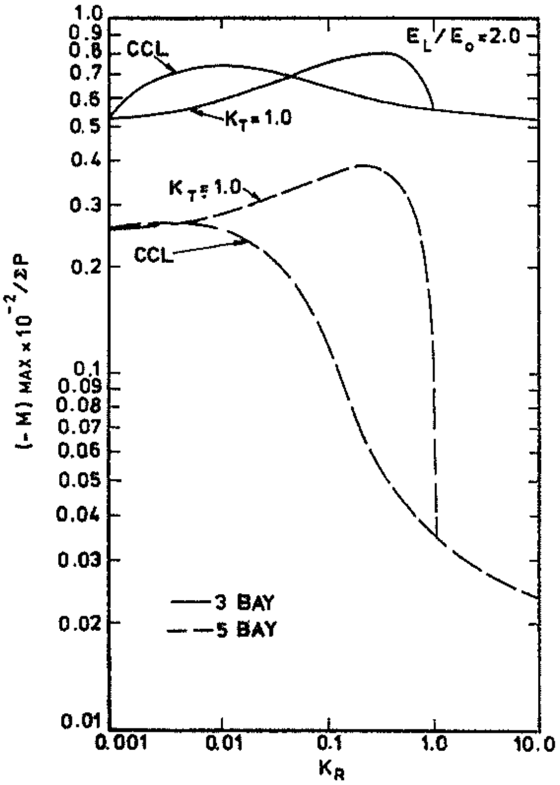

Figure 9. Maximum negative bending moments for $K_T = 1.0$ when $E_L/E_o = 2$

This maximum value is essentially constant regardless of K_T. As K_R tends towards 0.001 the K_T contour asymptotes to the CCL curve.

6. CONCLUSIONS

The interaction behaviour of a three dimensional frame and a raft foundation can be predicted if the relative stiffness of the components is expressed by the parameters presented herein. Consideration of a large number of analyses of 3 and 5 bay frames indicates the following trends:-

(i) most redistribution of column loads occur for $0.1 < K_S/K_R < 10.0$.

(ii) differential settlements of a structure raft system are always less than those for a raft with K_R equal to K_T.

(iii) for a given K_T the larger the value of K_S, then the smaller the maximum positive raft bending moments.

(iv) for a given K_T the largest negative bending moments occur when $K_R = K_S$.

(v) differential settlements increase with the number of bays in the structure and reduce as the rate of increase of the soil modulus with depth increases.

(vi) the reduction in maximum positive bending moment that occurs because of interaction increases with the number of bays in the structure and reduces slightly as the rate of increase of soil modulus with depth increases.

(vii) the increase in maximum negative bending moment that occurs because of interaction increases with the number of bays in the structure and reduces as the rate of increase of soil modulus with depth increases.

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