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The Shear Strength of Rock Masses

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Summary: The Hoek-Brown criterion, first published in 1980 and updated several times since, is now in widespread use throughout the geotechnical community. The authors have an interest in predicting the shear strength of weak rock masses for slopes and have thus assessed the criterion for its applicability to this field. It has been found that the Hoek-Brown criterion, as currently formulated, does not adequately predict the shear strength behaviour of a rock mass during its transition from intact rock to a disintegrated rock mass. The authors have used triaxial testing on intact rock, rockfill and rock mass to develop new equations for the parameters in the Hoek-Brown criterion. These equations are valid for both low stress applications (e.g. slopes) and high stress applications (e.g. tunnels).

INTRODUCTION

The most commonly used strength criterion, having received widespread interest and use over the last two decades, is the Hoek-Brown empirical rock mass failure criterion, the most general form of which is given in Equation 1. Hoek and Brown (1980) developed this rock mass criterion as they "found that there were really no suitable criteria for the purpose of underground excavation design" (Hoek, 2001). The equation, which has subsequently been updated by Hoek and Brown (1988), Hoek et al. (1992), Hoek et al. (1995) and Hoek et al (2002), was based on their criterion for intact rock. The original criterion for 'rock mass' was based on model studies, rockfill and 152mm core samples of Panguna Andesite from Bougainville in Papua New Guinea (Hoek and Brown, 1980). The validation of the updates of the Hoek-Brown criterion have been based on experience gained whilst using this criterion. To the authors' knowledge, the only data published supporting this experience has been two mine slopes cited in Hoek et al (2002).

$$\sigma_1' = \sigma_3' + \sigma_c \left(m \frac{\sigma_3'}{\sigma_c} + s \right)^a \qquad (1)$$

For intact rock, For rock mass,

For intact rock,	For rock mass,
$m = m_i$	$m = m_b$
$s = s_i$	$s = s_b$
$a = a_i$	$a = a_b$

The authors have assessed the Hoek-Brown criterion in detail and modified it into a more generalised form to account for various inconsistencies in the current version. This paper contains a discussion and development of these modifications. The authors have concentrated mainly on developing the criterion for use at low confining stresses (e.g. slopes) however, the resulting criterion should be applicable for the full stress range.

RATIONALE FOR PARAMETER MODIFICATION

Table 1 shows the equations used to estimate the parameters in the Hoek-Brown criterion. These equations assume that the exponent a is a constant equal to 0.5 for intact rock and that a and m are independent. Mostyn and Douglas (2000) demonstrated that the exponent a in the Hoek-Brown criterion is not always 0.5 for intact rock and that the parameter m_i is not a function of rock type and as such the Hoek-Brown tables relating m_i to rock type should not be used. Mostyn and Douglas (2000) also showed that a_i can be approximated by a function of m_i. The authors suggested that for intact rock the parameters should preferably be measured from triaxial tests on intact rock samples. Alternatively an approximation can be made using the uniaxial compressive strength, σ_{ci}, and tensile strength, σ_{ti}, of the intact rock and Equations 2 and 3.

Table 1. Estimation of the Hoek-Brown Coefficients

<table>
<tr><th colspan="2">Parameter</th><th>Hoek et al (pre 2002)</th><th>Hoek et al (2002)</th></tr>
<tr><td colspan="2">m_b</td><td>$\frac{m_b}{m_i} = \exp\left(\frac{GSI-100}{28}\right)$</td><td>$\frac{m_b}{m_i} = \exp\left(\frac{GSI-100}{28-14D}\right)$</td></tr>
<tr><td rowspan="2">s</td><td>GSI>25</td><td>$s = \exp\left(\frac{GSI-100}{9}\right)$</td><td rowspan="2">$s = \exp\left(\frac{GSI-100}{9-3D}\right)$</td></tr>
<tr><td>GSI<25</td><td>$s = 0$</td></tr>
<tr><td rowspan="2">a</td><td>GSI>25</td><td>$a = 0.5$</td><td rowspan="2">$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right)$</td></tr>
<tr><td>GSI<25</td><td>$a = 0.65 - \frac{GSI}{200}$</td></tr>
</table>

Where GSI is the Geological Strength Index (Hoek et al, 1995)

$$m_i \approx \left|\frac{\sigma_{ci}}{\sigma_{ti}}\right| \tag{2}$$

$$a_i \approx 0.4 + \frac{1.2}{1+\exp\left(\frac{m_i}{7}\right)} \tag{3}$$

The Hoek-Brown equations for the rock mass parameters m_b, s_b and a_b, were derived from an extension of the Hoek-Brown parameters for intact rock, in particular $a = 0.5$. As the authors have found that a_i is a function of m_i and varies from 0.4 to 0.9, new equations for the rock mass parameters are required. These equations must adequately predict the changing behaviour of a rock sample from an intact rock to a heavily broken rock mass. A discussion of each of the individual rock mass parameters is given in the following paragraphs.

The Hoek et al (2002) equation for the exponent a results in a maximum a of 0.62 using a GSI of five (minimum possible value of GSI). A rock mass can be considered a transitional material between intact rock and soil. At the soil limit, it is expected that a_b would approach unity (Mohr-Coulomb material). Using a statistical analysis of 929 triaxial tests on rockfill, which could be considered as a heavily broken poorly interlocked rock mass, Douglas (2002) showed that a_b for rockfill is approximately 0.9 and that m_b is approximately 2.5.

The parameter m (in association with the exponent a) predominately affects the friction angle of the rock mass at low stress. Therefore, as GSI drops it can be expected that the rock mass will become less interlocked and the frictional strength of the rock mass will reduce (predominately through a reduction in dilation). The parameter m_b should therefore reduce from m_i to a limiting value as a function of GSI. The limiting value for a non-cohesive rock mass could be taken as 2.5 from the analysis of rockfill tests.

The parameter s contributes predominately to the cohesiveness of the rock mass. Thus, as GSI decreases s should also decrease as per the Hoek-Brown criterion. The limiting value should be zero where the 'soil-like' rock mass is non-cohesive. Once a rock mass no longer behaves as 'intact' it can be expected that cohesion and thus s should decrease rapidly with a decrease in GSI. Hoek and Brown use an exponential drop in s versus GSI as shown in Table 1.

The Hoek-Brown criterion was developed for hard rock masses. As such, at the 'soil limit' (GSI$\rightarrow$0) the parameter s in the Hoek-Brown equation approaches zero. This is reasonable for a non-cohesive or frictional rock mass. However, where the rock mass is controlled by cohesion (i.e. the rock mass is controlled by the clay in the mass) this assumption is incorrect. The authors believe that where there is sufficient clay material in the rock mass, such that there is no rock to rock contact during shearing, the shear strength of the soil should be used. The Hoek-Brown criterion is inappropriate in this instance, as the properties of the intact rock (e.g. σ_c) will have, at most, a minor effect on the strength of the mass.

DEVELOPMENT OF NEW EQUATIONS FOR THE HOEK-BROWN PARAMETERS

The previous section discussed the development of modifications to the parameters of the Hoek-Brown criterion. These were based on the authors' work on intact rock, rockfill and rock masses. The bounds can be well quantified. The limited information available on failures in rock masses and the additional challenge of a variable a dependent on m, means good quality triaxial tests were needed to provide equations for intermediate rock masses. The analysis of triaxial testing on rock masses could also give confidence to the theory developed in the previous section. Habimana et al (2002) published four sets of triaxial tests on quartzitic sandstone. These were part of an extensive laboratory testing program carried out by the rock mechanics laboratory of the Swiss Federal Institute of Technology Lausanne (EPFL) for the hydroelectric power plant of Cleuson-Dixence and the reconnaissance gallery for the Lotschberg Tunnel in the Swiss Alps. New sampling techniques were developed to ensure as little disturbance as possible. To the authors' knowledge, these are the best set of published triaxial tests on rock mass available. The quartzitic sandstone had varying degrees of tectonic crushing. Habimana et al (2002) classified the rocks used into four GSI groupings (GSI = 15, 25, 50 and 80).

The authors analysed each data set statistically using the Hoek-Brown equation. Table 2 shows the results of the statistical analysis of the Habimana et al (2002) test data. During this analysis, the loss function used in the statistical analysis had to be modified to allow the solution to converge and to avoid local minimums outside the bounds of the Hoek-Brown parameters i.e. $0 \le s \le 1$; $0 \le m \le 40$; $0 \le a \le 1$.

Table 2. Results of statistical analysis of Habimana et al (2002) test data

GSI	m_b	s_b	a_b	σ_c (MPa)	Variance explained (%)
15	2.46	0	0.84	16	93.4
25	3.9	0.016	0.65	16	99.3
50	14	0.10	0.62	16	99.996
80	20	0.7	0.55	16	93.3

The results from the statistical analyses were used, together with the discussion and models in the preceding section, to develop equations for the parameters s_b, a_b and m_b.

The results for the parameter m_b showed that a linear equation gave the best fit to the data. The best-fit equation (variance explained = 95.5%) was:

$$m_b = \frac{GSI}{4} \tag{4}$$

If this line was extrapolated to GSI = 100 then $m_b = 25$. The equation was therefore rewritten as:

$$m_b = m_i \frac{GSI}{100} \tag{5}$$

A lower limit of m_b was set at 2.5 as indicated from the analysis of rockfill. Thus the equation becomes:

$$m_b = \max\begin{cases} m_i \dfrac{GSI}{100} \\ 2.5 \end{cases} \tag{6}$$

The linear relationship is different in form to the exponential Hoek-Brown relationship given in Table 1. The authors do not see this as a problem as: frictional strength may not decrease as rapidly with GSI as indicated by the Hoek-Brown equation for m_b; a is no longer almost constant and as it is directly related to m_b, the Hoek-Brown equation will not be appropriate; and an exponential relationship is not supported by the data.

The results from the statistical analysis of the Habimana et al (2002) data showed a rapid decrease of s with a decrease in GSI as predicted above. Therefore an exponential relationship, similar to the Hoek-Brown relationship for s, was deemed appropriate. One issue raised from the test results was the relatively high value for s for a GSI of 80. This result was taken to indicate the possibility of a plateau in the value of s at high GSI. This can be justified by the argument that at high GSI (>85) the rock mass is very strongly interlocked and thus the cohesive strength is similar to that of intact rock and therefore s should be close to unity. This is supported by the GSI table by Hoek (1999) that includes a new row for 'intact or massive' rock that has a GSI ranging from 80 upwards for very good defect quality.

A statistical analysis of the data in Table 2 using an exponential curve together with a plateau (maximum) of s = 1 for GSI>85 was performed and produced the following equation (variance explained = 99.97%):

$$s_b = \min\begin{cases} \exp\left(\frac{(GSI-85)}{15}\right) \\ 1 \end{cases} \tag{7}$$

The exponent, a_b, varies with GSI and m_b. The limits, as discussed above, on a_b are when GSI = 100, $a_b = a_i$; and as GSI $\rightarrow$ 0, $a_b \rightarrow$ 0.90. A statistical analysis of the data using these limits produced the following equation (variance explained = 83.3%).

$$a_b = a_i + (0.9 - a_i)\exp\left(\frac{75-30m_b}{m_i}\right) \tag{8}$$

It should be noted that, for this analysis, m_i was taken as that extrapolated from the equation derived earlier for m_b (Equation 6). The exponent for intact rock, a_i, was also estimated from the test data.

Figure 1 shows the data from the statistical analysis of the Habimana et al (2002) triaxial tests (Table 2) together with the fits to the data (Equations 6, 7 and 8).

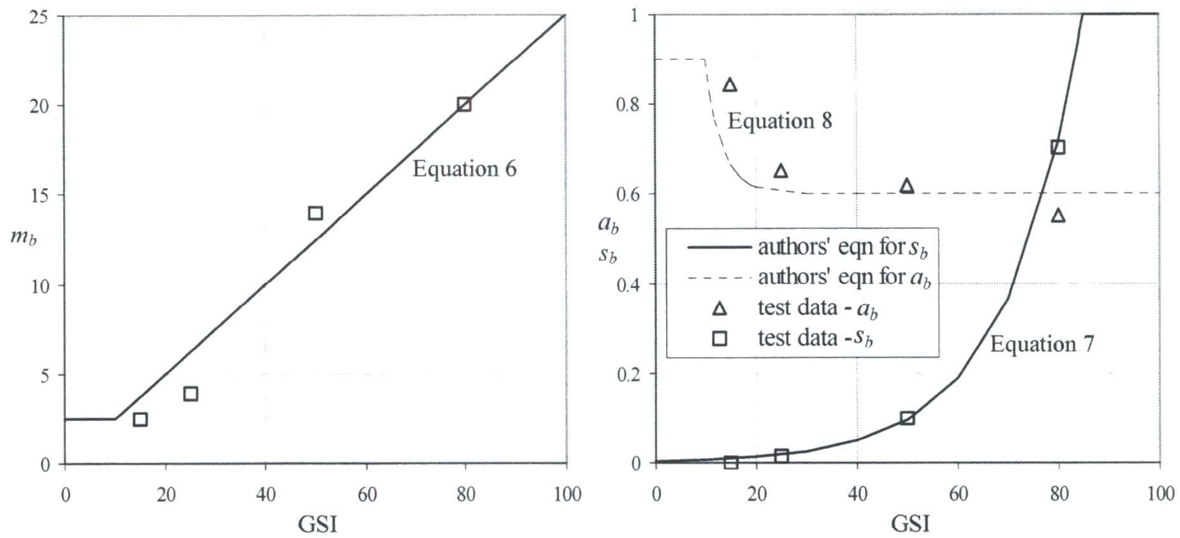

Figure 1. m_b, a_b and s_b versus GSI derived with data points analysed from Habimana et al (2002)

Figure 2 shows diagrammatically the transition from intact rock to rock mass. The fine line represents the relationship for all intact rocks (Equation 3). The bold line shows the transition from a GSI=100 (intact) to a GSI=0 (rock mass limit) for a particular rock sample.

Figure 3 shows the equations derived for the interrelationship of m and a with GSI. The figure shows that the increase in a occurs rapidly over a short range of m_b. This increase starts at approximately $m_i/4$ or GSI $\approx$ 20-25. Interestingly, this is very similar to the Hoek et al (1995) relationship for a where a remains constant for GSI approximately 25-100 and increases below a GSI of 25.

A final global statistical analysis of all the Habimana et al (2002) data was performed using the general Hoek-Brown criterion together with the equations developed in this paper. This was carried out to check whether any errors were introduced due to each parameter equation being derived separately and then being recombined into the Hoek-Brown equation. The results from the statistical analysis showed good fits to the data with a variance explained of 95.6%.

Figures 4 and 5 compare the author's modified criterion with that of the current Hoek-Brown criterion for an m_i of 40 and an m_i of 4 respectively. The plots show that the modified criterion gives a higher strength in the compressive region for an m_i of 4. For an m_i of 40, the modified criterion gives a lower strength for a GSI of 100, a similar strength for a GSI of 80 and a higher strength for a GSI of 10. There is a larger relative drop in strength between GSI 100 and 80 for the Hoek-Brown criterion compared with the modified criterion. This is in accordance with the earlier discussion. It should be noted that the two criteria use different methods of approximating m_i. The Hoek-Brown criterion uses either regression of triaxial results using a_i = 0.5 or a correlation with rock type. The authors' formulation of the criterion using either regression of triaxial results employing a variable a_i or the ratio of the unconfined compressive strength to tensile strength to estimate m_i.

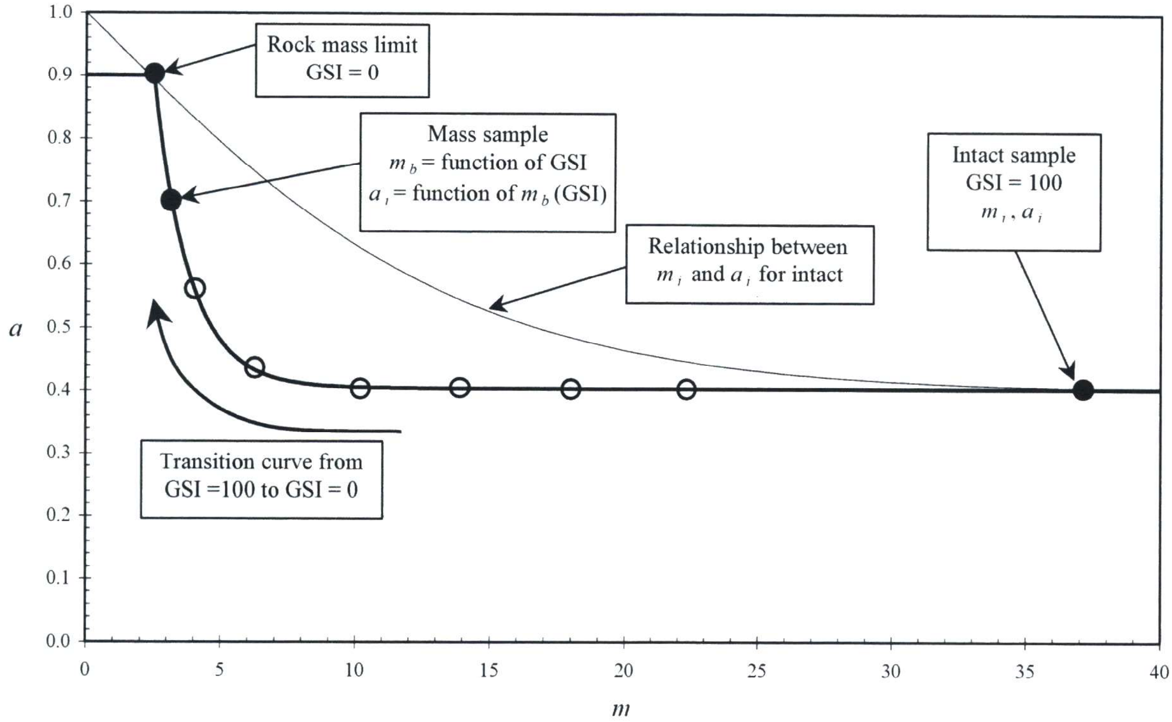

Figure 2. Transition of a and m from intact rock to rock mass

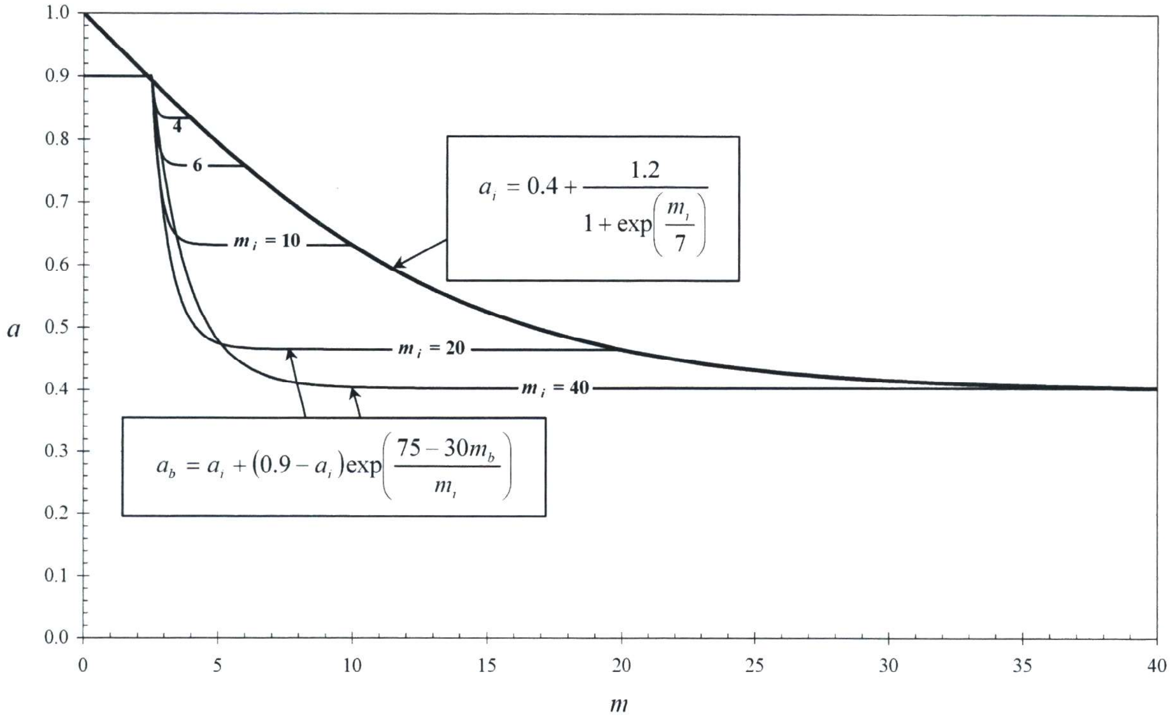

Figure 3. Equations relating a and m

Figure 6 compares the authors' curves with those of the Hoek-Brown criterion for the Habimana et al (2002) data for a GSI of 25 and a GSI of 80. The Hoek-Brown curves were created using an assumed m_i of 19, which is appropriate for sandstone (Hoek et al, 1995). The graphs show that, in this case, the authors' curves give a much better fit to the data. It should be noted that the Hoek-Brown curves could be improved by using regression analysis on intact rock samples to determine m_i for $a_i = 0.5$.

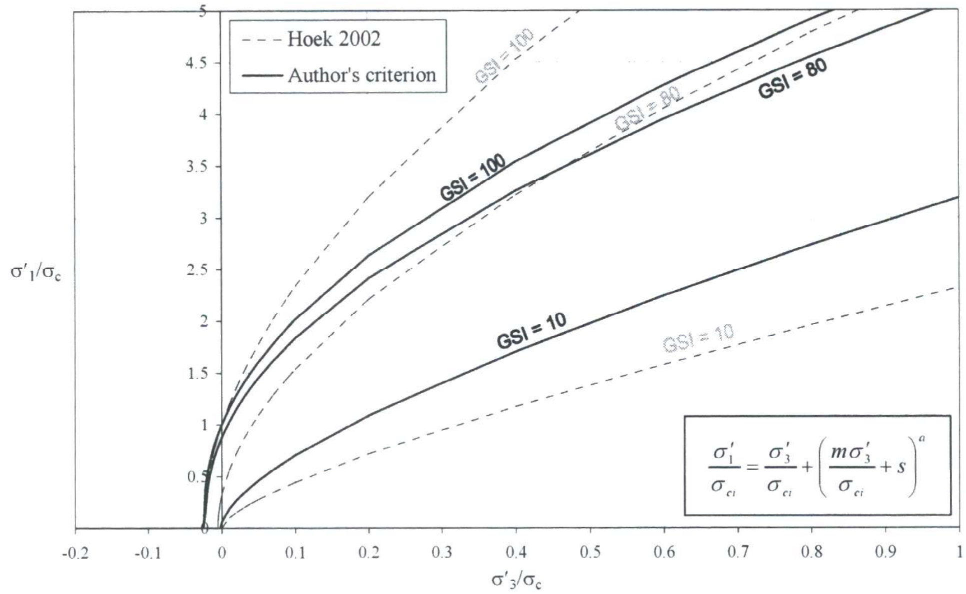

Figure 4. Comparison of the author's criterion and the Hoek-Brown criterion (Hoek et al, 2002) for $m_i = 40$

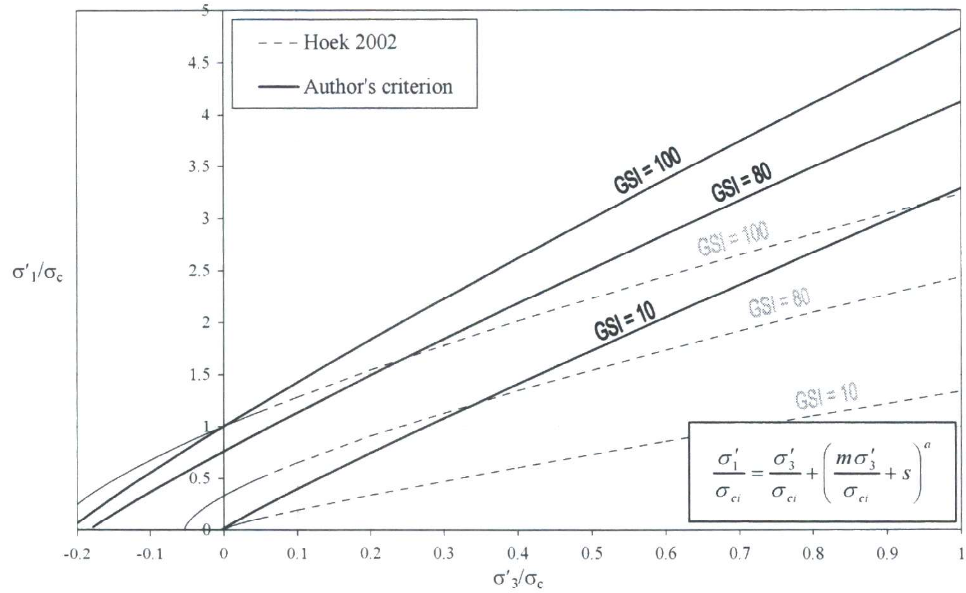

Figure 5. Comparison of the author's criterion and the Hoek-Brown criterion (Hoek et al, 2002) for $m_i = 4$

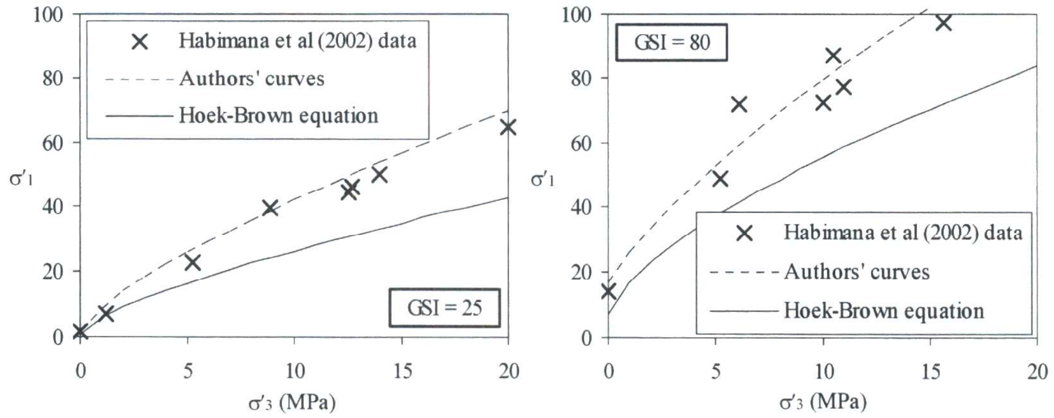

Figure 6. The authors' curves and the Hoek-Brown curves fitted to the Habimana et al (2002) data

APPLICATION

The following is a summary of the authors' method. Determine a_i and m_i using a statistical analysis of triaxial tests on intact rock samples. The authors recommend the approach presented in Mostyn & Douglas (2000) Alternatively, an approximation can be made using Equations 2 and 3. The GSI of the rock mass together with a_i and m_i can be input into Equations 6, 7 and 8 to determine m_b, s_b and a_b respectively. These values can then be input into Equation 1 to determine the shear strength of the rock mass under different confining stresses. The equations presented by Hoek et al (2002) for estimating the cohesion, c, and friction angle, ϕ, of the rock mass from σ_1,σ_3 space can be used with the authors' equations, as the general form of the Hoek-Brown equation has not been changed.

It should be remembered that a rock mass criterion should only be used where "there are a sufficient number of closely spaced discontinuities that isotropic behaviour involving failure on discontinuities can be assumed" (Hoek and Brown, 1997).

Slope failures in which the failure surface is entirely through the rock mass are not common. This is due to the low stresses typically acting in a slope. Large-scale (relative to the slope) defects concentrating stresses into regions of weak rock mass are usually required for failure through rock mass to occur. For example, a long vertical joint or subvertical fault may lead to over stressing of weak material at the toe of the slope. In this case, the authors caution that a rock mass criterion is only applicable to the region of rock mass at the toe of the slope.

CONCLUSION

The current version of the Hoek-Brown criterion does not adequately predict the strengths of intact rock and very poor rock mass (rockfill). The authors present new equations to estimate the parameters in the Hoek-Brown criterion that allow for a good prediction of the shear strength of a rock mass during its transition from intact rock to very poor quality rock mass.

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