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Bearing capacity under combined loading - a study of the effect of shear strength heterogeneity

S Gourvenec
PhD
Research Fellow, Centre for Offshore Foundation Systems
University of Western Australia

Summary: Many of Australia's marine oil and natural gas reserves are found in areas with superficial deposits of soft, normally consolidated clayey soils with linearly increasing undrained shear strength with depth. It is on these deposits (and often in several hundreds of metres of water) that the offshore facilities to harvest these resources must be constructed. The bearing capacity of shallow foundations under combined loading (i.e. simultaneous vertical, lateral and overturning loads) is a question of fundamental concern in offshore geotechnical design. Unfortunately conventional bearing capacity theory (on which the offshore industry design methods are based) falls far short of predicting realistic collapse loads for complex combined loading conditions even of strip footings on homogeneous deposits. Numerical studies have been carried out by various researchers (e.g. Salencon & Pecker, 1995; Tani & Craig, 1995; Ukrichon et al., 1998; Bransby & Randolph, 1998; Taiebat & Carter, 2000; Gourvenec & Randolph, 2003) addressing various uncertainties surrounding the response of shallow foundations to complex loading but to date none have integrated the issues of general combined loading, three-dimensional foundation geometry and soil heterogeneity. This paper presents results of a program of three-dimensional finite element analyses of circular footings founded on both homogeneous and heterogeneous soft clays subjected to combined vertical, moment and horizontal loading. The numerical results are presented in conjunction with predictions from the offshore industry design guidelines based on conventional bearing capacity theory indicating some significant shortcomings of relying on the traditional method.

INTRODUCTION

Conventional bearing capacity theory of shallow foundations is based on Terzaghi's solution for a vertical bearing failure of a strip footing founded on a homogeneous deposit as given in equation 1 (Terzaghi, 1943).

$$V_{ult} = A\, s_u N_c \qquad (1)$$

Where V_{ult} is the design unit vertical bearing capacity
A is the area of the foundation
s_u is the unchained shear strength of the soil, and
N_c is the bearing capacity factor for vertical loading of a strip footing for which there is an exact solution (Prandtl, 1921)

Non-verticality of load, three-dimensional foundation geometry and non-uniform shear strength are then accounted for by applying various empirical and semi-empirical factors to the basic equation. Most industry guidelines are based on this principle, including those used by the offshore industry which are set down by the International Organisation for Standardisation (ISO). The expressions adopted herein are those recommended by ISO in DIS 19901-4 (2002). The ISO guidelines suggest the unchained failure of a shallow foundation on a deposit with a linearly increasing shear strength with depth can be approximated by:

$$V_{ult} = As_u(Nc + kB'/4)KcF \qquad (2)$$

Where K_c is a correction factor to account for foundation shape and load orientation, k is the gradient of the undrained shear strength profile and F is the correction factor to account for the heterogeneity (given as a function of a non-dimensional non-homogeneity factor kB'/s_{uo} after Davis and Booker (1973)).

$$K_c = 1 + s_c - i_c \qquad (3)$$

Where s_c is a shape factor and i_c is an inclination factor (both after Brinch Hansen (1970)) defied by

$$s_c = s_{cv} (1-2i_c)B' \quad (4)$$

and

$$i_c = 0.5 - 0.5\sqrt{(1 - H/A's_u)} \quad (5)$$

where s_{cv} is a shape factor coefficient which is a function of of kB'/s_{uo} (Salencon and Matar, 1982).

The footing geometry is defined in terms of the 'effective width', B', as proposed by Meyerhof (1953) to account for combined vertical and moment loading. B' is defined by B-2e. where B is the width of the footing and e is the eccentricity of an applied vertical load. The guidelines define the coiresponding area A'=B'L, where L is the length of a rectangular footing, such that for the case of a circular footing the dimensions of an equivalent rectangular footing must first be calculated (by ensuring equivalent area and areal moment of inertia of the two geometries). Under zero moment A'=A and the inclination factor i_c reduces to an algebraic fit of Green's (1954) exact solution for a strip footing under inclined (i.e. combined vertical and horizontal) loading.

For conditions of soil strength homogeneity, conventional theory (as outlined in equations 1-5) adequately predicts bearing capacity under either centrally applied inclined loads (i.e. combined vertical and horizontal loading) ***or*** vertical eccentric loads (i.e. combined vertical and moment loading), but breaks down under superposition of the solutions for inclination and eccentricity (i.e. VMH loading) (Gourvenec & Randolph, 2002a). For non-uniform shear strength profiles conventional bearing capacity theory is unsuitable even for simple eccentricity with no lateral load (i.e. VM loading) (Ukritchon et al, 1998).

The purpose of the study reported here is to identify the influence of soil strength heterogeneity on bearing capacity of circular footings founded on both homogeneous and heterogeneous clays subjected to combined vertical, moment and horizontal loading. Results from three-dimensional finite element analyses are presented in conjunction with predictions from the ISO guidelines, based on conventional bearing capacity theory and highlight the oversight of available load capacity with the traditional approach.

ANALYSES

Conditions for this study were restricted to undrained failure of a circular footing founded on the surface of Tresca material with either a uniform or linear shear strength profile. Although long or strip footings are used in various engineering applications circular or quasi-circular footings are the more common in offshore applications which provides the motivation for the study. Furthermore, previous work, e.g. Gourvenec & Randolph, (2003b) and Colreavy (2002). indicates similar trends are observed for strip and circular footings. The influence of footing geometiy on bearing capacity in heterogeneous soils may be addressed in a subsequent study, but the focus of this paper is to address the effect of the degree of soil heterogeneity on footing response.

A uniform undrained strength profile and two cases of linearly increasing strength with depth were investigated. The degree of heterogeneity is defmed in terms of a non-dimensional non-homogeneity factor denoted by:

$$\kappa = kD/s_{um} \quad (6)$$

where k is the gradient of the undrained shear strength profile
D is the diameter of the footing, and
s_{um} is the undrained shear strength at the mudline.
Values of κ=0 (i.e. homogeneous), κ=2 and κ=6 were considered.

FINITE ELEMENT MODEL

The finite element mesh and the modelled material properties are shown in Figure 1. Displacement boundary conditions prevent out-of-plane movement of the vertical faces and restrict any movement of the base. The mesh area represents a semi-cylindrical section through a diametrical plane of a circular footing in fully bonded contact with the soil surface. Separation of the footing from the soil was prevented to represent the effect of a foundation skirt, used in reality to enhance moment capacity of shallow foundations by enabling suctions to develop within the cavity of the foundation during undrained moment loading.

Approximately 150 analyses were carried out; 50 for each soil strength profile investigated. Each analysis followed a single load path to failure in vertical, moment and horizontal (VMH) load space. A constant vertical load was imposed as a direct force and the horizontal and moment load components were applied at a fixed displacement ratio. 5 discrete levels of vertical load were modelled, V=0, $0.25V_{ult}$, $0.5V_{ult}$, $0.75V_{ult}$ and $0.9V_{ult}$. The ultimate vertical load, V_{ult}, for each soil case was calculated from a separate finite element analysis imposing a uniaxial vertical displacement to failure. The terminating points of the individual load paths were used to construct a continuous bounding envelope of horizontal and moment load (H:M) at a constant vertical load (V). Representation of the failure loci in terms of combined H:M at constant V was adopted since in reality vertical foundation load is quasi-constant, largely due to the self-weight of the superstructure and foundation system while the horizontal and moment components result from the environmental forces i.e. wind, wave and current forces and are inter-dependent and variable.

Soil:
Linear elastic Tresca plastic
E_u/s_u=500, ν=0.49

Footing:
Linear elastic
E=10^7 E_u(soil), ν=0.15

(All the finite element modelling was carried out with the software ABAQUS (HKS, 2002).)

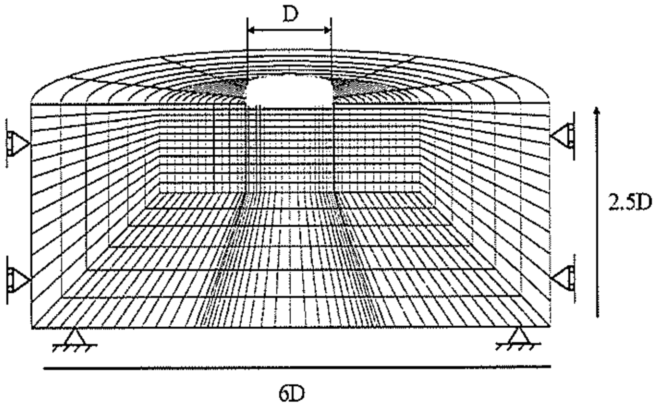


Figure 1. Finite Element Mesh and Modelled Material Properties

RESULTS

Established theoretical solutions of uniaxial limit states are shown in Figure 2, tabulated for the soil conditions investigated in this study and graphically over a broader range in terms of load normalised by the ultimate capacity under conditions of homogeneity. The theoretical solutions provide useful performance indicators for the finite element predictions and confirm good agreement, to within 2% for uniaxial vertical and horizontal loading and 6% under pure rotation. Uniaxial vertical bearing capacity of a circular footing was defined exactly by Shield (1955) for homogeneous conditions and extended to heterogeneous conditions by Martin (2001) and an upper bound solution for the moment capacity under pure rotation of a circular footing was proposed by Murff and Hamilton (1993). The theoretical ultimate horizontal load is based on the assumption that failure occurs as sliding along the foundation/soil interface with the shear stress exceeds the undrained shear strength of the soil, which is dependent only on the magnitude of s_u at the surface and therefore not affected by the increase in s_u with depth.

κ =kD/s_{um}	V/As_{um}	H/As_{um}	M/ADs_{um}
0	6.05	1.0	0.67
2	7.63	1.0	0.88
6	9.69	1.0	1.24

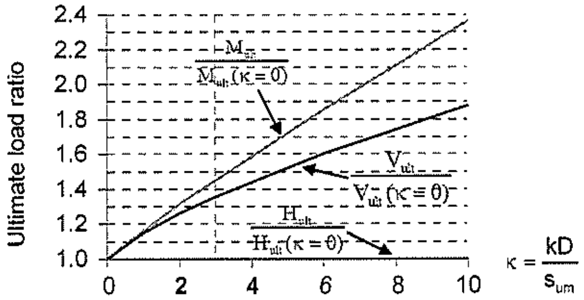


Figure 2. Uniaxial Ultimate Limit States as a Function of Heterogeneity (κ)

Failure loci from the finite element results are shown in Figure 3 for each of the shear strength cases modelled (i.e. $\kappa=0$, $\kappa=2$ and $\kappa=6$). Each plot shows five individual loci corresponding to the five vertical load cases $V=0$. $0.25V_{ult}$, $0.5V_{ult}$, $0.75V_{ult}$ and $0.9V_{ult}$. Increased bearing capacity with increased degree of heterogeneity is indicated by the expansion of the loci under equivalent load states, while the reduction in lateral and moment capacity of the footing as the vertical load increases is indicated by the contraction of the failure loci. The complex shape of the loci reflects the complexity of the interaction of vertical, lateral and overturning load components contributing to failure of a shallow foundation. The observed asymmetry results from mobilisation of the maximum moment in conjunction with a horizontal load (as opposed to zero horizontal load), and it can be further observed that the eccentricity diminishes both with increases in vertical load and increasing degree of heterogeneity (i.e. increasing κ).

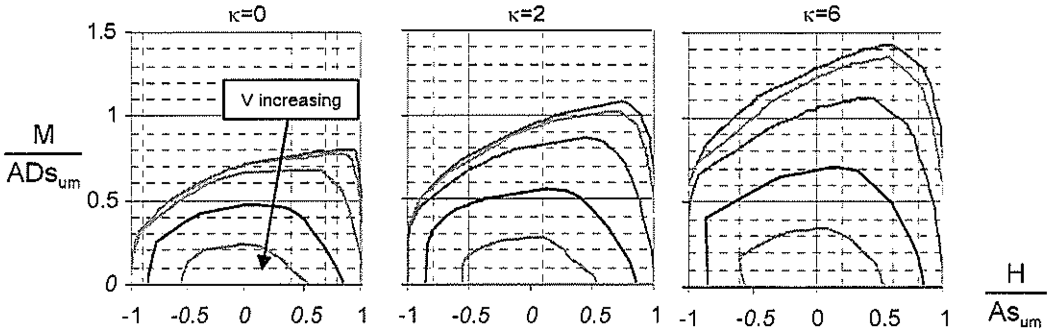


Figure 3. Failure Loci from Finite Element Analyses

The differences in the shape of the failure loci observed in Figure 3 are physically represented by the kinematic mechanisms that accompany failure of the soil mass. Displacement vector fields at failure at maximum moment for each of the shear strength profiles are illustrated in Figure 4 for $V=0$ and $V=0.5V_{ult}$. Under zero vertical load, for the homogeneous soil conditions ($\kappa=0$) nearly all the lateral resistance of the soil is mobilised and a near-perfect scoop mechanism accompanies failure. As shear strength increases with depth (κ increases) the maximum moment is mobilised in conjunction with less horizontal load and some lateral displacement at the edges of the footing accompanies the rotational failure. The increase in shear strength with depth is also reflected by a reduction in the depth of the failure mechanism as the soil displacements are confined to the weaker soil nearer the surface. An increase in vertical load component, corresponding to the contraction and backwards rotation of the failure loci in Figure 3, is accompanied by transformation of the scoop mechanism (for the homogeneous case) or the wedge-scoop-wedge mechanism (for the heterogeneous cases) to a scoop-wedge configuration (shown in the second row of Figure 4). The shallower mechanisms continue to reflect the increasing strength with depth.

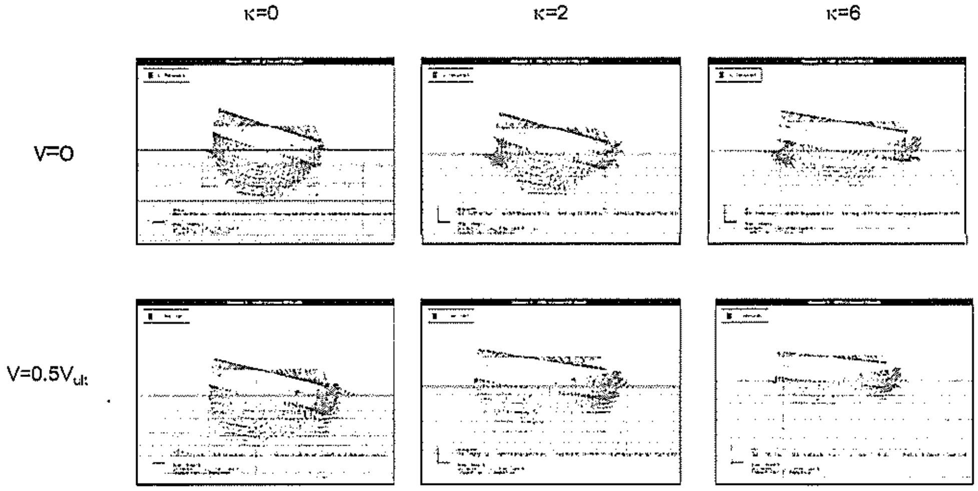


Figure 4. Failure Mechanisms Under Maximum Moment

Figure 5 illustrates explicitly the trend indicated by the failure loci shown in Figure 3 of diminishing lateral load accompanying mobilisation of maximum moment with increasing heterogeneity and vertical load. The ratio of horizontal load to moment at maximum moment is plotted against the normalised vertical load for each of the soil cases modelled and shows the reducing effect of κ as the component of vertical load is increased. The dependence of the VMH interaction as a function of heterogeneity is reinforced in Figure 6 which shows the failure loci from Figure 3 re-plotted in terms of normalised loads. For each of the soil cases the moment loads are normalised with respect to the appropriate uniaxial ultimate moment (i.e. when V=0 and H=0) and it can be seen that at each level of vertical load the size of the normalised loci reduce with increasing degree of heterogeneity. This is unfortunate as it prevents the possibility of a simple scaling procedure as an alternative design approach, as loci derived for conditions of homogeneity and scaled to the apex values relevant to a given degree of heterogeneity would be unconservative, i.e. err on the unsafe side.

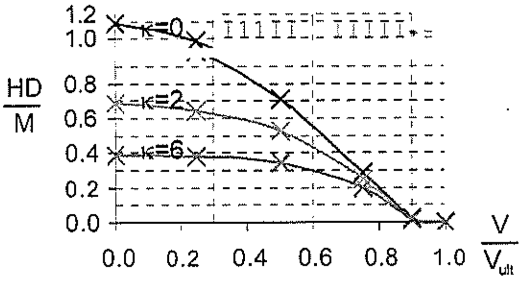


Figure 5. Variation of Horizontal Load Mobilised at Maximum Moment (as a function of V and κ)

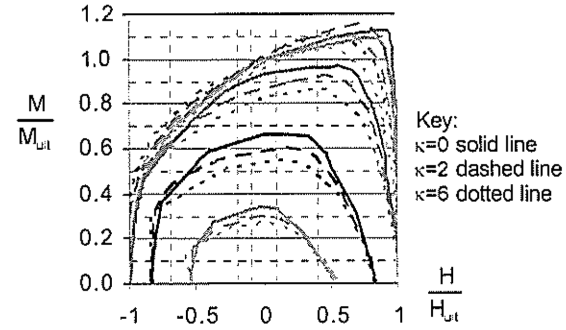
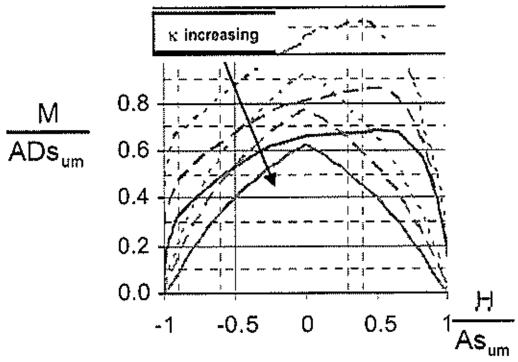


Figure 6. Normalised Failure Loci

Results from the finite element analyses are compared with conventional bearing capacity theory in Figure 7 for the case of $V=0.5V_{ult}$. (The 'conventional' calculations follow the recommendations of the offshore industry guidelines ISO (2002) as set out in equations 1-5.) The results presented in Figure 7 clearly show the traditional bearing capacity calculation is ill-suited to predicting the ultimate limit states of a circular skirted footing under general combined VMH loading irrespective of soil heterogeneity. The disparity between the predictions is a function of the symmetry and quasi-linearity of the loci derived from the conventional approach which cutting inside the arcs of the finite element loci result in considerable under-prediction. The symmetry of the loci results from overlooking the physical reality that a combined lateral load and moment acting in the same direction is not equivalent to the same loads acting in opposition to each other. The quasi-linearity reflects that the conventional calculation is based on solutions for either inclined or eccentric loading and does not represent well the two acting simultaneously.



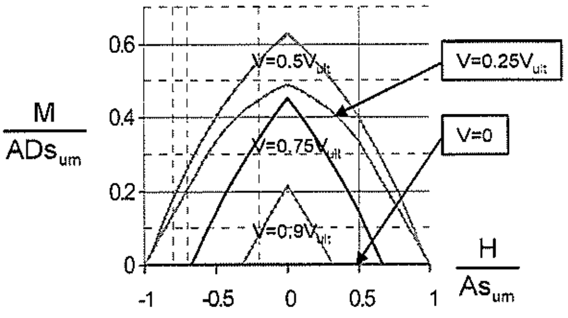


Figure 7. Failure Loci from Finite Element Analyses and Conventional Calculation for $V=0.5V_{ult}$

Figure 8. Failure Loci from Conventional Calculation (All V)

A further shortcoming of the conventional approach for bearing capacity calculation of shallow foundations is that for low vertical loads (less than $0.5V_{ult}$) the approach looses relevance to skirted footings which are commonly employed offshore. Conventional bearing capacity theory assumes that under moment loading at low vertical loads the footing lifts off the surface of the soil reducing the bearing area and hence predicts diminishing moment capacity. This is illustrated in Figure 8 by loci calculated from the ISO guidelines (for the homogeneous case) for each of the vertical load levels modelled in the finite element analyses, showing the locus at $V=0.25V_{ult}$ falling within that at $V=0.5V_{ult}$, and that at $V=0$ no load capacity is predicted (the calculation yields a straight line along the $M=0$ axis). Comparison of the loci predicted in the finite element analyses shown in Figure 3 ($\kappa=0$) reinforces the potential performance enhancement of employing skirts with shallow foundations, or conversely the loss of design efficiency by overlooking their benefit.

CONCLUSIONS & RECOMMENDATIONS

The finite element results have shown failure of a shallow foundation under combined moment and horizontal loading is complex even in the absence of vertical load, manifested graphically by an asymmetric failure locus. Inclusion of a vertical load component further complicates the interaction reflected by the degree of eccentricity of the asymmetry. The results of the finite element study have quantified the available additional load capacity of a shallow foundation under combined vertical, moment and horizontal loading in soils with increasing undsained shear strength with depth with respect to homogeneous conditions in terms of failure loci in VMH load space. Significantly, the results have also shown that the response of a footing changes as a function of heterogeneity, i.e. the kinematic mechanism governing failure under a given load state is not a scaled version of a unique form. As such combined load capacity for heterogeneous soil conditions cannot be estimated by simply scaling a failure locus for homogeneous conditions by a factor related to the degree of heterogeneity.

Comparison of the finite element results with predictions from conventional bearing capacity theory show the traditional methods are ill-suited to representing failure of a shallow foundation under combined vertical, moment and horizontal loading, typical of conditions encountered are offshore. Industry guidelines for offshore foundation design have developed based on a framework established from theory and empiricism relevant to onshore conditions despite significant differences between the operating conditions of onshore and offshore foundations. In particular the substantial proportion of lateral load and overturning moment (from wind and wave forces) that contributes to failure of an offshore foundation. The conventional approach fails to represent complex load combinations because it is based on solutions for inclined *or* eccentric loading and is not versatile enough under superposition to accommodate the large lateral and moment load components relevant to offshore conditions. In addition because of the large overturning forces that must be designed for offshore the oversight of tensile capacity by the conventional approach leads it to be inapplicable to a wide range of loading conditions likely to be encountered in reality. For the conditions considered in this study a considerable proportion of available load capacity could potentially be overlooked by following the recommendations of the current industry design guidelines.

This study has used finite element analysis to identify the bearing capacity for foundation geometry, soil and loading conditions likely to be encountered offshore and has highlighted the necessity of finding a practical design alternative, The relationships presented here are a preliminary step towards finding a rigorous and reliable yet practical design alternative. Various associated research projects in pursuit of this are underway at the Centre for Offshore Foundation Systems.

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