Nonlinear Rocking Compliance of a Rigid Footing

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Summary: A nonlinear constitutive relationship for the cyclic response of lightly overconsolidated soil is implemented in *FLAC* and its application in investigating the dynamic rocking response of a rigid footing is illustrated. The results are compared with elastic analysis. The nonlinear response of the footing is very similar to that of elastic soil for very small levels of cyclic loading. It is found also that at high frequencies the nonlinear response is very close to the elastic result. This means that for applications such as foundations for vibrating machines nonlinear soil deformation may not be significant, but in the lower frequency range associated with earthquake excitation nonlinear soil deformation is found to be significant. The results also show that the nonlinear compliance is no less frequency dependent than the elastic compliance.

INTRODUCTION

This paper reports on the implementation in the *FLAC* software (Fast Lagrangian Analysis of Continua) (Itasca 2000) of a simple constitutive relationship for static and cyclic deformation of lightly overconsolidated soil and its application in investigating the dynamic rocking stifhess of a rigid footing. The stress-strain model of Pender (1977a, 1977b and 1978) is formulated within the critical state framework but, unlike the classical critical state models, Cam clay and modified Cam clay, it postulates the occurrence of plastic deformation whenever there is a change in shear stress. Because of this, realistic modelling of cyclic behaviour is possible. With the addition of the small-strain elastic shear modulus, the soil parameters needed are those for the Cam clay models, readily obtained from standard soil testing.

In general the equations for incremental strain and stress changes cannot be expressed in closed form, so numerical integration is necessary. However, for one special case, the shear strains can be expressed in closed form. This is when the mean principal effective stress is constant at the value on the critical state line corresponding to the current void ratio of the soil. The existence of this closed form stress-strain relationship has been used to verify the implementation of the model as a User Defined Model in the *FLAC* software. An earlier paper by Pender (1977a) showed how this version of the model generates the well-known variations, with cyclic shear strain amplitude, of apparent shear modulus and equivalent viscous damping ratio, consequently this model can be applied to modelling the dynamic response of foundations.

Presented herein are data for the nonlinear rocking compliance of a rigid footing subject to sinusoidal rocking excitation. The results are compared with those for the behaviour of a footing on an elastic half space. It is verified that at very small cyclic loads the nonlinear model gives essentially the same result as the elastic case. As the level of excitation increases the compliance curves depart from the elastic curves at low frequencies but are very close to elastic for high frequency sinusoidal excitation. The frequency range investigated was from 0.1 to 20 hertz, so covering the range of interest in earthquake geotechnical engineering. Although the loading is applied directly to the footing, and does not come through the underlying soil as one would expect for earthquake excitation, the results are still relevant to earthquake design as feedback from structural response to earthquake loading may impose cyclic rocking loading on foundations.

BACKGROUND AND DESCRIPTION OF THE MODEL

The model is formulated within the framework of work hardening plasticity but, as it is assumed that plastic strains occur for all changes in shear stress, the yield surface is carried along with the stress point. This makes for simple computations, as it is not necessary to check for yielding at each step. In addition at small strains the elastic stiffness of the soil is important, so the shear strain is the sum of elastic and plastic parts. The detailed derivation of the equations of the model is given elsewhere, Pender (1977b and 1978).

The stress and strain variables used in the model are: q = 3/42 (octahedral shear stress), p = mean principal effective stress, η = the stress ratio, q/p, and E = 1/42 (octahedral shear strain). The model discussed here is for the special case when $p = p_{cs}$, for which there is no volume change and thus undrained deformation. For this case a closed form equation for the plastic distortion is available:

$$\varepsilon^p_{p_o=p_{cs}} = \frac{2\kappa}{M^2(1+e)}\left\{(AM-\eta_o)\ln\left[\frac{AM-\eta_o}{AM-\eta}\right]-(\eta-\eta_o)\right\}+\varepsilon^p_o \qquad (1)$$

where

A ÷1 for compression and −1 for extension

A4 critical state friction parameter

κ - slope of the swelling line in the $e - \ln p$ plane

e voidratio

p_{cs} p on the critical state line corresponding to e

η_0 value of η at the last turning point

ε^p_0 accumulated distortion at the last turning point

Cyclic loading is controlled by the A parameter and the value of η_o. A indicates the direction of the loading, that is whether the soil is being deformed in compression or extension. Each time there is a change in direction of the loading, that is the commencement of a new half cycle, the values for A and η_o (or q_o if equations 2 or 3 are in use) are reset.

IMPLEMENTATION OF THE MODEL IN *FLAC*

In principle, the model should be formulated in terms of work-hardening plasticity. However, as there are no volumetric strains for the $p = p_{cs}$ condition, an attractive initial step is to use a pseudo-elastic approach in which the tangent shear modulus is a function of the current stresses; that is indeed what is done here. The provision of the simple hyperbolic constitutive model, HYP.FIS. in the library of FISH functions for *FLAC*, was a particularly helpful starting point for developing the *FLAC* version of the model; the main extension from the HYP.FIS User Defined Model was the handling of cyclic loading. Further details were given by Pender (1999).

As *FLAC* supplies the user defined constitutive model with strain increments an incremental form of equation 1 is needed. This is:

$$d\varepsilon^p = \frac{2\kappa}{p_{cs}M^2(1+e)}\frac{(q-q_o)dq}{(AMp_{cs}-q)} \qquad (2)$$

where: q_o is the value of q at the last turning point.

From this the tangent shear modulus for plastic distortion is obtained from:

$$G_p = \frac{p_{cs}M^2(1+e)}{6\kappa}\frac{(AMp_{cs}-q)}{(q-q_o)} \qquad (3)$$

The equivalent tangent modulus, including both elastic and plastic contributions, is:

$$G_{\text{tangent}} = \frac{G_e G_p}{G_e + G_p} \qquad (4)$$

Where G_e is the small strain shear modulus of the soil.

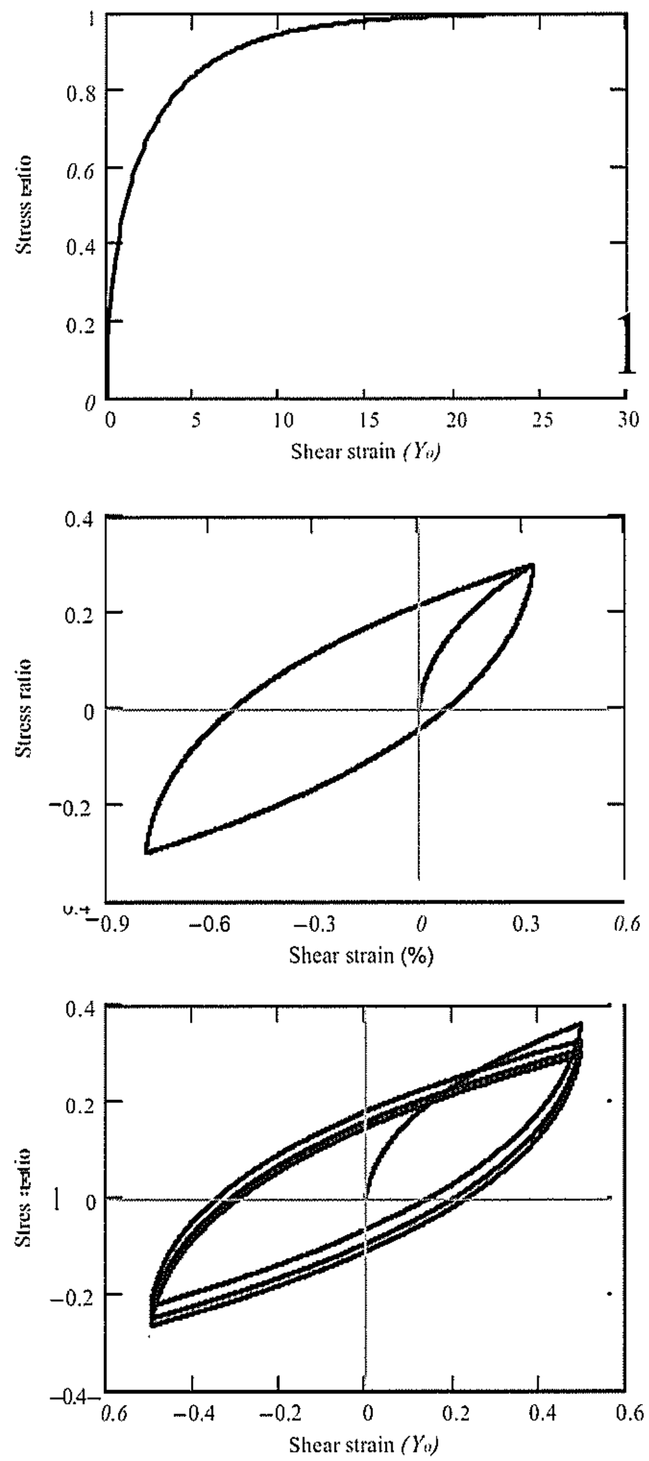

Figure 1. Stress-strain Behaviour Obtained from Equation 1.

Figure 1 illustrates the stress-strain behavior obtained from the constitutive model. Stress ratio, η, is plotted against shear strain for monotonic loading (upper), cyclic loading between fixed shear stress limits (centre) and cycling between fixed strain limits (lower).

FLAC requires a bulk modulus (K) as well as a maximum value for the shear modulus (G,) of the soil. Although the tangent shear modulus varies with stress it is assumed in the *FLAC* implementation of the model that the bulk modulus is constant; this is appropriate, as the mean principal effective stress remains at p_{cs} in the special case of the model considered in this paper.

For all the calculations discussed in this paper the following soil parameters were used: e = 1.0, M = 1.0, κ = 0.05, p_{cs} = 200 kPa (thus the undrained shear strength = $\frac{1}{2}Mp_{cs}$ = 100 kPa), density = 1.8 t/m^3, G, = 40 MPa, K =

200 MPa. The ratio of the small strain shear modulus to the undrained shear strength obtained from these input values is about 400, which is consistent with values of this ratio which have been found for overconsolidated soil by Weiler (1988).

DESCRIPTION OF THE RIGID FOOTING MODEL

The calculations reported below are for the dynamic response of a strip footing (2D plane strain) on the surface of a soil layer. These calculations are based on work reported by Ni (2001) and Pender (2000). The footing was 1.0 m in thickness and 5 m in width. It had a density of 2.4 t/m^3. The footing was subject to sinusoidal moment loading and it was modelled as rigid using a large Young's modulus (100 GPa). It was assumed that the underside of the footing was rough. As a static loading was applied to the footing prior to the dynamic loading no separation or sliding was considered between the footing and the underlying soil layer, hence no interface elements were necessary in the numerical modelling.

The soil layer was 10 m in height and 40 m in width with quiet boundaries at its sides and bottom. The soil layer had a small strain shear modulus of 40 MPa. which corresponds to a shear wave velocity of 150 m/sec. and an undrained shear strength of 100 kPa. These properties represent a "good" foundation material for which a shallow foundation would be appropriate. The values of the bulk modulus and small stain shear modulus give a Poisson's ratio of 0.4.

The details of the footing, finite difference mesh, and dynamic boundary conditions are shown in Fig. 2. The quiet boundaries in *FLAC* are less effective at handling Rayleigh waves, so in addition to the hysteretic damping implicit in the model 2% local damping was applied. This ensured a smooth response for the footing during the dynamic loading; without this damping reflected waves are likely to mask the foundation behaviour.

Before the dynamic response can be calculated there are two preliminary calculation steps. First the in situ stresses in the ground must be introduced. This is done by having the ground surface free of load and applying gravity to the soil mass. Once these in situ stresses are installed the static loading is applied to the foundation. This is done for the required static bearing capacity factor of safety. In the case presented herein the factor of safety is 2.4. For this stage the *FLAC* calculations are continued long enough to ensure that for static conditions the model is at equilibrium. Only after these two steps have been completed can the dynamic loading be applied. The amplitude of the cyclic moment applied to the foundation ranged from very small values up to 160 kNm/m. Footing response calculations were done for a range of values for a, between 0.01 and 2.5. This corresponds to frequencies of about 0.1 Hz to 20 Hz.

The standard frequency parameter for characterising footing response is:

$$a_o = \frac{\omega B}{V_s} \qquad (5)$$

where

a_o is the dimensionless frequency parameter
ω is the excitation frequency in radians per second
B is the half width of the footing
V_s is the shear wave velocity for the soil.

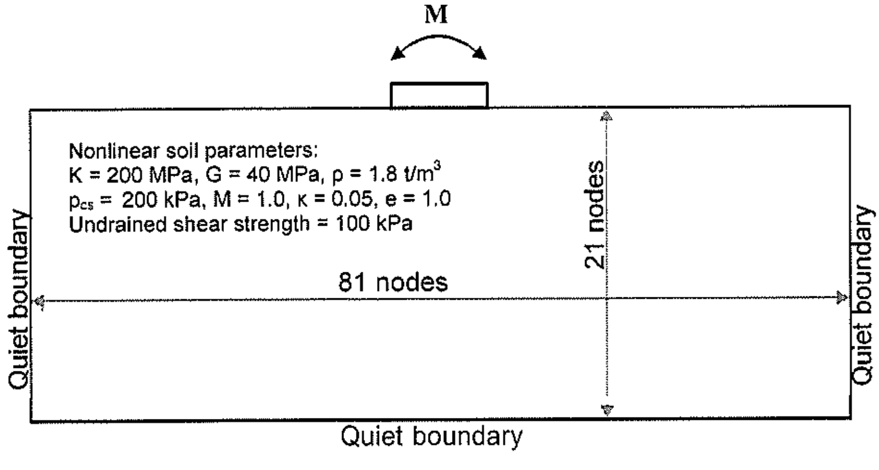

Figure 2. Details of the Dynamically Loaded Shallow Strip Footing on Nonlinear Soil

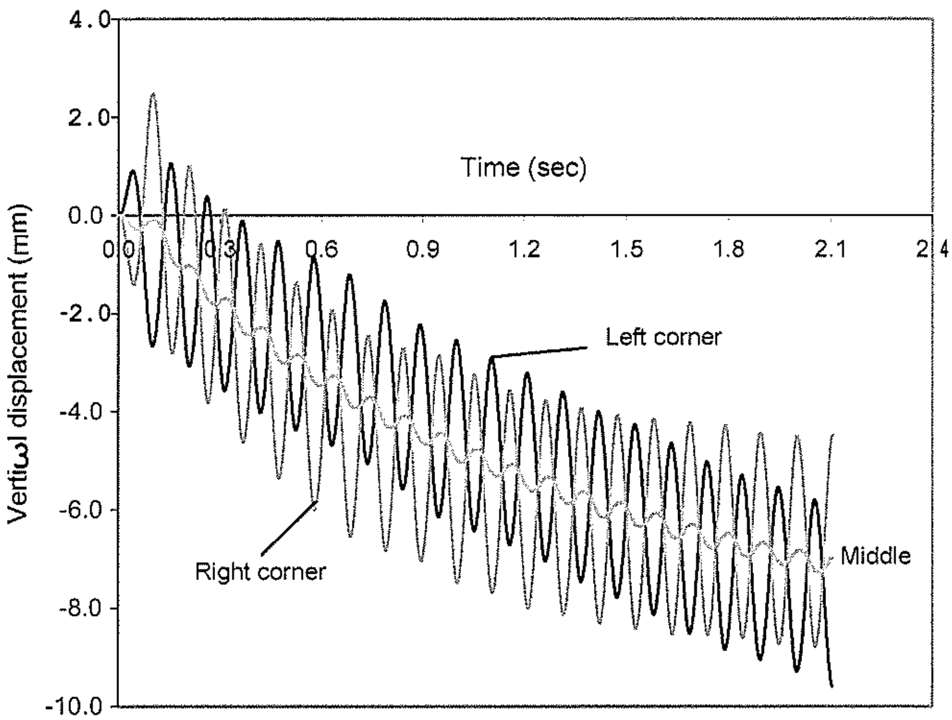

Figure 3. Typical Results of Displacement of Footing
(250 kPa static pressure was applied to footing prior to dynamic loading at $a_0 = 1.0$)

TYPICAL RESULTS

Figure 3 presents a typical result of the nonlinear dynamic analysis of a rigid footing subject to sinusoidal rocking vibration. The static pressure is 250 kPa (FOS = 2.4). The driving frequency is 9.5 Hz (a_0=1.0). The dynamic moment amplitude is 80 kNm/m. Zone size is 0.5m by 0.5m. i.e. the ratio of zone size to the shear wavelength is about 1:32. It is seen that the cyclic settlement of the footing reaches a steady state after about 20 cycles. Also apparent from the figure is that the attainment of the steady state is accompanied by permanent settlement of the footing.

ROCKING COMPLIANCE

Figure 4 illustrates the nonlinear response to rocking of the footing. The amplitude of the angle of rotation is plotted respectively against the frequency parameter (left) and the moment amplitude (right).

A resonant response is observed for each loading level. It is associated with the natural frequency of the footing. It is seen that as the cyclic moment amplitude increases from 40 kNm/m to 160 kNm/m, the resonant frequency parameter decreases from about 1.0 to 0.5, corresponding to a range of frequencies from 9.5 to 4.8 Hz. This illustrates how increasing nonlinear soil behaviour reduces the natural frequency.

Nonlinear results are compared with those obtained from an elastic soil layer. Figure 5 shows that if the level of loading is in a lower range the resonant frequency for the nonlinear soil layer is very close to that for the elastic soil layer.

It is also apparent from Figures 5 and 6 that although the deviation in the response of nonlinear soil from that of the elastic soil increases with the level of loading, nonlinear results are very close to elastic results in the high range of frequency. This means nonlinearity of soil becomes insignificant to the dynamic response if the excitation frequency is sufficiently high.

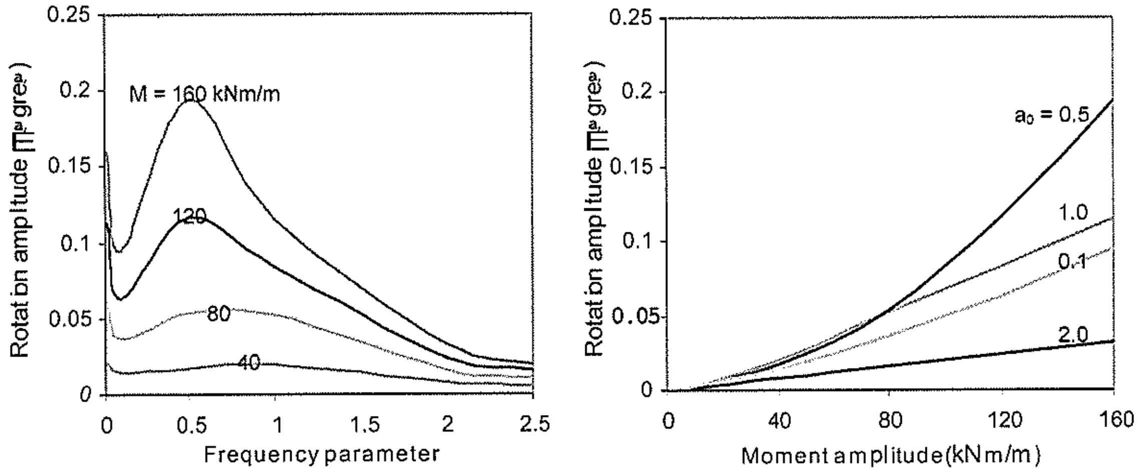

Figure 4. Nonlinear Response to Rocking of a Rigid Footing

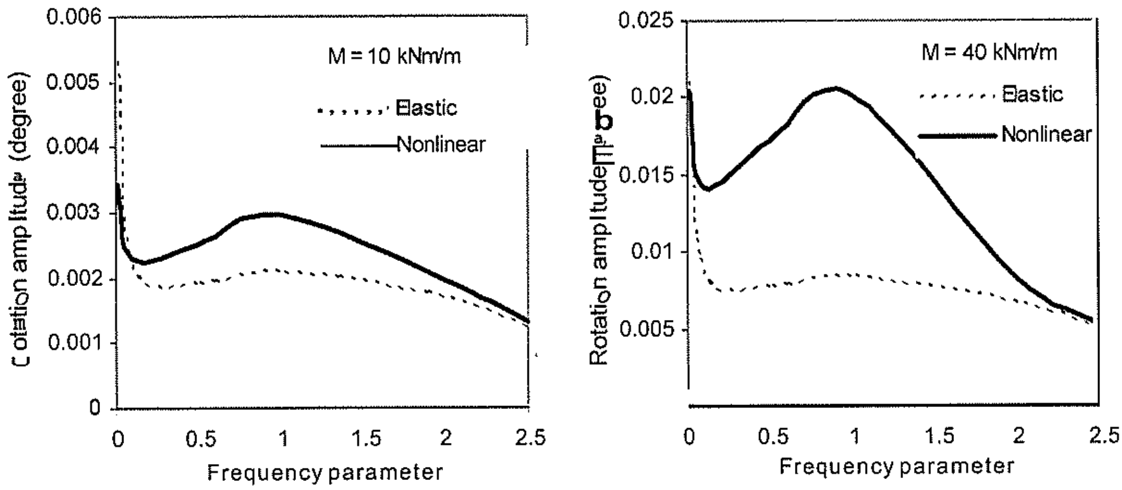

Figure 5. Comparison between Nonlinear and Elastic at Different Levels of Loading

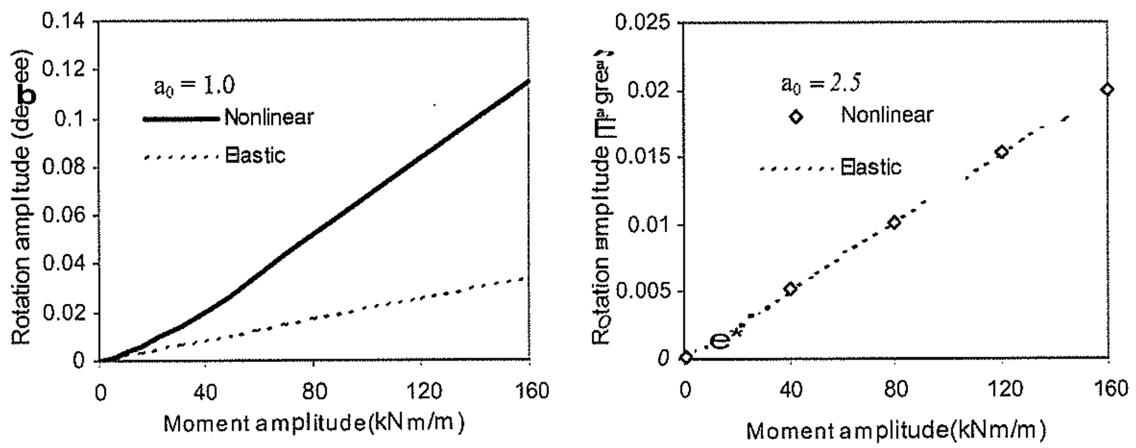

Figure 6. Comparison between Nonlinear and Elastic at Different Frequencies

CONCLUSIONS

The following conclusions have been reached:

- The response to rocking vibration of a rigid footing on a nonlinear soil layer is found to be very close to that of the footing on an elastic layer when the amplitude of the cyclic loading level on the footing is small.
- At larger cyclic moment amplitudes nonlinear soil behaviow becomes significant and the footing stiffness is reduced with consequent increased displacement amplitude.
- At low frequency excitation nonlinear stress-strain behaviour of the soil beneath the footing has a significant effect on the dynamic response.
- At high frequency excitation the dynamic response of a footing is very close to that of the footing on an elastic soil layer, regardless of soil nonlinearity.

REFERENCES

Ni, B. (2001) "Nonlinear Dynamic Soil-Structure Interaction Analysis With *FLAC,*" *M E thesis,* University of Auckland, New Zealand.

Itasca Consulting Group (2000) "Optional Features," *FLAC version 4.0 Manual.*

Pender, M. J. (1977a) "Modelling Soil Behaviour under Cyclic Loading," *Proc. 9th International Conference of Soil Mechanics and Foundation Engineering, Tokyo:* (I) pp. 325-331. Tokyo: Japanese Society for Soil Mechanics and Foundation Engineering.

Pender, M. J. (1977b) "A Unified Model for the Stress-Strain Behaviour of Normally and Overconsolidated Soil," *Proc. 9th ICSMFE, Tokyo, Specialty Session No.9, Constitutive Relationships for Soils:* pp. 213-222. Tokyo: Japanese Society for Soil Mechanics and Foundation Engineering.

Pender, M. J. (1978) " A Model for the Behaviour of Overconsolidated Soil.," *Geotechnique.* Vol. 28, pp. 1-25.

Pender, M.J. (1999) "Implementing a soil stress strain model with hysteretic damping in *FLAC*". *Proceedings of the Symposium: FLAC and numerical modelling in Geomechanics,* Minneapolis. Balkema, pp.475-482.

Pender, M. J. (2000) "Dynamic compliance of shallow footings on nonlinear soil," *Proc. 12th World Conference on Earthquake Engineering,* Auckland. Paper 2739.

Weiler, W. A. (1988) "Small strain shear modulus of clay," Proceedings, ASCE Conference on Earthquake Engineering and Soil Dynamics 11: Recent Advances in Ground Motion Evaluation. Geotechnical Special Technical Publication 20: pp. 331-335. New York: ASCE.