The paper was published in the proceedings of the 9th Australia New Zealand Conference on Geomechanics and was edited by Geoffrey Farquhar, Philip Kelsey, John Marsh and Debbie Fellows. The conference was held in Auckland, New Zealand, 8 – 11 February 2004.

Nonlinear finite element modelling of triaxial extension effect of marine clay

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Summary: A relatively simple nonlinear finite element model was developed for predicting the triaxial extension type of behaviour and its details are presented in the paper. The model was first verified against data from triaxial extension test on marine clay and it predicted the deviator stress versus axial strain behaviour quite well. Despite some discrepancies between the predicted and measured pore pressure responses when the soil approached critical state, the stress paths followed during both triaxial compression and extension tests were reasonably well predicted. To assess the proposed model further, the geotextile reinforced test embankment constructed at Sackville, Canada was also analysed using the proposed model and MCC model. A fully coupled Biot consolidation formulation was adopted for the foundation soil in both these analyses. The proposed model predicted both the settlement at the centreline and the maximum horizontal displacement at the toe more accurately than the MCC model. The applicability of the proposed model is promising and further research is continuing

INTRODUCTION

The importance of modeling the triaxial extension type of behaviour at critical locations of a soil structure was recognized a long time ago (e.g., Poulos, 1972; Tavenas, et al., 1979). Although it had been possible to model the compression behaviour of the soil quite accurately, predicting the extension behaviour had proved difficult. Behaviour of embankments on soft soils have been predicted well considering the triaxial extension and related effects more recently but the models were complicated and involved several additional parameters (Zdravkovic, et.al., 2002). A relatively simple but novel approach, using the finite element method, for predicting the behaviour of the soil in a triaxial extension test is reported in this paper. The numerical model is validated against experimental data obtained from carefully performed laboratory tests on a marine clay deposit. In particular, a nonlinear finite element model was developed for predicting the behaviour of the soil under K_o consolidated triaxial extension tests. In this numerical analysis, the soil is modelled as a Modified Cam Clay (MCC) type material. However, provision was made to account for the change of the yield surface when the principal stress direction is reversed. Details of the numerical model and the predicted responses of a typical marine clay under triaxial extension test from the finite element analysis are discussed in this paper in comparison with the experimental data.

There are several practical situations where the rotation of principal stresses takes place during the construction of soil structures and a typical example would be the soil near the toe of an embankment. Inclinometers are often installed at the toe of embankments to provide warning of any potential failure/instability problems as suggested by Kirby and Lambe (1972) and predicting the lateral displacements accurately at the toe of an embankment are important for efficient design and construction. Therefore, the proposed model was used to predict the horizontal displacement behaviour at the toe of a typical embankment also and the results are compared with the field performance.

MODEL DESCRIPTION AND BACKGROUND DETAILS

The shear strength parameters required for the analysis of a soil structure are typically determined from K_o consolidated triaxial compression and/or extension tests. It has been shown that the normalised undrained shear strength, c_u/σ_o' (where, c_u = undrained shear strength and σ_o' = effective vertical consolidation stress) of natural and remoulded clays depend on the effective stress friction angle, ϕ', and pore pressure coefficient at failure, A_f (Leonards, 1962; and Nakase and Kamei, 1988). It has also been observed that the consistency characteristics of marine clays closely follow the *'2-line"* of the plasticity chart, a feature well predicted by Cam-clay model (Schofield and Wroth, 1968). These observations suggest that a "generalised" incremental plasticity based effective stress model would be appropriate for predicting the soil behaviour. One obvious advantage of using such a model is that the soil response for a wide range of conditions can be predicted from a relatively few tests.

The MCC model has the advantage of requiring a small number of soil constants that could be determined from simple tests. For a sample in a state of virgin loading in the sense of incremental plasticity theory (i.e., plastic strain increment > 0 and the current yield surface is expanding),

$$d\varepsilon_v = \frac{1}{1+e}\left[\lambda\frac{dp'}{p'} \pm \frac{2\eta(\lambda-\kappa)}{M^2+\eta^2}d\eta\right] \tag{1}$$

$$M = \frac{6\sin\phi'}{3 \mp \sin\phi'} \tag{2}$$

where, $p' = (\sigma_1' + \sigma_2' + \sigma_3')/3$

$\eta = (\sigma_1' - \sigma_3')/p'$

e = void ratio

λ = slope of $e - ln$ p' isotropic consolidation line

κ = slope of e - lnp' isotropic rebound line

The $\pm$ (and $\mp$) signs refer to compression and extension shearing mode respectively.

For undrained compressive shearing (both CIUC and CK_oUC tests) and isotropically consolidated undrained extension shearing (CIUE test), the effective stress paths followed correspond to virgin loading (in the sense of plasticity theory) and thus Equation (1) is applicable. This, in conjunction with the fact that $d\varepsilon_v = 0$ during an undrained condition, leads to:

$$dp' = \frac{\mp 2\eta(1-\frac{\kappa}{\lambda})p'}{M^2+\eta^2}.d\eta \tag{3}$$

Thus, the effective stress path for undrained compressive shearing (CIUC and CK_oUC tests) and isotropically consolidated undrained extension shearing (CIUE test) can be computed using Equation (3). The undrained shear strength corresponds to the point where the effective stress path intersects the *M-Line*. Using this approach, the undrained shear strength of marine clays was predicted well with the Cam-Clay type plasticity model for isotropically consolidated triaxial compression and extension tests as well as for K_o consolidated compression test (Lo, 1989). However, it was found that the model underestimated the undrained shear strength for the K_o consolidated undrained extension test (CK_oUE). Lo (1989) proposed the following postulates for predicting the undrained shear strength values in CK_oUE tests and were adopted in the current studies:

(i) Anisotropic effective stress state (with $q > 0$) will produce hardening in accordance with the Cam-Clay model only in the effective stress domain defined by $q > 0$.

(ii) Isotropic effective stress state will produce isotropic hardening.

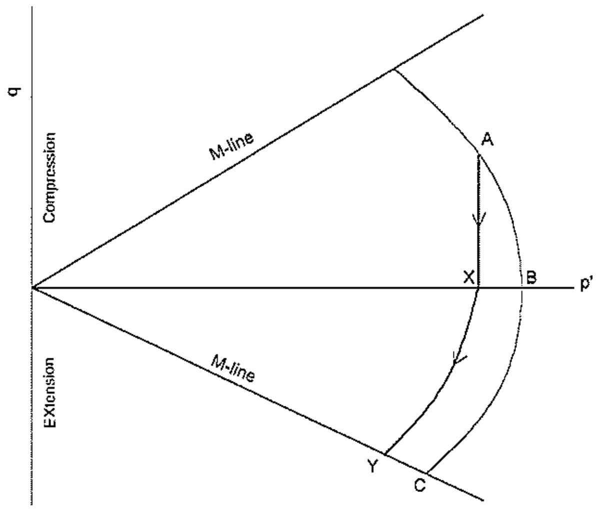

Figure 1 Detailed stress path of the proposed model

According to the above postulates, the effective stress path of CK_oUE test follows the path AXY where response along path AX would be elastic and yielding commences at point X. The advantages of such an idealisation are:

(i) The response along path XY can still be predicted by Equation 1.
(ii) There is continuity at point **X**.
(iii) There is continuity in response between isotropically and anisotropically consolidated samples.

Appropriate modifications were made to the MCC model by shrinking the yield surface when q becomes negative so that the stress path would follow along AXY for the analyses reported in this paper.

MODEL VERIFICATION WITH TRIAXIAL TEST DATA

To verify the reliability of the model, the CIUC and CIUE triaxial test results for normally consolidated Hong Kong marine clay reported by Zhu and Yin (2000) were numerically predicted using the above model and the results compared with the experimental data. The analyses were performed using the authors' modified version of the program AFENA originally developed by Carter and Balaam (1995) with the above model implemented in it.

An undrained axisymmetric finite element model was adopted in which the soil was modelled as an elasto-plastic MCCmaterial using 6-noded linear strain triangular elements. The details regarding the MCC material behaviour can be found elsewhere (e.g. Schofield and Wroth, 1968; Roscoe and Burland, 1968; Britto and Gunn, 1987). MCC uses four critical state parameters: λ , the gradient of the consolidation line in the e - $ln\,p$ 'space; κ, the gradient of the swelling line in the e - $ln\,p$ 'space; M, the value of the stress ratio q/p 'at the critical state condition; and e_{cs}, the void ratio at $p' = 1$ kPa on the critical state line in the e - $ln\,p'$ space. Where p' = mean normal effective stress = $(\sigma_1' + \sigma_2' + \sigma_3')/3$: q = generalized deviator stress = $(1/\sqrt{2})\,[(\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_1' - \sigma_3')^2]^{1/2}$: and σ_1', σ_2', σ_3' are principal effective stresses. The procedure for determining these parameters is discussed by Britto and Gunn (1987). The parameters of the Hong Kong marine clay used in the analyses are $\phi' = 31.5°$, $\kappa = 0.0443$, λ 0.2, $\upsilon = 0.3$, $\gamma = 20$ kN/m^3, $p_c' = 400$ kPa, and $e_{cs} = 2.466$.

Analyses were carried out to predict the behaviour of normally consolidated (OCR = 1) Hong Kong marine clay under isotropically consolidated triaxial compression and extension tests. The relationship between the deviator stress and axial strain predicted by the model are compared with the experimental data of Zhu and Yin (2000) in Figure 2. For easy reference, the deviator stress has been normalized with the consolidation vertical stress, σ_o'. It could be observed that the model predicted both the triaxial compression and extension q versus *strain* behaviour reasonably well. However, it is worth noting that the model predictions for triaxial compression are identical to that of the original MCC model and the modifications wouldn't have had any effect in the compression test prediction. The difference between the predicted and experimental maximum q was about 10% for triaxial compression and 15% for triaxial extension. It could therefore be concluded that the model is capable of predicting both triaxial compression and extension *deviator stress - strain* behaviour quite well. This observation suggests that the proposed model could be used to predict the lateral displacements near the toe of embankments more accurately. The lateral displacement results of the analysis carried out for a typical embankment using this model is discussed later.

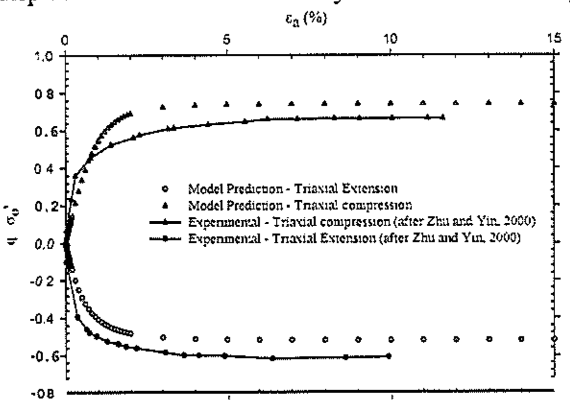

Figure 2 Comparison of predicted and experimental deviator stress versus axial strain

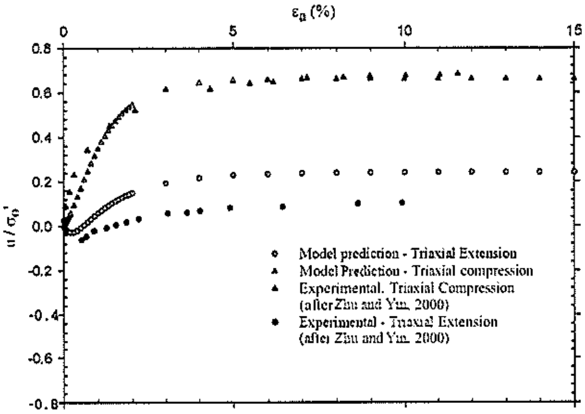

Figure 3 Comparison of predicted and experimental pore pressure versus axial strain

To further examine the ability of the model to predict the other characteristics of soil behaviour, the excess pore water pressure response with axial strain and the stress paths followed in $p' - q$ space during the triaxial compression and extension tests are compared with the experimental data in Figures 3 and 4 respectively. Here again, non dimensional form of the pore pressure, p' and q are used by dividing them with the consolidation vertical stress σ_o'. It could be observed from Figure 3 that the predicted excess pore pressure agreed closely with the measured values for triaxial compression. For the triaxial extension test, the agreement was less satisfactory. The analysis over-predicted the excess pore pressure at failure quite significantly. However, considering the difficulties involved in modeling principal stress rotation, the excess pore pressure prediction could be deemed reasonable.

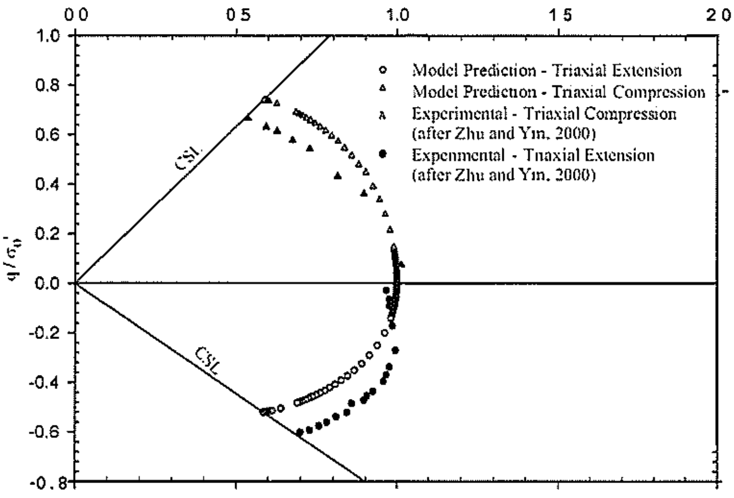

Figure 4 Comparison of predicted and experimental stress paths in $p' - q$ space for triaxial compression and extension tests

MODEL VERIFICATION WITH EMANKMENT ANALYSIS

To verify the ability of the proposed model for predicting the behaviour of a prototype embankment where the in-situ soil is K_o consolidated, the geotextile reinforced test embankment constructed on an organic clayey silt deposit at Sackville, New Brunswick, Canada (Gnanendran, 1993; Rowe et al., 1995) was analysed using the proposed model. For comparison purposes, analysis was performed using the MCC model as well. A fully coupled Biot consolidation formulation was adopted in both analyses for the foundation soil. Details of the instrumentation, construction and field performance of this embankment could be found in Gnanendran (1993) and Rowe et al. (1995) test but for easy reference the cross section of this embankment is shown in Figure 5. The foundation soil and the embankment fill were discretized using 3243 six-noded linear strain triangular elements. The foundation soil properties adopted in the analysis are summarized in Table 1. Also, $\phi' = 28.2°$.

The vertical hydraulic conductivity (k_v) of the foundation soil was allowed to vary with the void ratio (e) using two different equations. In the over-consolidated state, $k_v = A1\,(e^{B1})$, where $A1 = 0.00864$ and $B1 = 0.0$ as reported by Gnanendran (1993). When the soil becomes normally consolidated, $k_v = A\,(e - C)^{B}$, where A, B and C are constants (see Table 2). The ratio of the horizontal to vertical hydraulic conductivity (i.e. k_h/k_v) was assumed to remain constant in both the overconsolidated and normally consolidated states. To obtain reasonable stresses and strains in the embankment fill, it is necessary to consider both stress dependent stiffness characteristics of granular materials as well as plastic failure. The following Janbu's equation was used to account for the stress-dependent stiffness of the embankment fill, $(E/P_a) = K\,(\sigma_3/P_a)^m$, where E is the Young's modulus of the soil, P_a is the atmospheric pressure, σ_3 is the minor principal stress, and $K = 100$ and $m = 0.5$ are the empirical parameters adopted in this paper. Plastic failure of the fill was modelled using a Mohr-Coulomb failure criterion and a non-associated flow rule with a cohesion

Table 1. Foundation soil properties (adopted from Gnanendran, 1993)

Depth (m)	γ (kN/m^3)	k	λ	e_{cs}	OCR	K_o'
0.0-1.1	17.8	.055	.242	2.210	1.0	.68
1.1-1.8	17.8	.021	.111	1.300	3.6	.68
1.8-2.7	17.8	.027	.154	1.589	1.2	.68
2.7-4.4	17.0	.045	.224	1.799	1.0	.74
4.4-5.8	17.0	.027	.154	1.590	1.2	.79
5.8-10	17.0	.027	.154	1.590	1.2	.82
10.0-14	17.0	.027	.154	1.590	1.2	.88

Table 2. The constants to describe the variation of permeability (adopted fiom Gnanendran, 1993)

Depth (m)	A	B	C	k_h/k_v
0.0-1.0	0.5769E-3	5.1033	0.1006	10
1.0-3.5	0.5769E-3	5.1033	0.1006	4
3.5-5.0	0.5769E-3	5.1033	0.1006	10
5.0-14	0.7413E-4	4.8574	0.0000	4

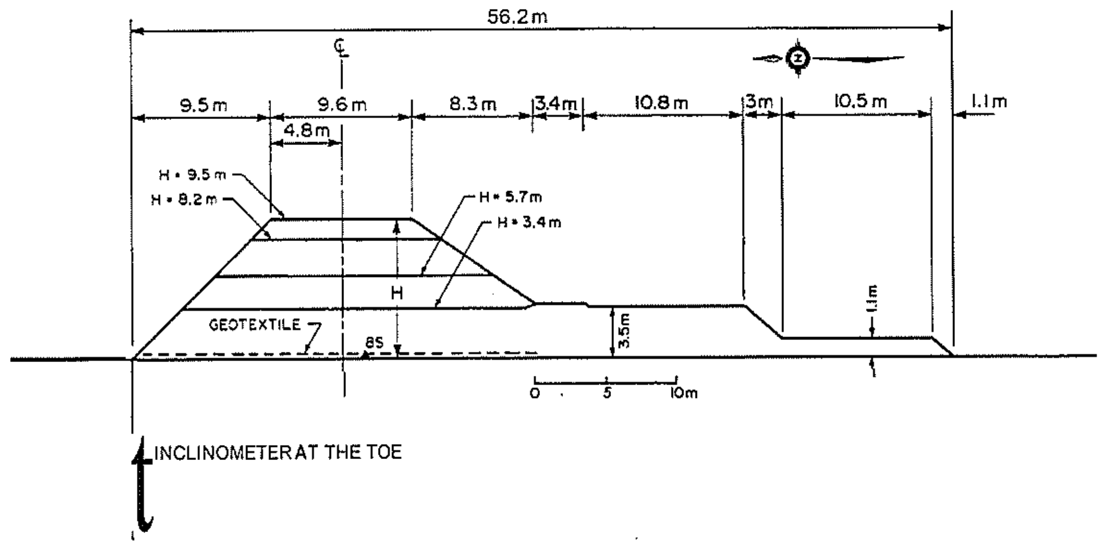

Figure 5 Details of the Sackville test embankment (extracted from Gnanendran, 1993)

intercept c′, fiiction angle ϕ' and dilatancy angle ψ. The properties of the fill material used in the analyses were: y = 18 kN/m^3, c′ = 0, ϕ' = 43°, ψ = 8" and v = 0.35 for the frst 0.7 m thickness of fill (i.e., for the 0.3 and 0.4 m thick layers of fill just below and above the geotextile reinforcement) and γ = 19.6 kN/m^3, c′ = 17.5 kPa, ϕ' = 38°, ψ = 7" and v = 0.35 for the rest of the embankment. The geotextile reinforcement was modelled as a series of linear elastic bar elements, whose axial stiflness (J) was taken as 1920 kN/m and the tensile strength, T_f = 216 kN/m. The reinforcement-soil interface was modelled using nodal compatibility joint elements, assumed to be rigid plastic and non-dilatant (i.e.. ψ = 0). The geotextile-fill interface fiiction angle was taken to be 41.9". Provision was made for slip between the reinforcement and the soil by incorporating interface slip elements above and below the reinforcement. Thus, slip could occur independently above and/or below the reinforcement.

It is common practice to monitor the settlement of the embankment and pore water pressure in the foundation soil near the centerline, and horizontal displacement at the toe since they are critical indicators of any potential instability problems. However, this test embankment was instrumented with several inclinometers, settlement plates, heave plates, settlement augurs and piezometers in the foundation soil and three types of strain gauges on the geotextile. Due to space limitations, only the settlement near the centerline of the embankment and the horizontal displacement along the inclinometer placed at the toe are examined here.

The predicted settlement at the settlement plate 8S located near the centreline of the embankment (see Figure 5 for the location) with time from the analysis using the proposed model and the MCC model for the foundation soil are compared with the field measurements in Figure 6. Also shown in this figure is the embankment thickness with time which illustrates the embankment construction sequence. It could be observed that both models over-predicted the settlement at low embankment thicknesses up to about 2.4 m thickness. However, the predicted settlement from both models agreed well with the field measurements for construction of the embankment up to about 7 m thickness. In particular, the proposed model predicted the settlement more accurately than the MCC model beyond 3.4 m thickness until failure of the reinforced embankment at 8.2 m (Rowe et al., 1995). Very large increases in settlement observed in the field at about **8.2** m thickness and until failure of the embankment could not be predicted by the proposed model and this failure was attributed to time dependent viscoplastic characteristics of the foundation soil. Further research is continuing to capture this behaviour.

Shown in Figure 7 are the variations of horizontal displacements with depth at the toe of embankment for 5.4 m thickness obtained from the analyses using the proposed model and MCC model compared with the field data (Note: 5.4 m was the highest thickness for which field data is available). It could be observed that the predicted horizontal displacement profiles by the two models were significantly different from the field measurements. However, the proposed model predicted the maximum horizontal displacement quite accurately, better than the MCC model. The proposed model over-predicted the maximum horizontal displacement by about 4% whereas the MCC model under-predicted the same by 6.6%. Therefore, the proposed model appears to predict both the settlement at the centreline and the horizontal displacement at the toe more accurately than the MCC model. The applicability of the proposed model is promising and further research is continuing.

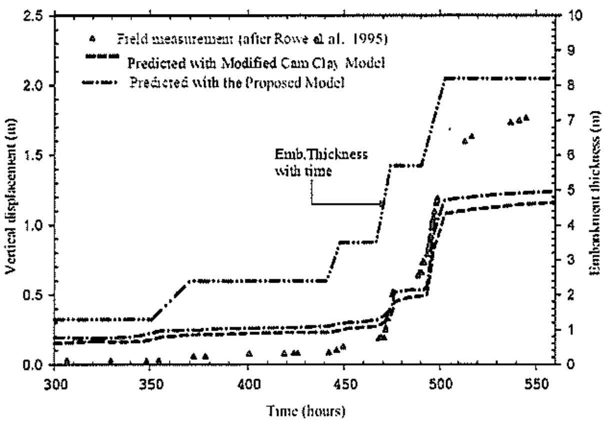

Figure 6 Variation of settlement with time at 8S: comparison of predicted and measured displacements

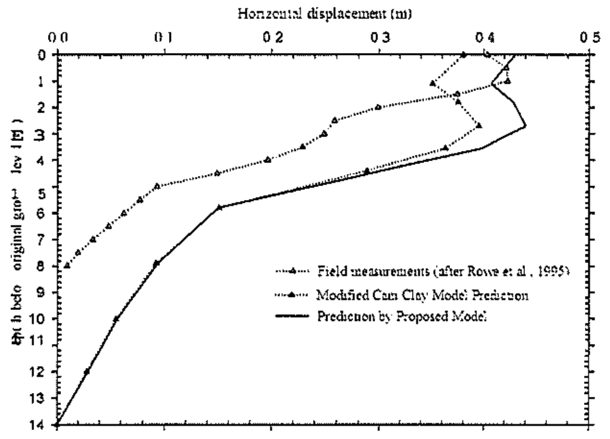

Figure 7 Variation of horizontal displacement with depth at the toe: comparison of predicted and measured displacements

SUMMARY AND CONCLUSIONS

A nonlinear finite element model was developed for predicting the triaxial extension type of behaviour of a soil. In this model, the soil is considered as a MCC type of elasto-plastic material with provision to account for the change of the yield surface when the principal stress direction is rotated. Details of the numerical model are presented in the paper.

The model was first verified against isotropically consolidated triaxial extension test data on normally consolidated clay. The predicted deviator stress versus axial strain, excess pore water pressure versus axial strain, and the stress path followed during both triaxial compression and extension tests on a typical marine clay with the proposed model were examined in comparison with the experimental data. Although there were some discrepancies between the predicted and measured pore pressure responses when the soil approached critical state, the stress paths followed during both triaxial compression and extension tests were reasonably well predicted in the analyses. The difference between the predicted and experimental maximum deviator stress was about 10% for triaxial compression and 15% for triaxial extension tests and it was concluded that the model is capable of predicting both triaxial compression and extension behaviour quite well.

To assess the ability of the proposed model for predicting the behaviour of a prototype embankment, the geotextile reinforced test embankment constructed on an organic clayey silt deposit at Sackville, New Brunswick, Canada was analysed using the proposed model as well as MCC model. A fully coupled Biot consolidation formulation was adopted in both these analyses for the foundation soil. Details of the material properties of the foundation soil, embankment fill and geotextile and the models adopted for these components as well as the interfaces between them were briefly discussed. Due to space limitations, only the settlement near the centerline of the embankment and the horizontal displacement along the inclinometer placed at the toe which are often considered as critical instability indicators of embankments on soft soils were examined.

The predicted settlement at the settlement plate located near the centreline of the embankment with time from the analysis using the proposed model and the MCC model for the foundation soil were compared with the field measurements. Both models over-predicted the settlement at low embankment thicknesses up to about 2.4 m thickness but well predicted the settlement beyond 2.4 m thickness up to about 7 m thickness. The proposed model, in particular, predicted the settlement more accurately than the MCC model beyond 3.4 m thickness until failure of the embankment at 8.2 m thickness. Very large increases in settlement observed in the field at about 8.2 m thickness and until failure of the embankment could not be predicted by the proposed model which was attributed to time dependent viscoplastic characteristics of the foundation soil. Further research is continuing to capture this behaviour.

Since 5.4 m was the highest thickness for which field data was available, the variation of horizontal displacement with depth at the toe of embankment obtained at 5.4 m thickness predicted by the analysis using the proposed model and MCC model were compared with the field data. Although the predicted horizontal displacement profiles by the two models were significantly different from the field measurements, both models predicted the maximum horizontal displacement quite accurately. The proposed model over-predicted the maximum horizontal displacement by about 4% whereas the MCC model under-predicted the same by 6.6%. In summary, despite some discrepancies the proposed model predicted both the settlement at the centreline and the maximum horizontal displacement at the toe more accurately than the MCC model. The applicability of the proposed model is therefore promising.

REFERENCES

Britto, A.M. and Gunn, M.J. (1987). *"Criticalstate soil mechanics via finite elements".* Ellis Horwood, England.

Carter, J.P. and Balaam, N.P. (1995). "AFENA - A general finite element algorithm - User's Manual", *School of Civil and Mining Engineering, University of Sydney,* N.S.W. 2006, Australia.

Gnanendran, C.T. (1993). "Observed and calculated behaviour of a geotextile reinforced embankment on a soft compressible soil". *Ph.D. thesis. University of Western Ontario,* London, Ontario, Canada.

Kirby, C. R. and Lambe, T. W. (1972). "Design of embankment soft soils". *MIT Research Report,* R72-36, Soils Publication 307.

Leonards, G.A. (1962). "Engineeringproperties of soils". Chapt. 2. In: *Foundation Engineering,* McGraw-Hill Co., N.Y.

Lo, S.-C.R. (1989). "Undrained shear strength of remoulded marine clays" - Discussion. *Soils and Foundations.* Vol. 29, No. 1, pp. 182-184.

Nakase, A. and Kamei, T. (1988). "Undrained shear strength of remoulded marine clays". *Soils and Foundations.* Vol. 28, No. 1, pp. 29-40.

Poulos, H.G. (1972). "Difficulties in prediction of horizontal deformations of foundations". *ASCE J. of the Soil Mechanics and Foundations Division,* Vol. 98 (SM8), pp. 843-848.

Roscoe, K.H. and Burland, J.B. (1968). "On the generalized behaviour of 'wet' clay." In: *Engineeringplmticity.* Edited by: J. Heyman, and F. Lecckie. Cambridge University Press, London, England, pp. 535-609.

Rowe, R.K. and Gnanendran, C.T., Landva, A.O. and Valsangkar, A.J. (1995). "Construction and performance of a full-scale geotextile reinforced test embankment". Sackville, New Brunswick. *Canadian Geotech J.* Vol. 32: 512-534; Erratum Vol. 33:208, 1996.

Schofield, A.N. and Wroth, C.P. (1968). "Critical state soil mechanics". McGraw-Hill, London, England.

Tavenas, F., Mieussens. C. and Bourges, F. (1979). "Lateral displacements in clay foundations under embankments". *Canadian Geotechnical J.,* Vol. 16, pp 532-550.

Zdravkovic, L., Potts, D.M. and Hight, D.W. (2002). "The effects of strength anisotropy on the behaviour of embankments on soft ground". *Geotechnique,* Vol. 52, No. 6, pp. 447-457.

Zhu, J.G. and Yin, J.H. (2000). "Strain-rate dependent stress-strain behaviour of over consolidated Hong Kong marine clay". *Canadian Geotechnical J.,* Vol. 37, pp 1272-1282.