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Modelling of Moisture Transfer and Shrinkage in Crushed Basaltic Rock Stabilised with Cementitious Binders

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Summary: The prediction of moisture transfer within the cementitiously stabilised materials (CSM) is essential for assessing their durability against shrinkage cracking. A theoretical approach was presented for modelling of moisture loss during drying of cementitiously stabilised pavement materials. The moisture loss process was characterised by isotropic non-linear diffusion theory. Laboratory experiments were undertaken to measure the material properties and characteristics that included the coefficient of moisture diffusivity and the humidity isotherm. Independent laboratory tests were undertaken to validate the theoretical approach adopted, and the experimental and predicted results displayed close agreement. The laboratory results indicated that as the drying progressed, the rate of moisture loss became slower, which can be explained by the reduction in the coefficient of moisture diffusivity with the decrease of moisture content. A phenomenological relationship between moisture loss and free shrinkage strain is presented.

NOTATION AND UNITS

a=constant; b=constant; D=coefficient of moisture diffusivity; E_s=evaporation from the material surface; f =surface factor; i=interval indices; J=moisture flux; n =unit vector normal to the drying surface; R=universal gas constant; H=relative humidity; RH_{en} =environmental relative humidity; RH_S =pore relative humidity at the material surface; T=temperature; t =time; w=moisture content; W_v=molecular weight of the water; ψ =moisture potential; θ =relative moisture content; k = coefficient of proportionality; $\varepsilon(t)$ = shrinkage strain; W = water loss.

BACKGROUND

Recycling of degraded pavement materials by insitu stabilisation using cementitious additives has been carried out by many road authorities as a cost effective and environmentally friendly method of rehabilitation of degraded road pavements in Australia. The waste or by-products of other industries are also included in some cementitious additives.

Shrinkage cracking, which could occur when the pavement loses its moisture to adjacent materials or to atmosphere (*i.e.*, pavement drying) during and after its construction, has been identified as an issue that warrants detailed study (Chakrabarti et *al.*, 2002; Colombier, 1997). Cracks are considered to initiate when the tensile stress that develops when the resulting shrinkage is restrained exceeds the tensile strength of the stabilised material. Shrinkage cracks can deteriorate the pavement performance by reducing the overall stiffness of the pavement system, allowing water ingress into pavement base and subgrade, and by providing pathways for erosion of cemented materials (Bullen, 1994). Therefore, it is beneficial if the potential for shrinkage cracking can be predicted at the design stage of a pavement. It follows then that the modelling of the moisture loss and resulting shrinkage during drying is a prerequisite.

Theoretical Modelling of Moisture Loss During Drying

Theoretical Concepts

Modelling of moisture movement in porous materials during drying has received considerable research effort in geotechnology, soil science and concrete technology (Kodikara and Chakrabarti, 2001). In geotechnology and soil science, the moisture movement in surficial soils is considered under two separable components, namely, the bulk water flux and the vapour flux (*e.g.*, Wilson et al., 1994). In accordance with the Darcy's law, the bulk water flux is controlled by the hydraulic potential (***i.e.,*** pore water suction and elevation potential) gradient and the hydraulic conductivity, which is a non-linear function of suction potential. The vapour flux is governed by the Fick's law, which links vapour pressure gradient and the coefficient of vapour diffusion in unsaturated pore space. In concrete technology, however, the moisture movement during drying is considered solely as a diffusive process, where a single parameter of moisture diffusivity (D) is used to represent both bulk water and vapour diffusion. The latter approach is more convenient because only one material property, ***D*** ,is needed to characterise the moisture movement. This approach, however, is more suitable when the material is predominantly unsaturated and the moisture movement in the form of vapour flux is dominant (***i.e.,*** when the pore relative humidity, *RH* is in the range 95% and 15%) (Bazant and Najjar, 1972). The stabilised materials used in pavement construction also behave predominantly in unsaturated states, except when the bulk water is allowed to ingress into the pavement matrix (***e.g.,*** rainwater infiltration or capillary rise). Under these circumstances, the authors used the approach adopted by concrete technology for modelling the moisture movement in stabilised materials.

Mathematical Modelling

The moisture flux (*J*) can be considered proportional to the gradient of the gravimetric moisture content (***e.g.,*** Rahman *et al.* 1999; Torrenti *et al.* 1999):

$$J = -D\,\mathrm{grad}(w) \tag{1}$$

where, D is the coefficient of moisture diffusivity, which is a non-linear function of moisture content, w.
Conservation of moisture flux within the pore matrix gives:

$$\frac{\partial w}{\partial t} = -\mathrm{div}(J) = \mathrm{div}\big(D\,\mathrm{grad}(w)\big) \tag{2}$$

where, t is the drying time. Equation (2) presents the governing equation for moisture movement under isothermal conditions. One disadvantage of the use of this equation is that it is not suitable for modelling moisture movement across layered media, because the moisture content can be discontinuous at layer boundaries. If the equations, presented in terms of moisture potential (as in geotechnology), they can be readily applied to model moisture flow across layered systems because the moisture potential must be continuous at the layer boundaries. Bazant and Najjar (1972) used pore *RH* as the field variable because *RH* is linked to moisture potential ψ (excluding gravitational potential) through Kelvin-Laplace equation:

$$\psi = -\frac{\mathrm{RT}}{W_v}\ln RH \tag{3}$$

where T is the temperature, R is the universal gas constant and W_v is the molecular weight of the water. Assuming that ($\partial w / \partial RH$) is a constant, Equation (2) can be re-written as:

$$\frac{\partial RH}{\partial t} = \mathrm{div}\big(D\,\mathrm{grad}(RH)\big) \tag{4}$$

The gradient ($\partial w / \partial RH$) is the slope of a material characteristic, known as humidity isotherm, which gives the relationship of w with pore *RH* at water-vapour thermodynamic equilibrium. In geotechnology, this characteristic is usually presented in terms of moisture potential and is known as soil-water characteristic curve.

Initial and Environmental Flux Boundary Conditions

Initial conditions within the stabilised material can be considered as the initial moisture content at the beginning of drying (***i.e.,*** at mixed or cured state), and these moisture contents can be converted to equivalent *RH* using the humidity isotherm.

Environmental flux boundary condition at the surface defines the moisture evaporation (moisture loss) to the surrounding environment. The evaporation from the material surface *(E,)* can be characterised by the following equation:

$$E_S = -D\left(\frac{\partial w}{\partial n}\right)_S = f\left(RH_S - RH_{en}\right) \tag{5}$$

where: f is a surface factor, which takes account for wind velocity, surface temperature and surface roughness; n is the unit vector normal to the drying surface; and RH_{en} and RH_S are respectively the environmental relative humidity and pore relative humidity at the material surface. This approach of modelling flux boundary has been adopted by both concrete and soil researchers (Akita et al., 1997). The solution of Equation (4) or (5) subjected to initial and flux boundary conditions require a numerical approach because the coefficient of moisture diffusivity is a variable of moisture content.

LABORATORY EXPERIMENTS

Cementitious Binder

Binder used in these experiment is general blended cement (GB). Cement consisting Portland cement with more than 5% of one or more appropriate inorganic materials is classified as GB cement (Austroads, 1998; Neville, 1994). GB cement includes industrial waste product like fly ash and blast furnace slag, and is commonly used in Australia as relatively slow setting binders (Senuto and Pardo, 2001). An important advantage of such binders over traditional binders such as general purpose Portland (GP) cement is the possible availability of greater working time. Typical binder contents used in the current series of tests were 2%, 3% and 4% on *dry* weight of the overall mix.

Host Material

The selected host material was crushed basaltic rock, commonly used in road pavements in Victoria, Australia due to the abundant availability of natural basaltic rock deposits. Samples were collected from a local quarry and were split and stored in 20 kg airtight plastic bags. Samples from each four bags were mixed and again stored in bags to minimise any variation between the samples. Optimum Moisture Content (OMC) and Maximum Dry Density (MDD) were determined from standard Proctor compaction method as per Australian Standard AS 1289.5.1.1 – 1993, and were found to be 9.8% and 22.04 kN/m^3 respectively. Specific gravity, linear shrinkage, liquid limit and plasticity Index of the material were found to be 2.97, 0.8 %, 22 % and 3 % respectively.

Mix Preparation

The basaltic crushed rock was mixed with water to achieve optimum water content and even distribution within the mix. The binder was then added and was mixed for further two minutes. The mixture was kept in a container, covered to prevent moisture loss for 2 hours prior to compaction. Specimens were prepared with standard compactive efforts, proportional to the maximum *dry* density corresponding to optimum moisture content for all types of tests.

Measurement of Humidity Isotherm

Humidity isotherm, which gives the relationship between the equilibrium moisture content and the relative humidity for stabilised material at a given temperature, is an essential input for modelling of moisture migration. Specimens measuring 105 mm in diameter and 38.5 mm in height were cured by wrapping them with two layers of wet plastic film, and then keeping them under controlled environment with relative humidity more than 90% and temperature between 21^{0}C and 24^{0}C for 24 hours. Subsequent to curing, each cured specimen was then dried in a desiccator that maintained an air environment with relative humidity controlled by a salt solution placed at the base. The specimens were weighed periodically until their moisture contents became practically constant. The time duration required to achieve equilibrium moisture varied from 8 to 28 weeks. The humidity isotherm was established by plotting the moisture content or the relative moisture content (the moisture content normalised by the initial moisture content, θ) against the relative humidity.

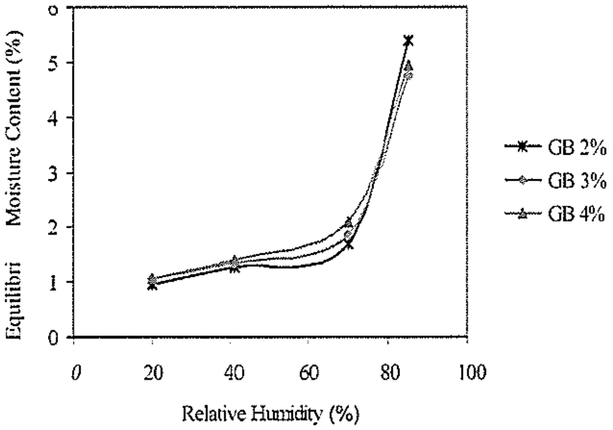

Figure 1. Humidity Isotherms for GB Cement

Typical measured humidity isotherms for GB cement are shown in Figure 1. It can be seen that the humidity isotherms are nonlinear, but can approximately be characterised by linear portion showing sharp decline in moisture content down to relative humidity of about 65% and a second linear portion showing only a small reduction in moisture content for relative humidity lower than 65%. Therefore, the slope of the isotherm can be considered constant down to about RH=65% approximately; which is a requirement for using governing equation in the form of Equation 4. As the binder content is increased, the isotherms seem to shift up by a small amount indicating that for a given relative humidity, the equilibrium moisture content becomes slightly higher.

Measurement of Coefficient of Moisture Diffusivity

An indirect method used in concrete technology (*e.g.*, Asad et al., 1997; Sakata, 1983) was used for the determination of moisture diffusivity. Three specimens with lengths 40 mm, 80 mm and 120 mm and each with cross-sectional dimensions of 75 mm in width and 75 mm in height were prepared by compacting the stabilised mix into steel moulds. The maximum Standard dry density at optimum moisture content were achieved in the specimens by compacting required mass of stabilised mix (according to the mould volume) in two layers. A steel tamping bar was used for compaction of materials and for obtaining a levelled surface. Specimens were covered with two layers of wet plastic film and cured for 24 hours at 90% or above relative humidity (RH) and air temperature that varied between 21 and 24°C. Subsequently, four sides, excepting the end sides (measuring 75 mm and 75 mm), of the specimens were covered with wax and plastic film, and the specimens were dried in a controlled environment with 50% RH and air temperature of 22°C.

Using the Boltzmann transformation as per methods described by Asad et al. (1997) and Sakata (1983), relationships between moisture diffusivity D (cm^2/day) and the relative moisture content θ (shown in Figure. *2)* can conveniently be presented using the following expression:

$$D(\theta) = D_0 + a\left(\frac{\theta}{1-\theta}\right)^b \qquad (6)$$

where, D_0 is the moisture diffusivity at the oven-dry condition (θ=0%), and a and b are parameters depending on binder type and quantity.

Determination of Surface Factor

The test results of the specimens used for the determination of coefficients of moisture diffusivity were back analysed to obtain values for surface factor. In other words, the surface factor f was determined by fitting the experimental moisture losses with the predicted values using the one-dimensional finite difference numerical model discussed later. The computed coefficients of moisture diffusivity were used for this purpose. The surface factors which ranged from 0.75 mmlday to 1.5 mm/day depending on the binder quantities. Sakata (1983) reported surface factors ranging from 0.75 mmlday to 7.5 mm/day for cement concrete with varying moisture contents.

Independent Tests for Measuring Moisture Loss, Shrinkage with Time

The known quantity of stabilised mix as per maximum standard dry density was compacted in two layers into a rectangular steel mould measuring 75 mm (wide) x 75 mm (high) x *285* mm (long) as per **AS** 1012.13. Standard Proctor hammer with maximum cross sectional dimension of 50 mm (65 mm including the guide) was used to compact the material. A clear gap of 5mm between the inner face of the mould and outer face of the guide of the hammer facilitated uniform compaction with compaction energy proportionate to the standard compaction. A steel tamping bar was used for compaction of local areas and for obtaining a levelled surface. Two gauge studs were placed at the middle of the end sections during compaction to facilitate shrinkage measurement. Specimens

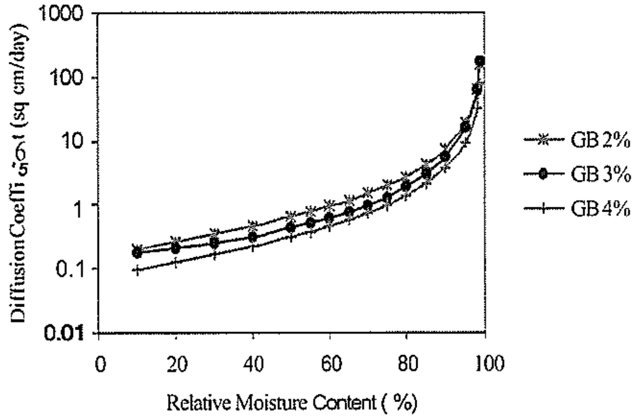

Figure *2*. Diffusion Coefficient versus Relative Moisture Content for GB Cement in duplicate were covered with two layers of wet plastic films and cured for 24 hours at 90% or above relative humidity (RH) and air temperature between 21°C and 24°C. For different drying experiments, two types of specimens were used: (1) drying from only one face simulating one-dimensional drying conditions and; (2) drying from all faces simulating three-dimensional drying conditions. The one-face drying was achieved by covering five sides with wax and plastic film (similar to the tests conducted for determination of *D*), but allowing dying from the top surface, measuring **75** mm x 285 mm. In the case of three-dimensional drying, all the six faces were exposed for drying. The specimens were dried in a controlled environment with 50% RH and air temperature of 22°C. Shrinkage strain between the gauge studs were recorded at regular time intervals with a digital micrometer of 0.001 mm sensitivity.

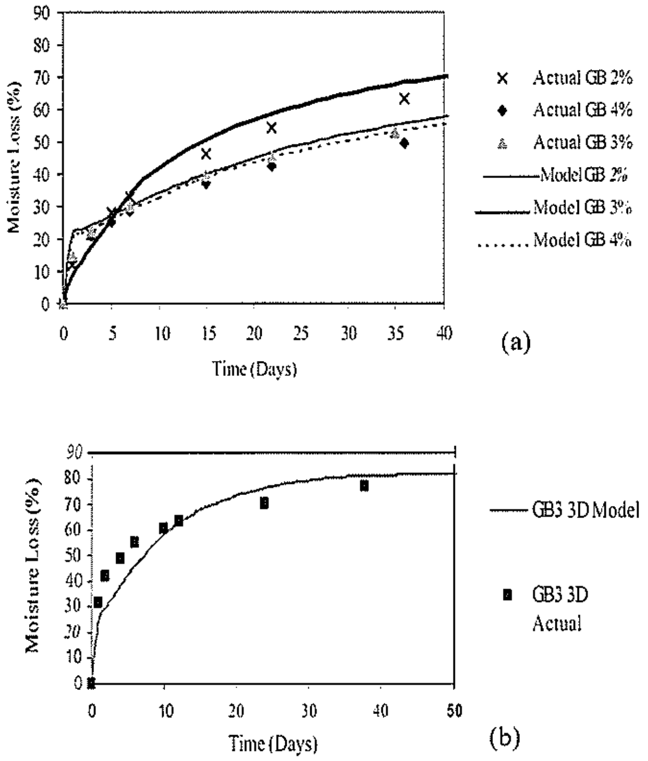

Figure 3. Moisture Loss versus Time for GB Cement (a) One-Dimensional Drying (b) Three Dimensional Drying

NUMERICAL PREDICTIONS

Numerical Solution Method

The non-linear Equations (2) or (4) can be solved with a suitable numerical approach. In this instance, explicit finite difference technique was employed by developing computer programs in Fortran language. Two programs were developed catering one- and two-dimensional forms of the equations. Non-linear relationships between coefficient of moisture diffusivity and relative moisture content were used at isothermal conditions. These relationships were assumed to be the same in both directions for two-dimensional analysis (*i.e.*, isotropic properties). Relationships between moisture content and relative humidity (*i.e.*, humidity isotherms) were used to convert *RH* to moisture content or visa-versa.

Numerical Predictions of Laboratory Results

The numerical model depicting Equation (4) was applied to predict the results of independent tests for moisture loss measurements, noted previously. Figures 3(a) shows comparisons of the experimental and calculated moisture loses for one-dimensional drying for different quantities of binders. All the input model parameters for these independent tests were determined from other tests as discussed previously. The experimental and predicted results show close agreement for the test drying periods of up to 40 days. It is clear that the modelling approach adopted is capable of simulating the moisture transfer within the stabilised mix under the conditions tested. The numerical model was also applied to predict the moisture loss results of three-dimensional drying tests, as shown Figure 3 (b). The experimental and predicted results show close agreement. In tests for measuring coefficient of moisture diffusivity, the moisture diffusion occurred perpendicular to the compaction direction. In one-dimensional drying tests, the direction of moisture diffusion was perpendicular to the compaction direction, whereas, in three-dimensional drying tests, the direction of moisture diffusion was in all directions. The close agreements obtained between the experimental and predicted results, therefore, appear to indicate that moisture diffusivity can be approximated to be essentially isotropic. Comparisons of corresponding graphs (in Figure 3) indicate that the moisture loss is substantially higher and faster for three-dimensional drying than for on-dimensional drying. The primary reason for this behaviour is the difference in diffusion lengths for moisture diffusion in the two test cases. According to the diffusion theory, the drying times are proportional to the square of the diffusion lengths (Bazant and Kim, 1991). Although the stabilised specimens predominantly featured non-linear diffusion, a somewhat similar dependence on the diffusion length can be expected.

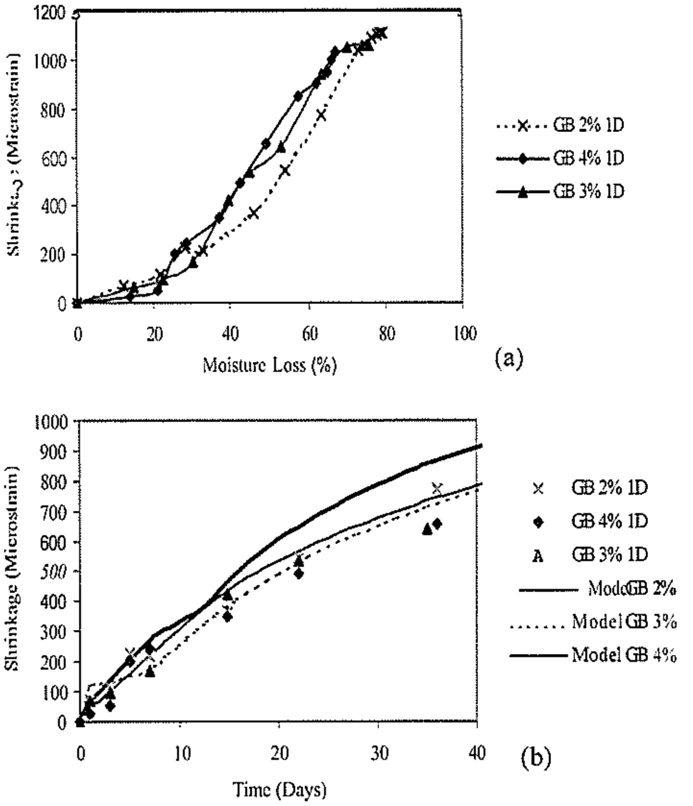

Figure 4. Relationships between (a) Shrinkage and Moisture Loss (b) Shrinkage and Drying Time

Relationship between Moisture Loss and Shrinkage

A number of authors have considered that the shrinkage strain is proportional to the loss of moisture (Torrenti et al., 1999; Sakata, 1983; Bazant and Najjar, 1972). Torrenti *et al.* (1999) reported why shrinkage may be considered to be proportional to the change of the moisture content for cement concrete and experimental confirmation of this consideration. Based on the experimental evidenced, the following equation has been proposed by Torrenti *et al.* (1999):

$$\varepsilon(t) = k\Delta W(t) \tag{7}$$

where $\Delta W(t)$ is the weight loss of the cross section being considered; k is the coefficient of proportionality; $\varepsilon(t)$ is the shrinkage strain. In the present investigation similar relationships between moisture loss and shrinkage were found as shown in figure 4(a). It should be noted that there might be an initial phase where substantial moisture loss takes place without resulting significant shrinkage. This appears to happen when the water is lost from macropores not exerting considerable suction on the material structure. These relationships could be successfully used in the numerical models to predict shrinkage of CSM for environmental conditions. Figure 4(b) shows comparison between actual and model predicted shrinkage with time relationships for different binder quantities. To apply this approach, however, k values need to be established by prior testing. Further research is underway on this topic.

A theoretical approach was presented for modelling of moisture loss during drying of cementitiously stabilised pavement materials. The moisture loss process was characterised by isotropic non-linear diffusion theory and its applicability was validated in the light of laboratory experimental results. All the parameters necessary for modelling were measured or computed. The moisture loss is directly related to the shrinkage but the appropriate relationships are currently being investigated. In the interim, it is possible to use a constant proportionality for change of shrinkage versus moisture loss obtained fiom experimental results.

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