Development of a simple, economical, and robust method of estimating G_{max} using Bender Elements

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Summary: A method for automating the measurement of shear wave velocity from triaxial tests, and hence of determining G_{max} the shear modulus at very small strains is presented. The method makes use of bender elements and cross correlation between input and output signals. Provided the correct peak in the cross correlation signal is identified and the frequency is above some critical level this method provides a robust and simple approach that is relatively independent of waveform and frequency. The approach is easy to integrate into existing test control programs, and it is shown how the technique may be implemented at relatively low cost.

INTRODUCTION

Shirley & Hampton (1978) first suggested the concept of using piezoceramic bender elements to measure the shear wave velocity in soils. Since that time many researchers have incorporated bender elements into triaxial, oedometer and direct shear apparatus. The increasing popularity of this procedure is because of greater importance attached to knowing the shear modulus at very small strains, its ease of implementation, and practical difficulties with the use of alternative on-sample measurements.

The principle behind the bender element test is very simple. An electrical pulse is applied to one bender element, which is converted to mechanical energy and causes waves to propagate through the soil. A second bender element detects the wave and converts the mechanical wave to an electrical pulse. By measuring the time delay, T, between the applied and received pulses a shear wave velocity, v_s can be calculated knowing the length, L, between the bender elements. Hence the shear modulus, G, can be determined knowing the bulk density, ρ. from

$$G = \rho \, v_s^2 \qquad (1)$$

Unfortunately, detection of the time delay is not straightforward and this has resulted in different researchers using a range of methods to obtain the "correct" arrival time. For example, Jovicic et al (1996) have recommended using the first arrival of the shear wave. But for this to work they stress that great care is required to reduce system noise, a sine wave pulse is recommended, and the pulse frequency must be adjusted to minimise near field effects. Other authors have suggested estimating the time delay from characteristic points (peaks, troughs) in the input and output signals, the second arrival of the output signal and the use of cross-correlation. Experimental and numerical results presented by Arulnathan et al. (1998) suggested that none of these methods were totally reliable. Other authors have suggested analysis in the frequency domain to determine the phase angle either indirectly (e.g. Viggiani & Atkinson, 1995), or directly using phase-sensitive detection techniques (e.g. Blewett et al. 1999).

The objective of this paper is to describe the procedures used to automatically obtain the small strain shear modulus, to show that the method is reliable, and to show how this can be easily implemented in commercial laboratory settings without the need for significant development making use of readily available equipment and software. The approach followed has been entirely empirical.

EQUIPMENT AND PROCEDURE

Bender elements have been installed in the rigid end platens of a triaxial apparatus as shown in Figure 1. This is achieved by encapsulating the bender elements (dimensions 13×10×0.5mm) in an epoxy for waterproofing, and locating this in a slot in the end platens so that the bender elements extend 3mm into the sample. The gap between the slot and bender is filled with silicone sealant. Tests have been performed on samples of sand with heights between 55mm and 200mm, and height to diameter ratios between 1 and 2. Two different approaches indicated schematically in Figure 1 have been used to activate and record the signals from the bender elements.

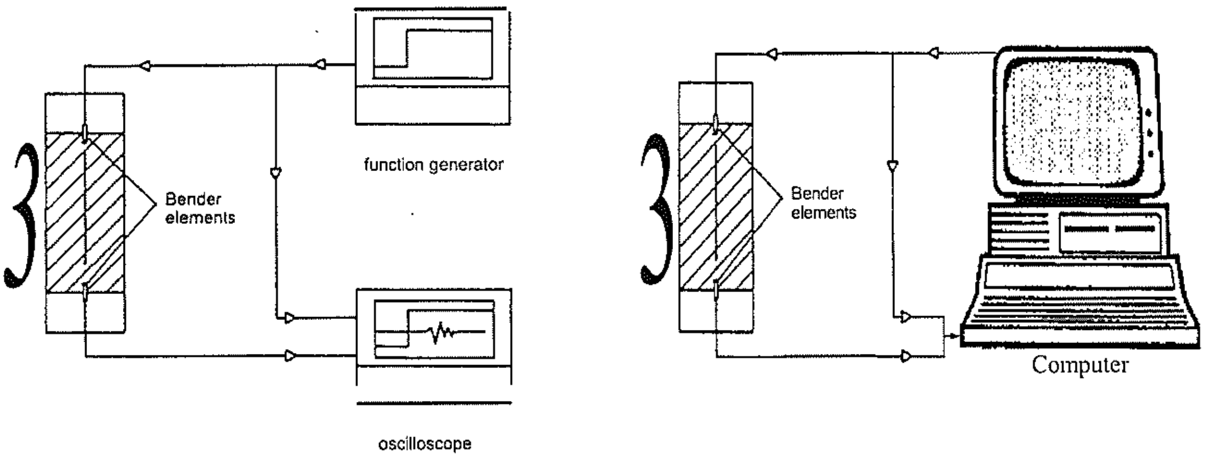

Figure 1. (a) Conventional Equipment Arrangement, and (b) Alternate Low Cost Arrangement.

The first, which may be considered as the conventional approach, uses a 15 MHz HP33120A function/arbitrary waveform generator to provide excitation of the bender elements, and a Yokogawa DL1520L digital oscilloscope to monitor the input and received waveforms from the bender elements. The function generator has been used to provide a variety of waveforms that include single sine pulses, sine pulses with two or three waves, a triangular pulse, a chirp signal comprised of three sine waves of different frequency, and continuous waves of various shapes. An input signal amplitude of ±10 V was used for most of the tests. After passing through the sample the signal was amplified to give a signal with peak amplitude of approximately 8 mV. The pulses, typically a sine wave with a frequency of 10 kHz, were sent repetitively from the function generator at a rate of 100Hz. To remove the effects of random noise the oscilloscope displayed the average of the last 256 signals. During a test the averaged responses on the oscilloscope were automatically downloaded to the computer at regular intervals. In addition the traces could be downloaded manually at any time.

In the second approach we have developed a system (Figure 1b) that makes use of already existing pieces of equipment to remove the need for an oscilloscope and function generator. A computer sound card has replaced the function generator. To achieve waveforms with frequencies of up to 20 kHz requires the latest generation of sound cards with high (96kHz) sampling rates. We are using a SoundBlaster Audigy 2 card although other sound cards have similar technical specifications. The sound cards can be programmed to output any waveform, however, the approach we have followed is to make use of a freeware program "Soundarb" that replicates a function generator. With this software package it is possible to specify your own waveform, which may run continuously. The software in combination with a stereo sound card allows one channel to be used as a synchronous (sync) output which can be used for triggering the recording device to mark the start of the analogue waveform. This emulates all the functions of the function generator that are needed for the bender elements. To remove the need for an oscilloscope we have made use of an A-D card installed within the computer that is also used to monitor and record the load, displacement and pressure data in the triaxial tests. A 16 bit A-D card with a 200kHz-sampling rate is more than sufficient to record the pulses from the bender elements. To overcome noise some averaging of the signals is required and this requires accurate timing, which is provided by the sync output from the sound card. By arranging the sync output to be sent slightly before the pulse it is possible to record the complete waveform without the need for sophisticated pre-trigger recording options. The input and output signals can be recorded when required. It is shown below that satisfactory traces can be obtained averaging 25 records of the output signal, and taking a single record of the input.

For both equipment arrangements after the computer receives the data the cross correlation of the input and received signals is computed. This is achieved by obtaining the fast fourier transforms, G(Q of the input signal (X(t)). and H(Q of the response signal (Y(t)). The cross-correlation CC(τ) can then be calculated using

$$CC(\tau) = IFFT(H^*(f)G(f)) \quad (2)$$

where IFFT indicates the inverse fast fourier transform, and H*(f) is the complex conjugate of H(f). The fast fourier transforms have been performed using freely available software routines (Press et al. 1986) that are linked into the Visual Basic program running the triaxial tests. The times of the peaks in the cross correlation signal are then recorded. There is no need to save the waveform traces, although as discussed below this may be desirable for quality control purposes.

When implementing bender elements there are a number of details that need to be checked. For instance, the elements must be aligned so that they bend in the same plane, and with the same polarity so that positive displacement gives a positive signal for both source and receiver. The polarity can easily be checked by placing the bender elements in contact. It has been our experience with sands that it is not possible to reliably determine the correct polarity from visual observation of the response signal. Putting the bender elements in direct contact has also been used (e.g. Dyvik & Madshus, 1985) to check for any the time lag in the electronic equipment. Various authors have reported time lags from zero to **5** μs (Gajo et al. 1997). Our observations using this approach indicated that the time lag was very small, but the value depended on the contact between the bender elements. An alternative procedure for estimating the time lag (Mohsin and Airey, 2003) indicated the time lag was less than 1 μs and could be ignored.

RESULTS AND DISCUSSION

For two similar waves shifted in time the peak in the cross correlation signal gives the time delay. However, due to resonance of the soil-bender element system, wave reflections and wave dispersion the peak in the cross correlation was rarely found to coincide with the correct time delay. Some authors have cited this, along with the dissimilar input and output waveforms, as a reason for not using cross correlation. Nevertheless, when dispersion is minimal and the soil is behaving as a linear system, as is assumed in interpreting the results, one of the peaks should correspond to the correct time delay (Oppenheim & Willsky, 1997). Therefore the procedure adopted has been to record the times of all the significant maxima in the cross correlation signal occurring at or before the peak. Typical waveforms showing the input and received signals at a confining stress of 100 kPa are shown in Figure 2. The cross correlation is also included on this figure. The peak is labeled 1, and the preceding maximum is labeled 2. In this case it can be seen that maximum 2 corresponds approximately to the time of the first arrival

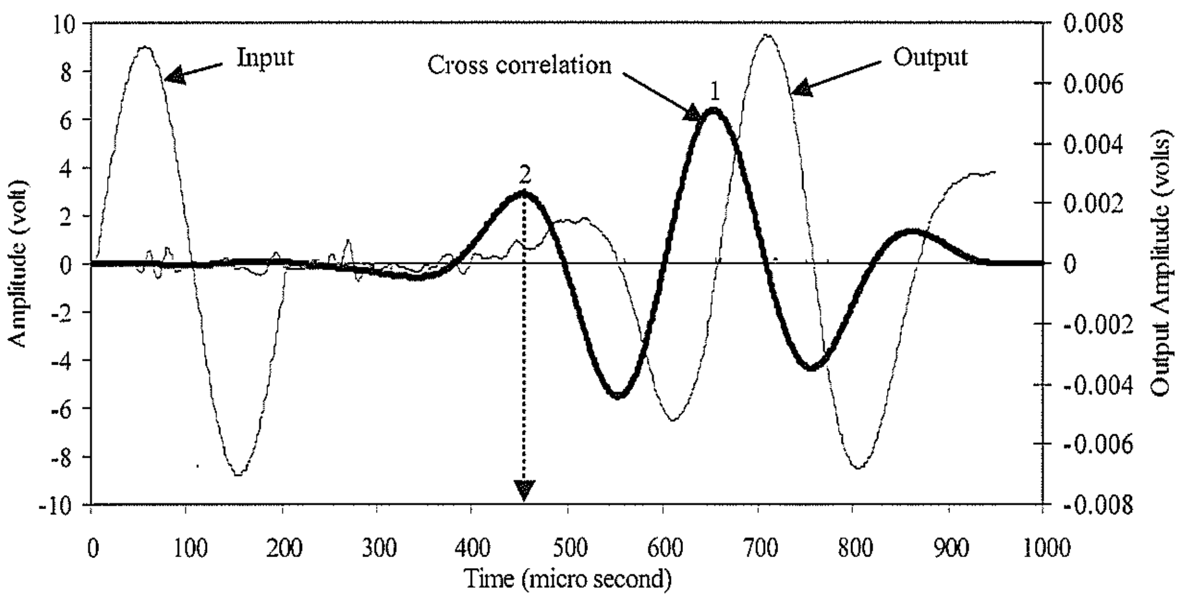

Figure 2. Typical bender element signals for Toyoura sand, p' =100 kPa, L = 102mm.

of the shear wave. In some of the other figures shown later it will be seen that there are many maxima in the cross correlation and the correct time delay can correspond to the third and occasionally fourth maximum preceding the peak. How the correct time delay can be determined is discussed in the next section.

To demonstrate that the low cost arrangement using the sound card and computer A-D card perform satisfactorily a comparison is presented in Figure 3 of the input and received waveforms together with their cross correlation produced by: (a) the function generator and oscilloscope; (b) the function generator and A-D card; and (c) the sound card and A-D card. All three plots are very similar with input and received signals appearing almost identical. The extra noise seen at the start of the output signals of Figures 3b and 3c is related to the extra cabling

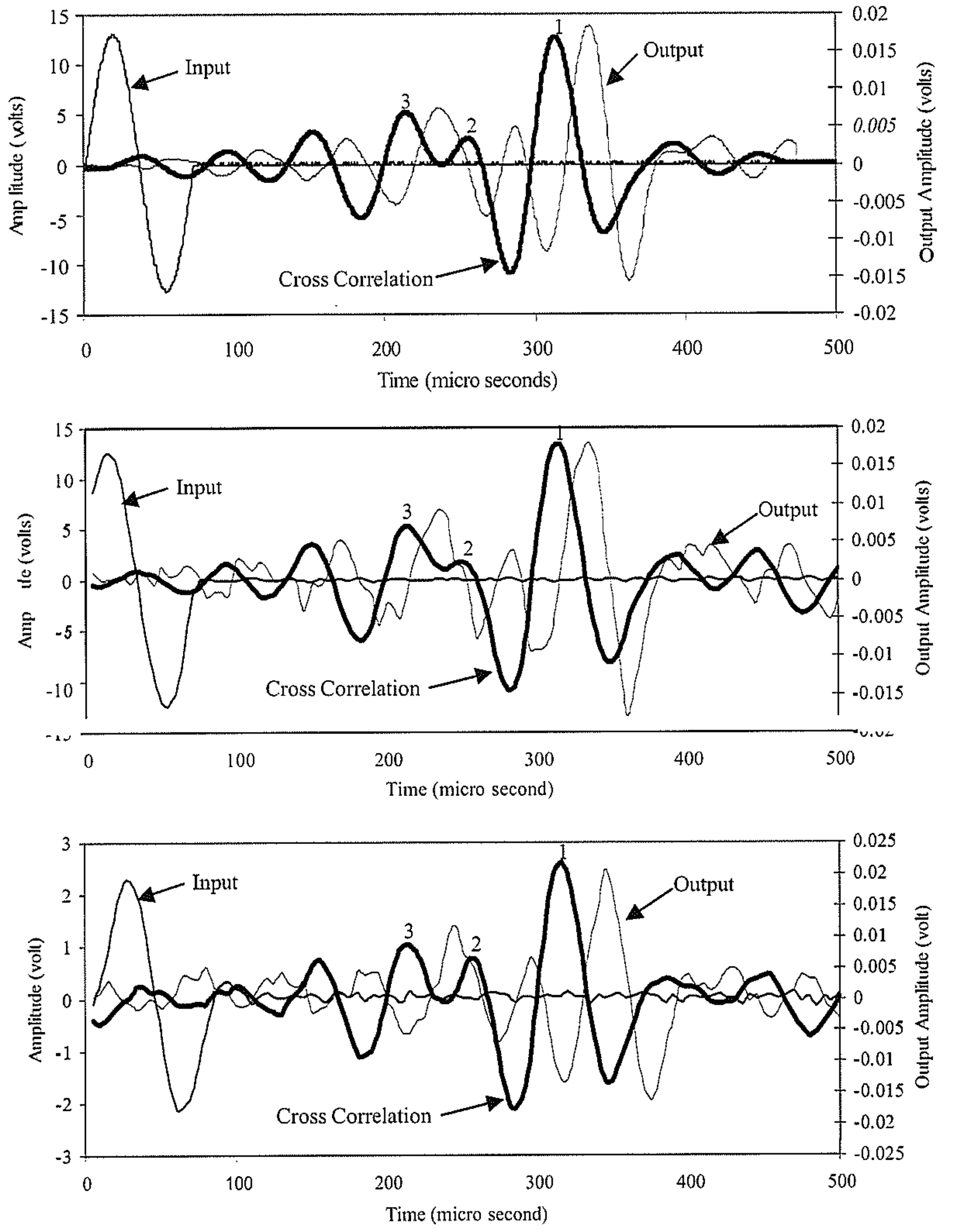

Figure 3. Traces from (a) Function generator/Oscilloscope. (b) Function generator/A-D, (c) sound card/A-D

required in transferring the signals to the A-D. However, the addition of this noise is seen to have no impact on the timing of the correlation maxima, which are given in Table 1. As shown in Figure 3b using the function generator and the A-D, the input waveform has not been fully recorded. The program uses an external trigger from the signal source, in this case the function generator, to initiate the scanning of the channels. It was found that in between the scan being activated by the trigger and the actual scanning of the channel a 10μs delay existed. This delay could affect the timing of the peaks in the correlation. In the case of the sound card the sync output is designed in such a way that it triggers the A-D long enough before the arrival of the input signal thereby overcoming the problem. Although not explored here it is possible to get the function generator to output the sync signal before the start of the pulse in a similar way to the sound card. The output voltage in Figure 3c is shown to be similar to that produced using the function generator, but to achieve this some amplification of the signal was required. For a linear system, such as provided by the soil sample, the output amplitude is directly proportional to the input and the lower output voltage of the sound card, ± 2.25V, compared to ± 10V for the function generator inevitably leads to a lower output signal.

Table 1. Comparison of Times of the Maxima in the Cross-Correlation Signals in Figure 3

Figure No.	Peak (μs)	Maximum 2 (μs)	Maximum 3 (μs)
3a	315	255	214.6
3b	320	255	220
3c	315	255	215

It has been found that the times of the peaks from the cross-correlation signal for the method using sound card and A-D agree very well with those determined from the oscilloscope. The timing of the first three peaks in the correlation are presented in Table 1. The timing of the peak (labeled 1) in the correlation in Figure 3a, downloaded from the oscilloscope, is at 315μs. The sound card/A-D combination, Figure 3c also records a peak in the correlation at a time of 315μs. The preceding maxima, labeled 2 and 3 on the figures were recorded at times of 255μs and 214.6μs for the oscilloscope and at times of 255μs and 215μs using the soundcard. The times of the peaks in Figure 3b were slightly different from those in the other two figures due to the inability of the A-D to fully record the input function generator signal. The input and output signals in figure 3a have been averaged 256 times by the oscilloscope. This compares with figures 3b and 3c where the input signal has not been averaged and the output signal has only been averaged 25 times. This appears to have no effect on the timing of the correlation peaks.

Selection of Correct Time

The major limitation of this cross correlation technique is that the times of several peaks in the cross correlation are recorded, and it can be difficult to determine which is the correct one. Other studies (e.g. Jovicic et al, 1996, Arulnathan et al. 1998) have reported that both the waveform and frequency can influence the identification of the correct time, however most of these studies have been concerned with identifying the first arrival of the shear wave. Mohsin and Airey (2003) have presented data showing the influence of waveform and frequency on the times indicated by the cross correlation method. They showed that the frequency needed to be greater than some critical value that depended on the shear modulus. As the modulus increased so the required frequency increased. In principle higher frequencies should eliminate near field effects and therefore give clearer first arrivals. However, the maximum frequency that can be used is limited by the bender elements, which cannot transmit pulses accurately at high frequencies, and also by increasing attenuation in the soil. For sands and a range of confining stresses between 20 kPa and 2000 kPa, corresponding approximately to shear moduli between 20 MPa and 400 MPa, a maximum frequency of 20kHz is sufficient. It is also observed that as the frequency increases the response becomes noisier and the first arrival becomes increasingly difficult to detect. However, the noisy signal does not significantly affect the timing of the cross correlation peaks. A variety of waveforms were investigated including single sine pulses, sine pulses with two or three waves, a triangular pulse and a chirp comprised of three sine waves of different frequency (Mohsin and Airey, 2003). All the pulsed waveforms gave essentially identical travel times when the frequency was greater than the critical value. The best waveforms were found to be the triangular input wave, which gave the least frequency dependent travel time, and the chirp waveform comprising a packet of sinusoidal waves with frequencies of approximately 20 kHz, 13 kHz and 8 kHz. It was concluded that having a range of frequencies in the input signal was an advantage as this resulted in less variability in the estimated times and avoided the need to adjust the input wave frequency during a test.

It can be seen from Figures 2 and 3 that the maximum in the cross correlation giving the correct time is not necessarily associated with the greatest correlation. In some stress states as the frequency is varied the number of maxima between the maximum giving the correct arrival time and the peak in the correlation can also vary. This creates a difficulty for anyone using cross correlation, because if a single cross correlation is taken it can be difficult to determine which of the peaks is giving the correct answer. This is particularly a problem when testing sand as the first arrival can be difficult to detect. To resolve this problem the signals are correlated at regular intervals so that a continuous variation of the times of all the maxima and hence G_{max} are evaluated. Figure 4 shows a typical result during isotropic compression of Toyura sand. At each confining stress 3 or more points are shown corresponding to the various maxima in the cross correlation signal. The different symbols on the plot refer to the different peaks in the cross correlation. In general it has been found that the lowest monotonically varying value gives the correct arrival time. But, if only a small range of confining stress is considered this may not be reliable. To overcome this uncertainty it is recommended that the correct time at low confining stress be identified by adjusting the frequency until the arrival time can be confidently estimated. As the input frequency is changed it is possible to observe a small range of frequencies where the response signal has at least one peak

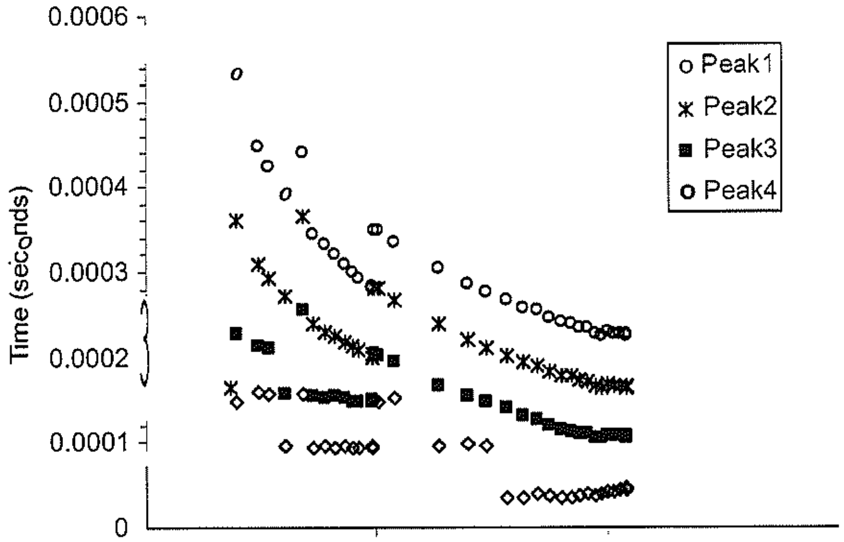

Figure 4. Influence of Confining Stress on the Timing of the Maxima in the Cross Correlation Signal.

-trough cycle at the same frequency as the input. In this "optimum" frequency range the time delay can be determined between corresponding peaks or troughs in the input and output signals. This can then be used to indicate which curve in Figure 4 is the correct one. Only with this continuous variation is it possible to confidently detect the correct peak from the cross correlation data at later points in the test.

Mohsin and Airey (2003) have shown that this method is capable of giving values of G_{max} that agree with values obtained by torsional shear and resonant column tests. As the method gives reasonable results and removes any operator dependence it is believed that the method outlined here is suitable for use in commercial laboratories. The cross correlation technique has been criticised because it is computationally intensive (e.g. Blewett et al. 1999) requiring the capture and analysis of complete waveforms and the use of fast Fourier transforms. However, with modern fast computers the time to capture the data and complete the calculations is not significant, and data storage is minimal as only the times of the peaks in the cross correlation need to be saved.

CONCLUSIONS

An empirical approach has been adopted to determine a methodology for automating the determination of G_{max} during triaxial tests. It has been found that the use of cross correlation between input and output signals can provide a reliable and robust means of estimating the shear wave travel time. However, it is important to realize that the time is obtained from one of the maxima in the cross correlation signal, and not necessarily the peak correlation as would be expected if the signals were simply shifted in time. It has been found that the frequency of the input pulse must be above some critical value that increases with effective confining stress. To avoid having to adjust the frequency during a test a Chirp waveform comprised of sine waves with frequencies of 20, 13 and 8kHz has been found to work satisfactorily for confining stresses from 20kPa to 2MPa for the sands tested.

The major limitation of this technique is that it is difficult to determine which is the correct peak in the cross correlation signal. The data show that the arrival times predicted by several peaks in the cross correlation vary monotonically during the test. To select the correct peak a visual check of the waveforms may be required at some stage of the test.

It has been shown that the bender element technique does not need any special equipment other than a sound card and computer A-D card, which are already present in many laboratories. The integration of the cross correlation into existing data-logging and control programs is relatively straightforward. There is thus no need for costly function generators and oscilloscopes. The technique is relatively cheap to implement and should be within the capabilities of commercial testing laboratories.

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