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The strength of Perth sands in direct shear

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ABSTRACT: This paper describes the results from a systematic study examining the shear strength parameters of two natural dune siliceous sands and two natural calcareous sands from the Perth area, as well as two commercially sourced uniform sands. Standard classification testing was performed including measurements of grading distribution, void ratio limits and shape parameters. A comprehensive series of direct shear tests was carried out on each sand to assess the strength and dilatancy characteristics. These results were used to evaluate the effects of particle shape as well as grading distribution on the peak friction angles.

1 INTRODUCTION

Knowledge of the shearing characteristics of granular soils at different stress level and densities is required to correctly predict the soil strength for different geotechnical applications. For example, for analysis of offshore pipeline stability and near-surface slip failures, knowledge of friction angles at low stresses is of great importance, since the peak friction angle φ'_p is usually significantly in excess of the critical state friction angle φ'_{cs}.

One of the most convenient descriptions of the strength of sands is the approach described by Bolton (1986). Following the early work on strength and dilatancy of Taylor and Rowe in the 1960s, Bolton observed the variation of (φ'_p - φ'_{cs}) with stress level and relative density D_r of granular soils. With extensive data, Bolton suggested a relative dilatancy index I_R. With this index, Bolton was able to fit the data of (φ'_p - φ'_{cs}) within a typical margin of error of about 2°. It was noted, however, that there is no one-to-one correspondence between φ'_p and D_r, even at a given stress level. A particular definition of a relative dilatancy index must therefore be entirely empirical, and a definition of (φ'_p - φ'_{cs}) is given as:

$$\varphi'_p - \varphi'_{cs} = AI_R = A\left[D_r\left(Q - \ln p'\right) - 1\right] \tag{1}$$

where p' = mean effective stress (expressed in kPa); A is an empirical coefficient with a suggested value of 3 for triaxial strain and Q is an empirical coefficient related to the effective stress required to ensure zero dilation (since zero dilation leads to no peak friction angle, and hence I_R = 0).

The calculated φ'_p values are highly dependent on the values of the A and Q coefficients, and yet, there is little published guidance to assist designers in the selection of appropriate values. It is, however, well understood that reducing the crushing strength of the grains reduces the effective stress at which dilatancy is suppressed, and hence reduces Q (e.g. Billam, 1972). Furthermore, attempts have been made to predict A – e.g. Liu & Lehane (2012) who found a correlation with the average particle shape and supports the

Table 1. Index properties of sands investigated.

Sand	Gradation		Void ratio limits		Particle shape			G_s	φ'_{cs}	Mineralogy
	d_{50}: mm	C_u	e_{max}	e_{min}	R	S	ρ			
UWA Superfine	0.18	1.74	0.78	0.49	0.46	0.76	0.61	2.66	31.0	Siliceous
UWA Coarse	0.82	1.76	0.71	0.48	0.58	0.73	0.66	2.66	31.0	Siliceous
Bassendean	0.33	1.92	0.74	0.44	0.53	0.75	0.64	2.65	31.4	Siliceous
Shenton Park	0.33	3.09	0.84	0.41	0.43	0.76	0.60	2.63	32.2	Siliceous
Kwinana 1	0.64	2.88	1.16	0.70	0.42	0.63	0.53	2.67	39.0	Calcareous
Kwinana 2	0.25	2.14	1.41	0.89	0.14	0.57	0.36	2.68	35.0	Calcareous

trends observed by Santamarina & Cho (2004), Cho et al. (2006) and others.

Several researchers have shown that Equation 1 can provide a reasonable description of direct shear data for sand (e.g. Simoni and Houlsby, 2006). However, in order to use this expression, the vertical effective stress σ'_v is used in place of p' in Equation 1, giving:

$$\varphi'_{p,ds} - \varphi'_{cs} = AI_R = A\left[D_r\left(Q - \ln \sigma'_v\right) - 1\right] \tag{2}$$

where $\varphi'_{p,ds}$ = peak friction angle in direct shear.

Given the uncertainties surrounding application of Equations 1 and 2, a systematic study examining the effects of particle shape, gradation and mineralogy on the A and the Q coefficients in direct shear was undertaken. Six different sands were tested, including two commercially sourced silica sands (UWA Superfine and UWA Coarse), two natural dune siliceous sands (Bassendean and Shenton Park) and two natural calcareous sands (Kwinana 1 and Kwinana 2). All four natural sands are retrieved from the Perth area.

2 CLASSIFICATION PROPERTIES OF SANDS INVESTIGATED

2.1 *Basic indices*

The basic classification indices of the six sands investigated are summarised in Table 1. All sands have very different particle shapes, but a moderately small range of uniformity coefficient, $C_u = d_{60}/d_{10}$, of 1.7 to 3.1 (where d_{60} and d_{10} are the mesh sizes through which 60 percent and 10 percent of the sand particles pass, respectively). The two UWA sands are commercially sourced silica sands with a distinctly different mean effective particle size d_{50}, and are the standard sands used in The University of Western Australia geotechnical centrifuges. Bassendean and Shenton Park are natural dune silica sands with similar d_{50}, but with the Bassendean sand having a wider grading variation. Kwinana 1 and Kwinana 2 are natural calcareous sands with different d_{50} values. These six sands altogether represent a good variation in gradation. The void ratio limits were determined following Australian Standard (Standards Australia, 1998).

2.2 *Particle shape quantification*

Typically, the particle shape is quantified by the roundness R (reflecting the average radius of curvature of the surface features), the sphericity S (reflecting similarity between a particle's length, width and height) and regularity ρ (being the average of roundness and sphericity, $\rho = (R + S) / 2$).

Wadell (1932) defined R as the average radius of curvature of surface features relative to the radius of the maximum sphere that can be inscribed in the particle, and S was quantified by Powers (1953) as the

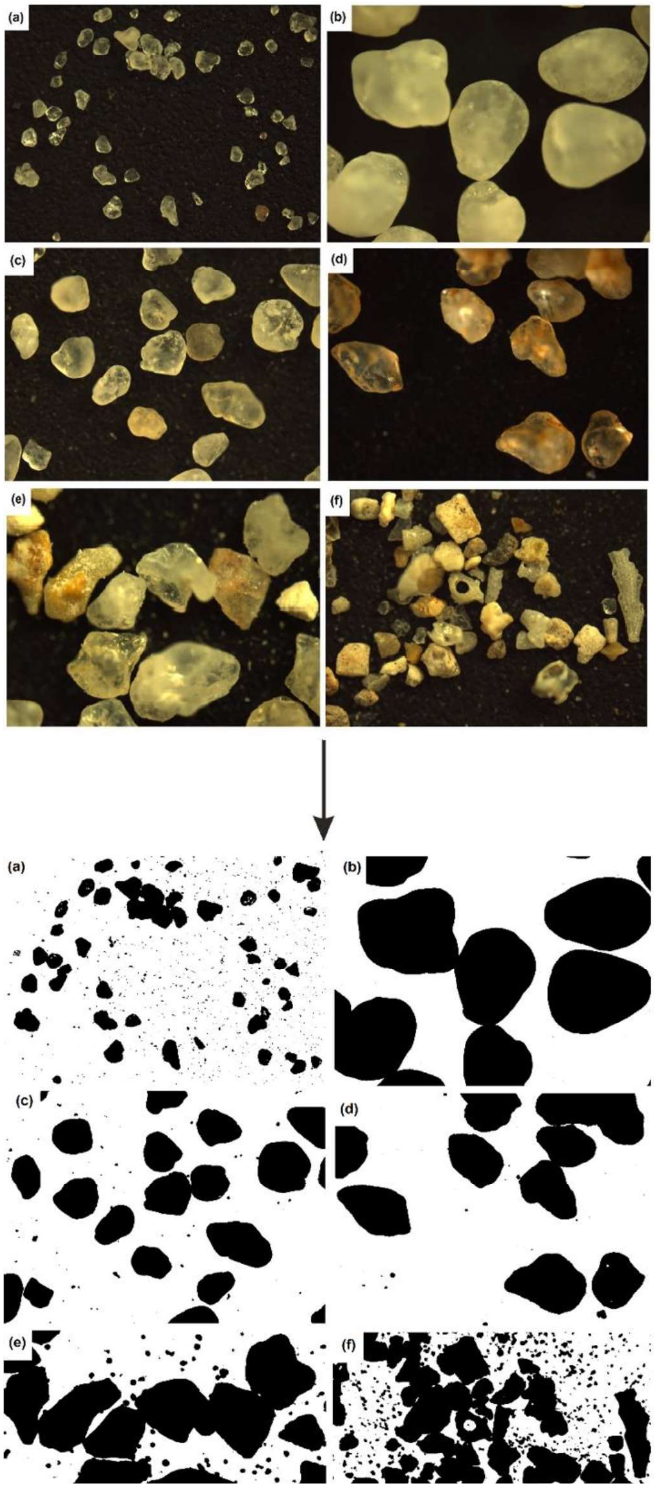

Figure 1. Conversion of light microscope images of typical grains to binary images: (a) UWA Superfine; (b) UWA Coarse; (c) Bassendean; (d) Shenton Park; (e) Kwinana 1; (f) Kwinana 2.

diameter ratio between the largest inscribed sphere and the smallest circumscribing sphere.

For the purpose of this paper, R and S (and hence also ρ) were determined as the average values of a visual assessment from light microscope images of at least 75 grains of each sand. The R values were determined with a chart presented by Krumbein (1941) and the S values from Krumbein (1963). In order to enhance the quality of the assessment of the grains, the light microscope images were converted to binary images with MATLAB. Light microscope images of typical grains of each of the sands and the conversion to binary images are shown in Figure 1. Note that the small dots in some of the binary images are caused by the background, and hence are not small sand particles. All estimated particle shape parameters are listed in Table 1.

On inspection, it was evident that the Kwinana 2 sand mainly comprises very angular particles, once living marine organisms, for which the R value was extremely difficult to quantify. A mean value for very angular particles of R = 0.14 suggested by Powers (1953) was therefore assumed for the Kwinana 2 sand. This value of R is significantly lower than the values of the other five sands. However, the very low R value is in accordance with the qualitative assessment of the particle shapes.

3 DIRECT-SHEAR TESTS

3.1 *Testing apparatus*

The shear box apparatus used in this study is a relatively basic device comprising a 63.5 mm diameter shear ring and sample depth of 25 mm. The upper box is restrained against horizontal movement, while the shear load is applied to the lower box. A dead weight loading system (including a 10:1 leverage) is utilized for applying the vertical load to the sample. The vertical and horizontal displacement is measured with dial indicators.

To examine the shear box size effect on the test results, results are compared to one test performed with a more sophisticated and larger shear box apparatus (sample measuring 71 mm in diameter and approximately 46 mm in height). UWA Coarse sand was used for this comparison, and the sample was prepared at D_r = 0.6 and tested at a vertical effective stress σ'_v = 192 kPa. The test performed in the larger shear box is shown in Figure 2 together with three tests performed at the same σ'_v and prepared at D_r values of 0.5, 0.7 and 0.9. The maximum shear stress of the tests performed at D_r values of 0.7 and 0.9 are seen to be higher than the one obtained with the larger shear box apparatus at D_r = 0.6, and the maximum shear stress of the test performed at D_r = 0.5 is seen to be lower. These observations confirm the suitability of the shear box apparatus employed.

3.2 *Experimental procedure and interpretation*

Direct shear tests were performed on all six sands, prepared at three different D_r values and with five different vertical stresses (12, 24, 72, 192 and 479 kPa).

The samples were tested in dry conditions. The

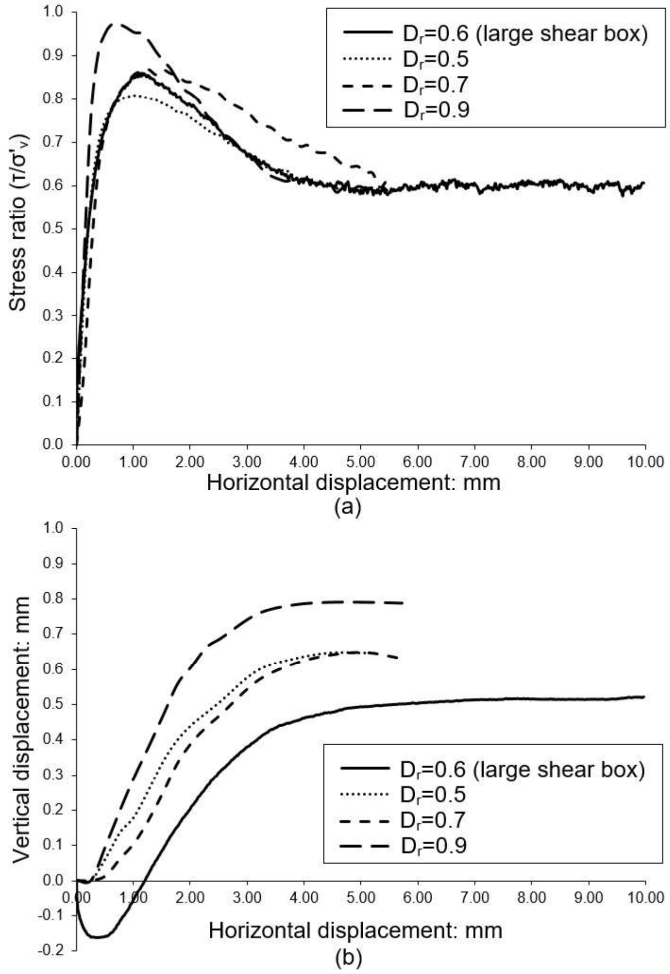

Figure 2. Comparison between direct shear machine used for this study and a larger shear box device tested at σ'_v = 192 kPa. Variation with horizontal displacement of: (a) stress ratio; (b) vertical displacement.

loose/medium dense samples were prepared by pouring the dry sand carefully into the box with a teaspoon, while the dense samples were vibrated to obtain the required densities. The density was monitored by measurement of sample height and weight. UWA Superfine and UWA Coarse were prepared at D_r values of 0.5, 0.7 and 0.9, and the four natural sands were prepared at three different D_r values ranging from 0.4 to 0.9.

$\varphi'_{p,ds}$ is derived as the arctangent of the maximum stress ratio i.e. $\varphi'_{p,ds} = \tan^{-1}(\tau_p/\sigma'_v)$ (with τ_p being the maximum shear stress). φ'_{cs} is derived as the arctangent of the stress ratio where the shear stress reaches a steady value i.e. $\varphi'_{cs} = \tan^{-1}(\tau_{cs}/\sigma'_v)$ (with τ_{cs} being the steady state shear stress).

3.3 *Direct shear results*

Initial tests with the direct shear testing device indicated that it did not provide repeatable and reliable data at shearing displacements in excess of 5mm; sample shearing was therefore generally terminated at a displacement of 5 mm. As a consequence, measurement of the critical state friction angle was not always achievable. Therefore, for consistency, it was decided

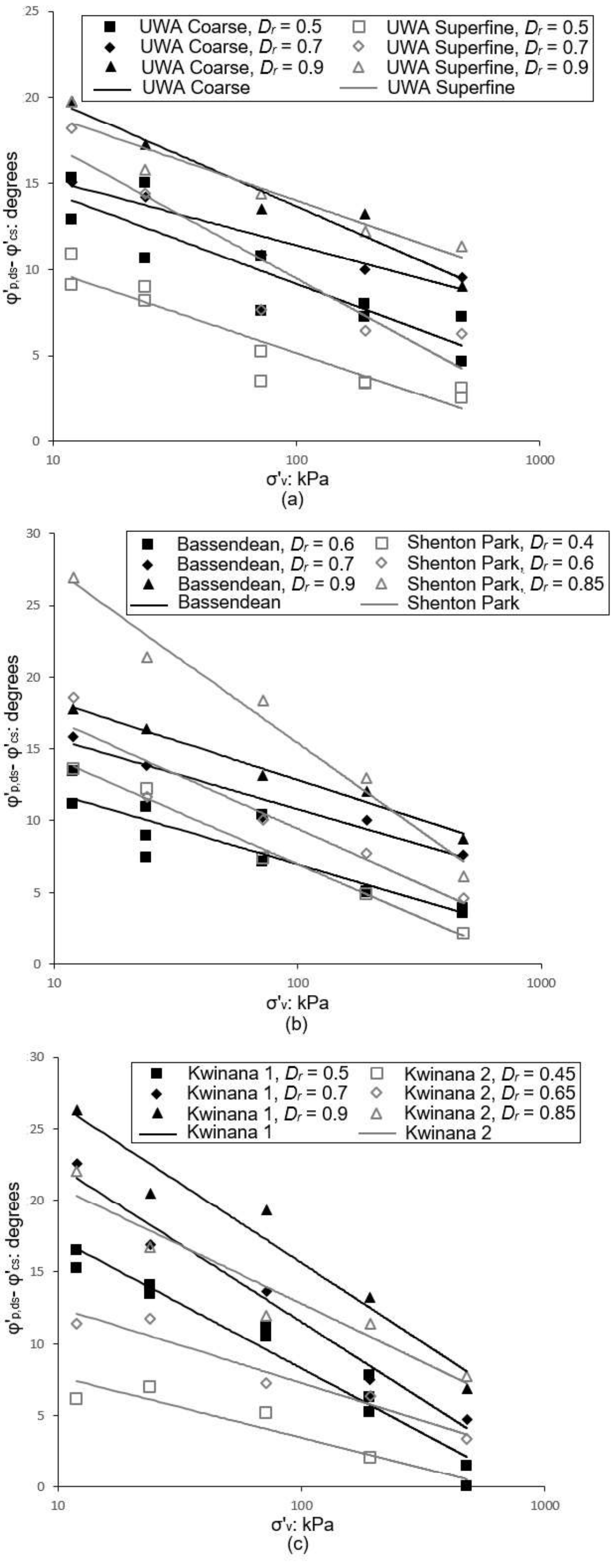

Figure 3. ($\varphi'_{p,ds}$ - φ'_{cs}) variation with stress level and D_r of (a) UWA Superfine and UWA Coarse; (b) Bassendean and Shenton Park; and (c) Kwinana 1 and Kwinana 2.

to estimate φ'_{cs} at the same stress level for all six sands (with three different D_r values for each sand). The chosen stress level was σ'_v = 192 kPa, since second order frictional effects should be more or less mitigated at a high stress level, and since this stress level generally gave the most consistent φ'_{cs} values. The estimations of φ'_{cs} are listed in Table 1.

Figure 3 shows the variations of ($\varphi'_{p,ds}$ - φ'_{cs}) for all the six sands. As suggested by Bolton (1986), it is evident that ($\varphi'_{p,ds}$ - φ'_{cs}) varies approximately linearly with the logarithm of σ'_v at a fixed D_r value.

However, when considering Equations 1 and 2, and keeping in mind Bolton's (1986) suggested Q value of 10 for silica sands, it may be expected that dilation will be suppressed, with ($\varphi'_{p,ds}$ - φ'_{cs})=0, at a high stress level when the sands are prepared at a loose/medium dense state. That this does not hold true for any of the sands tested is most likely reflecting the problem with the use of D_r as an index of sand behaviour. D_r is conventionally computed using the void ratio of the sample as it has been prepared, with no confining pressure. Consequently, it does not reflect the changes in void ratio that may occur when an initial stress state has been applied. This issue has been discussed previously by several researchers (e.g. Hamidi *et al.* 2013).

4 EFFECTS OF SOIL CLASSIFICATION PROPERTIES ON Q AND A

4.1 *Q and A quantification*

The Q coefficient for direct shear is estimated from the vertical effective stress at which dilatant behaviour is suppressed, and hence ($\varphi'_{p,ds}$ - φ'_{cs}) equal to zero. Q may therefore be calculated from Equation 2 as:

$$\ln \sigma'_{v,crit} = Q - \frac{1}{D_r} \tag{3}$$

where $\sigma'_{v,crit}$ = the vertical effective stress at which dilatant behaviour is suppressed. $\sigma'_{v,crit}$ values determined from extrapolation of the linear relationships indicated in Figure 3 for the sands prepared at D_r values of 0.85-0.9 (the dense samples) are used in Equation 3 to calculate the Q values presented in Table 2.

With the Q coefficients determined, a best-fit value of the A coefficient in Equation 2 was derived for each sand as the average value of the A coefficients from all the medium dense and dense samples; see Table 2.

Table 2. Q and A coefficients.

Sand	Q	A
UWA Superfine	12.3	2.27
UWA Coarse	10.8	3.21
Bassendean	11.1	2.84
Shenton Park	8.7	6.44
Kwinana 1	8.9	5.17
Kwinana 2	9.4	3.81

4.2 *Effects on the Q coefficient*

Billam's (1972) triaxial test data of initially dense samples of granulated chalk, crushed anthracite and limestone showed that reducing the crushing strength of the grains reduces the confining stress at which a transition from dilatant to contractive behaviour occurs and hence reduces the Q coefficient.

The crushability increases with increasing particle size (or d_{50}) and with greater angularity of soil particles (with lower ρ); see Nakata *et al.* (2001). Carbonate particles are also more crushable than silica particles. Based on these characteristics and the expectation that more grain crushing may be expected in more well graded sands (with high C_u), an index to reflect effects of ρ, d_{50} and C_u on Q was determined based on a regression of the data. This showed, as seen on Figure 4, that Q reduces systematically with a new coefficient, I_{sc} defined as: $C_u/\rho^{0.5}+1.9^{d_{50}}$, where d_{50} is expressed in mm. More suitable forms for I_{sc} can be investigated thoroughly and refined using data from other sands. The data for the 6 sands investigated here indicate a best-fit expression:

$$Q = 19 - 2I_{sc} \tag{4a}$$

$$I_{sc} = C_u / \rho^{0.5} + 1.9^{d_{50}} \tag{4b}$$

with d_{50} expressed in mm.

4.3 *Effects on the A coefficient*

The A coefficient increases systematically with I_{sc}, as seen on Figure 5 (the variations of the A coefficients are indicated with error bars). The data for the 6 sands investigated here indicate that a best-fit expression:

$$A = 1.5(I_{sc} - 2) \tag{5}$$

As for Equation 4, this equation can be refined when a large database of sands has been examined.

The increasing value of A with a reducing particle regularity (ρ) is in good agreement with a study performed by Liu and Lehane (2012). Liu and Lehane showed with a series of direct shear tests on four different granular materials with distinctly different particle shapes that a granular material at a given relative density and stress level is affected significantly by particle shape, i.e. a decreasing ρ leads to an increasing peak friction angle.

The increase of the A coefficient with decreasing C_u indicates that a uniform sand is expected to have a significantly smaller peak friction angle at low stress levels in comparison to a well graded sand. This characteristic is consistent with numerous studies, such as that of Igwe *et al.* (2007). Igwe et al. explained this behaviour to be a result of greater particle-to-particle contacts occasioned by the combination of a wide range of particle sizes.

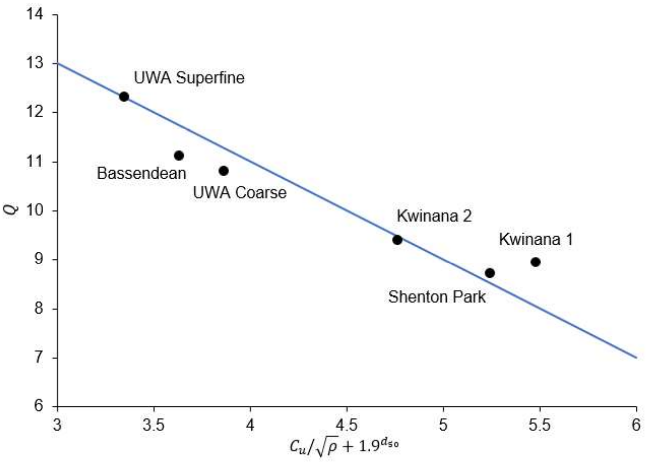

Figure 4. Q coefficient variation with $C_u/\sqrt{\rho} + 1.9^{d_{50}}$.

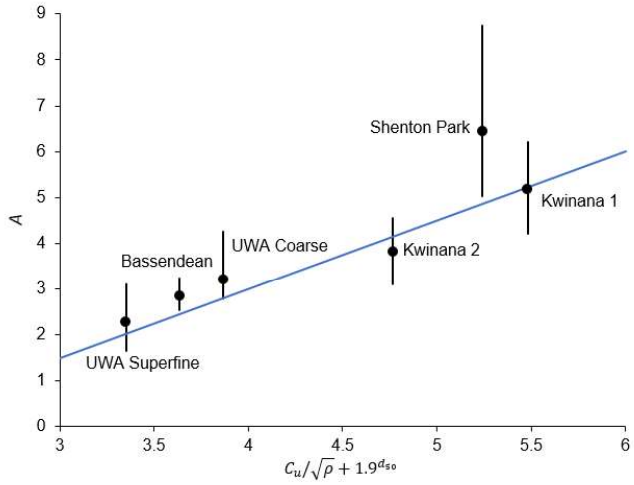

Figure 5. A coefficient variation with $C_u/\sqrt{\rho} + 1.9^{d_{50}}$.

4.4 *Correlation between Q and A*

Since both Q and A are strongly influenced by the same parameters for the test data presented in this study, these two coefficients are unsurprisingly also strongly correlated as seen in Figure 6. It appears that a decreasing Q coefficient leads to a considerable increase in the A coefficient, especially at Q values smaller than 10.

The A coefficients in this this study range from about 2.3 to 6.4 and the Q coefficient ranges from 8.7 to 12.3. It is therefore clear that taking A equal to 3 as suggested by Bolton (1986) for triaxial strain could lead to a significant underestimation of the strength of sand at low stress levels in direct shear.

The coefficient of variation of the product of the best estimate A and Q values for each sand (on which ($\varphi'_{p,ds}$ - φ'_{cs}) depends) was calculated as 0.27. This relatively low value indicates that compensating errors are the reason why reasonable estimates of peak friction angle are often obtained using Bolton's expression with constant A and Q values (Equation 2).

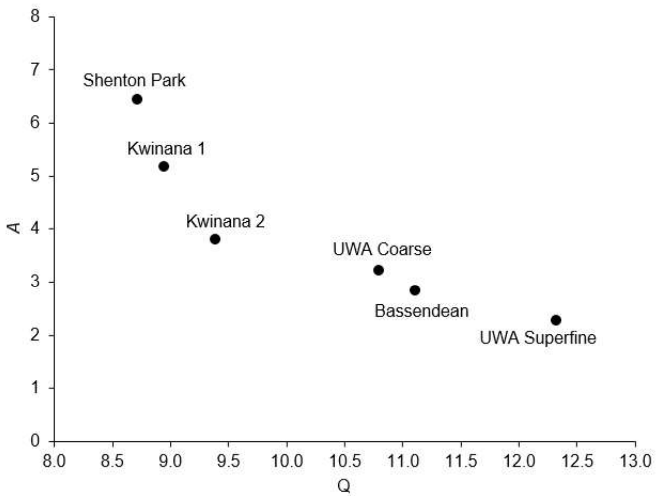

Figure 6. A coefficient variation with Q coefficient.

5 CONCLUSIONS

A comprehensive series of direct shear tests on six different sands with different particle shapes, gradation and mineralogy has shown:

(a) The Q coefficient in Equation 2, which is commonly considered to be a function of mineralogy only, reduces with increasing C_u, decreasing ρ and increasing d_{50}, i.e. higher effective stress is required to ensure zero dilation for uniformly distributed sand with small and rounded particles.
(b) The A coefficient in Equation 2 increases with increasing C_u, decreasing ρ and increasing d_{50}, i.e. higher levels of dilation are required by sands with large and irregular particles and a high value of C_u to accommodate shearing on the horizontal plane in the shear box.
(c) The Q coefficient increases systematically with a decreasing A coefficient, i.e. uniformly distributed sands with small rounded particles have lower values of ($\varphi'_{p,ds}$ - φ'_{cs}) at low stresses but require larger stress levels in order to suppress dilation.

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