13th Australia New Zealand Conference on Geomechanics – Acosta-Martínez & Lehane (Eds.)
© 2019 Australian Geomechanics Society, Sydney, Australia, ISBN 978-0-9946261-0-3

Experimental analysis of rock stiffness degradation due to the cyclic loading

M. Ghamgosar
Aurecon Group, Brisbane, Australia

ABSTRACT: As a critical aspect of rock mechanics for geotechnical and underground design, distributed pattern of initial and induced fractures plays a significant role by influencing on the rock mass stability and failure states. Failure modes and fracture mechanism are important factors to model the mechanical behaviour of rock materials under the different loading circumstances. Although the failure mechanism of the brittle material has been widely studied for decades, but heterogeneous and complex nature of the rock makes fracture analysis to be more complicated. Understanding of microfracture patterns and coalescence to macrofractures are very important for a vast variety of engineering applications involved in rock mechanics. The damage model was used in this research that comprehensively addresses degradation of rock stiffness. A series of uniaxial compression, Brazilian indirect tensile and fracture toughness tests were performed on monzonite and tuff samples according to the International Society for Rock Mechanics (ISRM) suggested method, aiming to determine stiffness degradation model based on the post-failure behaviour during the damaging progress. Brazilian test results revealed that plastic damage occurs in 35% of Ultimate Tensile Strength (UTS) to 70% UTS. In addition, findings indicated a significant decrease in monzonite stiffness over the determined plastic damage ranges under the cyclic loading test compared to the static failures. The methodological approach in this study provided a strong approach to investigate brittle damage states in rock, which induced under the cyclic and static loading tests and considered microfracture characteristics.

1 INTRODUCTION

1.1 *Microfracturing mechanics principals*

Rocks as a brittle material have different responses to various applied loading boundary conditions. The behavior of rocks under the cyclic loading and prediction of fracture propagation becomes critical for rock cutting industries. The conceptual models based on energy release and dissipation have been used to study a variety of brittle materials like rocks, which provides a constitutive relationship between the mechanical properties and external forces (Bazant and Oh 1985). The energy terms can be considered for the entire cracks in a rock body or a finite extension of the existing crack. In practical engineering applications, it can be an actual advantage to describe the failure condition based on the applied load, cross-sections of the new crack surface, plastic deformation (fracture process zone (FPZ)) and elastic compliance. Although axial load control tests make the loading machine and testing setup more flexible, displacement-an based control tests can be used to obtain detailed information about the post-failure behaviour in cyclic and static tests (Eberhardt 1998).

Tomographic analyzing of cracked chevron notched Brazilian disc (CCNBD) specimens revealed that there is a strong amount of microfractures were made on the second loading mode and damage evolution rate was significantly higher for cyclic loading which exposed for zero stress levels in its loading history (Ghamgosar, Erarslan, and Williams 2014).

A series of cyclic loading was carried out on brittle and monzonite rocks and laboratory investigations with two different types of cyclic loading first with keeping load levels in increasing mode for all loading steps and in the other mode, loading level returned to zero in every 100 cycles. The results demonstrated that there is a significant difference between these two types of loading related to the energy-releasing and absorbing modes in the same specimens.

1.2 *Brittle damage mechanics and energy balance theory*

Rock fracture studying is the fundamental aspect of rock engineering for different design practices, which is mainly studying the relationship between rock deformation and strength in externally applied loading. Violent and quick fracture coalescence results in rock degradation of strength which eventually leads to failure in large-scale mining or civil underground or on-the-ground constructions. Furthermore, investigations of rock fracture mechanics will allow a better understanding of rock cutting technologies. In modern rock cutting system, different combinations of loading applied to rock mass to achieve more efficiency in rock breakage. Postulating that hard rocks are brittle, so linear elastic is used for evaluation rock fracture propagation on a small scale; however, for practical purposes, rock engineers focused on the macro scales and rock mass under the different state of loading and deformation conditions. It is known that the rock fracture process is not simple phenomena, and therefore; propagation and coalescence of cracks from micro-fracture scales to macro-fractures stages could not be easily modelled by linear theoretical basis. Consequently, linear models could not predict the problems of crack propagation under the cyclic loading, considering pre-existing flaws or crack bifurcation and branching modes. It has been proved that the fracture process zone in cyclic and static loading is completely different, and mechanism of crack propagations is related to the energy absorption or stiffness degradation. In practical applications and even in standard literature, it is often useful to consider external energy which is the specific area of rock could tolerate until the complete damage is occurring in rock specimen. Well-known rock failure criterion mostly is used for rock fracture process; however, they cannot be expected to deal with the microfracture modelling and subcritical fractures for different loading modes. It is almost acceptable that rocks are weak in tensile loading and tensile strength of rocks is generally much lower than compression strength; however, tensile fractures could be generated in both compression and tensile loading. One of the fundamental aspects of rock fracture mechanics is its ability to establish the connection between the failure of rock and fracture toughness parameter. Moreover, different research and standardized methods have been developed to evaluate fracture toughness parameters as a principal parameter for rock breaking studying. Crack propagation is still one of the complicated phenomena which involving various conditions of initiation, coalescence and subcritical crack growing which lead to instability of cracks developing in a rock matrix. Cracks can be classified into three classes — micro, meso and macro fractures (Atkinson and Avdis 1980). The failure process in brittle rocks usually takes a very short time, whereas in microfracture process when the stress level reaches to the failure criterion, different micro-structures at local failure appears as micro-deformations and then cracks coalescence lead to ultimate failure (Hillerborg, Modéer, and Petersson 1976). By considering heterogeneity, anisotropy and discontinuities of rock masses, failure analyzing could be more challenging in rock fracture research. Furthermore, rock cracks and fractures are not subjected to the specific type of loading and will vary for various loading conditions. To describe the inelastic behavior of rock material in the fracture process zone, constitutive modelling could be used for describing fracture propagation in pure mode *I* which is achievable by having the crack opening distance and tensile stress in rock. With regard to the emerging microfractures in rock matrix depend on the cyclic loading, continuum damage mechanics is considered to be most appropriate tools for studying rock fracture propagation (Hillerborg, Modéer, and Petersson 1976). It is also seen that describing and even implementation of microfractures effects in the theoretical or numerical analysis is an often a difficult task. Therefore, main objectives of this research mainly focused on: (1) evaluate microfracture propagation through the different modes of cyclic loading with fixed mean level of frequency; (2) investigate effect of applying continues load returning to zero level after 100 cycles and without backing load to zero level to compare levels of damages occurs in rock; (3) compare numerical finite element (FEM) based on the conjugated brittle damage model with laboratory results.

1.3 *Brittle damage mechanics and energy balance theory*

It is generally accepted that rocks are quasi-brittle material, thus in the bases of the continuum damage mechanics, the evolution of existing flaws and damages of brittle material under the different types of loading represents the variations of microstructures and microfractures in the material (Atkinson and Avdis 1980). For brittle and elastic materials, basic state variables are second order tensors $[\mathrm{D}]$ and $[\Psi]$are used and defined as damage and continuity tensors, respectively. Consequently, the inelastic behavior of macrofractures and deformation in rock should depend on the damage and continuity tensors. In a brittle material, free energy density function (Π) is represented by thermodynamics state variables $([\epsilon], T, [\Psi])$ as the following:

$$\Pi = \Pi([\epsilon], T, [\Psi]) \quad (1)$$

where T is the absolute temperature, and $[\epsilon]$ is strain tensor of brittle materials. Considering that $[\sigma]$ is the Cauchy stress tensor, the equation of deformation and stress could be shown as follows:

$$[\sigma] = \frac{\partial \Pi}{\partial [\epsilon]} \quad (2)$$

Damage strain energy rate is defined as;

$$[D] = \frac{\partial \Pi}{\partial [\epsilon]} \quad (3)$$

In the case of the isothermal condition of rock elements free energy, Π', for static loading can be represented by following:

$$\Pi' = \frac{1}{2}[\epsilon]^T[D'][\epsilon] \quad (4)$$

where $[D'] = [D'([\Psi])]$ is the 3×3 matrix for isan otropic and 6×6 matrix for anisotropic materials, which represents damage state of rock in terms of the effective elastic tensor. Thus, having equation (2), the general form of effective damage matric for linear elastic rocks should be shown in Hook law as:

$$\{\sigma\} = [D'][\epsilon] \quad (5)$$

where the effective damage matrix $[D']$is a symmetrical matric. Generally, complementary form of free energy can be shown as:

$$\Pi' = \frac{1}{2}[\sigma]^T[D'][\sigma] \quad (6)$$

where is called stress energy density function for brittle damage. The most fundamental parameter for damage evolution is the rate of dissipating load which can be shown as the following:

$$[F] = \frac{1}{2}[\epsilon]^T \frac{\partial [D']}{\partial [D]} [\epsilon] \quad (7)$$

where by considering the elastic law for brittle rocks, once can represent dissipation load in terms of the stress tensor as:

$$[F] = \frac{1}{2}[\sigma]^T \frac{\partial [D']}{\partial [D]} [\sigma] \quad (8)$$

Therefore, by substituting the dissipating loads in Hook equation the following general form of damage load could be derived;

$$[F] = \frac{1}{2}[\epsilon]^T \left(\frac{\partial [D']^T}{\partial [D]} [D][D'] + [D']^T [\sigma] \frac{\partial [D']}{\partial [D]} \right) [\epsilon] \quad (9)$$

where $[D]$ is the damage continuum tensor that can be rewritten for isotropic rocks based on the Cauchy stress vector as follows:

$$[D'] = \begin{bmatrix} 1-D_1 & & & & & & & & \\ & 1-D_2 & & & & & & & \\ & & 1-D_3 & & & & & & \\ & & & \frac{1-D_3}{2} & \frac{1-D_2}{2} & & & & \\ & & & & & \frac{1-D_1}{2} & \frac{1-D_3}{2} & & \\ & & & & & & & \frac{1-D_2}{2} & \frac{1-D_1}{2} \end{bmatrix}$$

The principles of plasticity also provide confirmatory evidence to describe the crack condition at the FPZ for the onset of the development of a fatigue crack. Atkinson (Atkinson and Avdis 1980) showed that substantial stress develops at the FPZ exceeding the hardness of the material, and new cracks form during the unloading, which could be sufficient to cause an extension of the crack length. Some researchers observed severe local fatigue behavior in the FPZ of weaker material such as cement and that consequently much denser microfractures formed under cyclic and fatigue loading than in the statically loaded specimens (Schmidt 1980; Germanovich et al. 1997; Labuz, Shah, and Dowding 1985).

Direct observation of Brazilian tensile tests showed the fatigue cracks can develop in specimens at a ratio of the maximum peak fatigue stress to the UCS value of approximately between 0.6-0.8 (Bagde and Petroš 2005). It has been determined that the first visible cracks did not appear until the applied load reached 90% of the ultimate failure load for Berea sandstone and Tennessee marble (Bagde and Petroš 2005; Bazant and Oh 1985).

2 EXPERIMENTAL MODEL SET UP

2.1 *Sample preparation*

Tuff samples were the dominant rock formation in Brisbane's first motorway tunnel, CLEM7; with typically 97MPa of uniaxial compression strength (UCS) (Erarslan and Williams 2012), while monzonite UCS has been varied from 81Mpa to 271MPa from Cadia mining site in NSW (Ghamgosar and Erarslan 2014). Indirect Brazilian tensile test was conducted to determine the tensile strength of selected samples and results showed that tensile strength of tuff was 8MPa in average; however, monzonite showed variable tensile strength ranges changed from 7Mpa to 17.78MPa. Cracked chevron notched Brazilian disc (CCNBD) specimens were also prepared according to the international society of rock mechanics suggested method (ISRM) to evaluate the effect of cyclic loading on tensile failures for tested samples. Technically, the CCNBD method is carried out to determine Mode *I* fracture toughness of rocks (Fowell 1995) and has advantages over the other ISRM or ASTM methods in terms of the simplicity of sample preparation and the reduced material required (Erarslan and Williams 2012). Another advantage of CCNBD method is having the high precision and resulting from a higher level of load capacity achieving by having a sharp chevron notched tip instead of embedded straight through notch cracks (Ghamgosar, Erarslan, and Williams 2017). A fully-digital cutting machine with a circular 40 mm diamond saw was used to prepare notched cracks according to the ISRM (Ghamgosar, Erarslan, and Williams 2017). All geometrical dimensions were selected with respect to the specification radius and diameter suggested by ISRM (Fowell et al. 1995; Fowell 1995). In order to achieve valid results, there are two important dimensions; that is, notched crack length and the ratio of specimen thickness to diameter, which must fall within the range outlined in ISRM (Fowell 1995). In this study, the thickness of monzonite rock, was 22.3mm, inner chevron crack notch half-length, was 5.8mm, outer chevron notched crack half-length, was selected 15.6mm (Fig. 1c). The diameter of the tuff CCNBD specimen was different thus; geometrical dimensions for tuff specimen were: $B = 25mm$, $\alpha_0 = 8mm$ and $\alpha_1 = 18mm$. An Instron loading machine with a capacity of ±100kN was used for static and cyclic loading. Instron 2670-132 series crack mouth opening displacement (CMOD) gauge with the optimum length of 4mm travel was used to measure crack moth displacement (Fig. 1b).

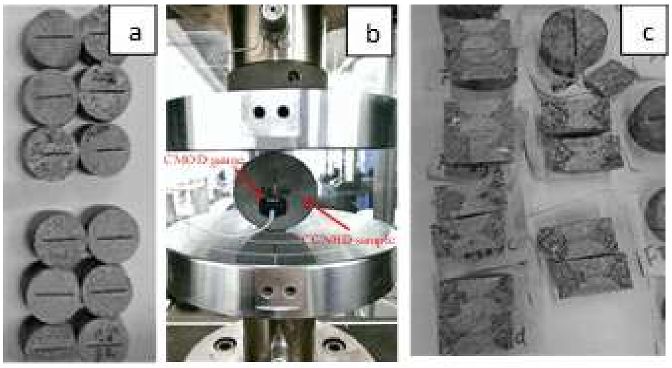


Figure 1. Cyclic loading set up in fracture toughness test; (a) monzonite, (b) test setup, (c) tuff samples

The CMOD gauge was set up and mounted on CCNBD specimen according to ASTM 399 70T.

In diametrical loading tests (CCNDB and Brazilian tests), cracks propagations develop in a direction transverse to the maximum tensile stress, and by increasing cyclic amplitudes, the available load-carrying area reduced, which in return the effective stress level increases at FPZ. Fracture toughness values (CCNBD) were calculated and summarized in Table1.

Table 1. CCNBD fracture toughness values under applied loads

Replication	K_1 $MPa\sqrt{m}$	FPZ (mm)		
Monzonite				
		Static	SCL	CCL
1	1.89	13.69	33.43	21.4
2	1.87	14.03	32.37	21.06
3	1.98	15.91	38.83	24.85
4	1.97	15.11	36.63	23.44
5	2.01	17.34	42.05	26.91
6	1.93	13.97	34.38	22.0
Standard deviation	0.05	1.42	3.66	2.27
Average	1.94	15.01	36.28	23.28
Tuff				
Replication	K_1 $MPa\sqrt{m}$	FPZ (mm)		
		Static	SCL	CCL
1	1.73	16.0	37.06	24.54
2	1.84	19.78	43.84	31.78
3	1.76	17.0	35.04	26.34
4	1.78	17.96	54.38	28.24
5	1.74	16.84	39.85	26.44
6	1.79	17.31	34.25	26.27
Standard deviation	0.04	1.29	7.54	2.50
Average	1.77	17.48	40.74	27.27

2.2 *Cyclic loading types*

Instron 8800 series machine equipped with a fast-tracking system was used for conducting a precise testing. The WaveMatrix™ dynamic testing platform was used to perform a wide range of the cyclic tests. The software is capable to support 24 channels at the same time and can control various loading types such as sinusoidal, square, triangle, trapezoidal, and the combinations of hold-ramp function facilitated by user-defined turning points and data recording display modes. In this study, two different cyclic tests were designed and performed on CCNBD specimens; Stepped Cyclic Loading (SCL), and Continuous Cyclic Loading (CCL). In SCL, loading path follows zero-to-max-to-zero fashion of loading, while in CCL diametrical force was applied partially in loading and unloading steps (Figure 2). The initial cracks size and density are difficult to measure directly by using the standard testing equipment. Therefore, stiffness reduction can be a practical and cheap approach to investigate the microfractures role in fracturing progress. In order to apply damage theory in cyclic loading tests and implement experimental data into numerical analyses, degradation variables such as crack tip deformation and cyclic loading magnitudes were measured from CCNBD tests.

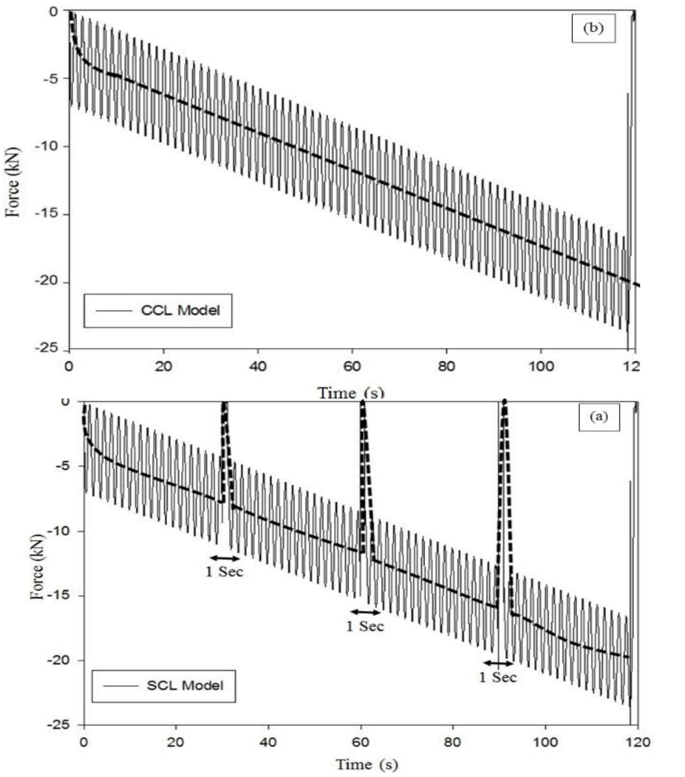


Figure 2. Cyclic loading tests; CCL (top) and SCL (down)

In strain and stress curves (Figure 3), elastic stiffness was determined by conducting a series of uniaxial compression tests. The slope of the damage evolution rate also experienced a sharp increase by applying the SCL loading type. In SCL also plastic deformation which indicates the maximum dimension of the FPZ was obtained larger compared to CCL loading type.

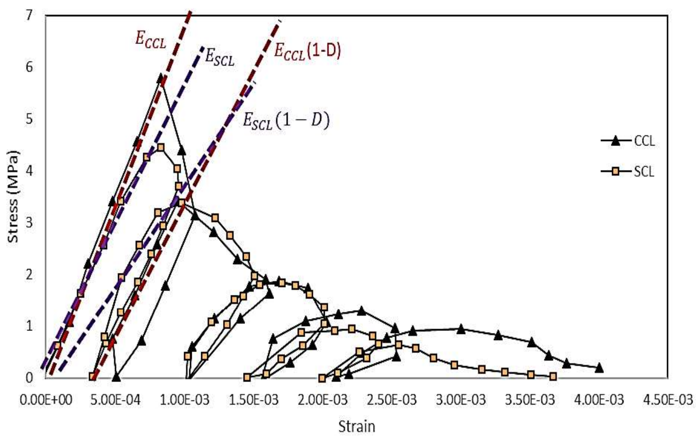


Figure 3. Damage evolution based on the stiffness reductions rate

3 NUMERICAL MODELLING

3.1 *Damage evolution model*

Two different methods are used to model damaging behavior in fracture mechanic tests; namely are the discrete crack model and the smeared crack model, where the last model was employed in this research.

In the smeared model material stiffness is the main indicator of deformation performance and directly related to the microfracture development (Figure 3). Tracking the experimental results from uniaxial compression strength, tensile and fracture toughness tests were used to calibrate finite element modelling in this study. In this model, an energetic approach for solving the fracture problem based on total potential energy (including surface energy) is utilized for modelling damage development (Figure 4).

The rate of the damage change versus the total plastic strains in numerical results and stiffness degradation models in Figure 4, revealed that stepped cyclic loading can create extra damages. The proposed model in this study confirmed previously published results (Ghamgosar, Erarslan, and Williams 2017).

According to the results obtained from the fracture toughness test, cyclic loading creates excessive microfractures in the FPZ by developing fatigue damage and subcritical fractures. Almost 30–40% of the ultimate strength of rocks was found to decrease when applying cyclic loading compared to the static test. Moreover, Stepped Cyclic Loading (SCL) generated extra microfractures in the FPZ compared to Continuous Cyclic Loading (CCL) (Figure 4). Comparison of the determined damage quantity showed that by applying SCL approximately 30% extra damage has been taken placed extra damage obtained under SCL loading can be integrated into developing rock-cutting technology to increase rock fragmentation and breakage, especially for hard and coarser rock types by using less energy. Damage factor in Figure 4 was numerically obtained as the stiffness reduction during the loading process under two loading types.

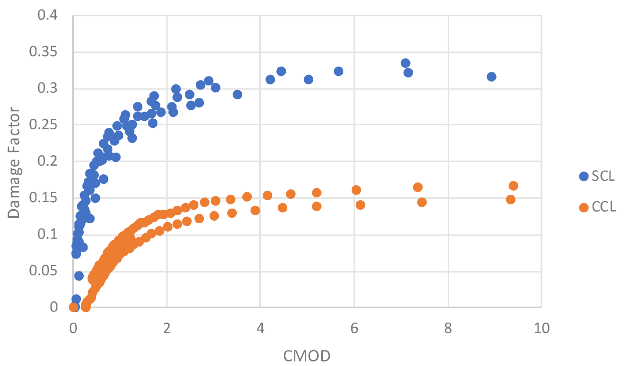
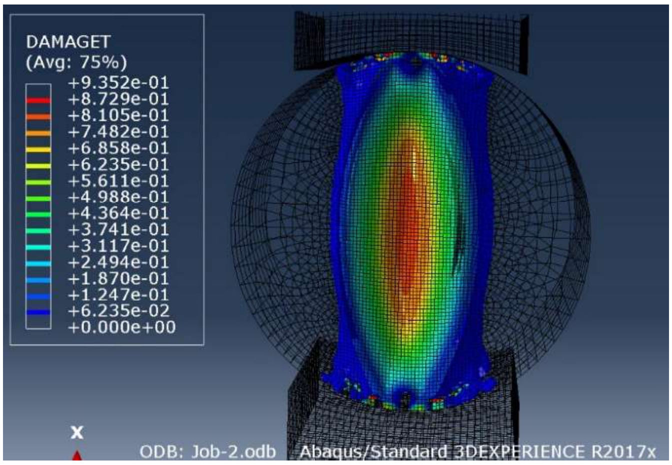
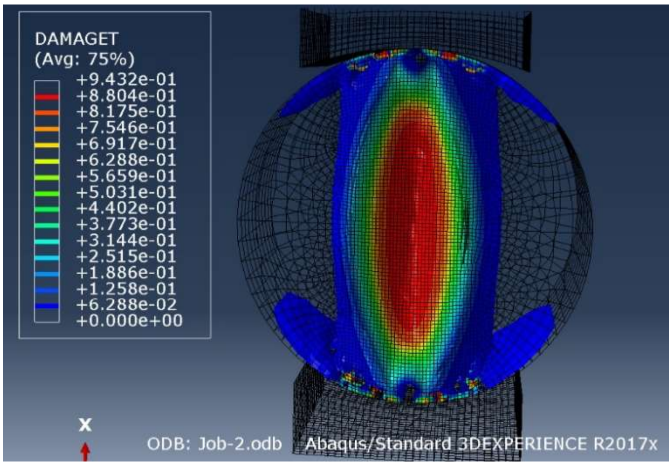


Figure 4. Stiffness degradation models under SCL and CCL

As it can be seen from Figures 5 and 6, different color contours demonstrate that the areas are in different tensile damages states with the damage value increasing at the critical area (middle of the sample). The incremental calculation for both loading tests was checked against the damage evolution values in Figure 3&4, which it was concluded that when the damage factor increases from 0.6 to 0.9, ultimate failure will be reached. After exceeding damage factor beyond 0.6, the rapid decline of bearing area along the maximum tensile stress in the middle of the samples taken place, where for SCL happen quicker compared to CCL type. Furthermore, damage contour in SCL was observed almost 35% further in density compared to CCL, which confirmed the laboratory test results. Figure 5 and 6 show tensile damages across the truncated cross section for CCL and SCL loading tests. Plastic strains also were obtained, and results indicated severe damages (extra 30% plastic strains) by applying SCL loading compared to damage level taken placed in CCL, as shown in Figure 7 and 8.





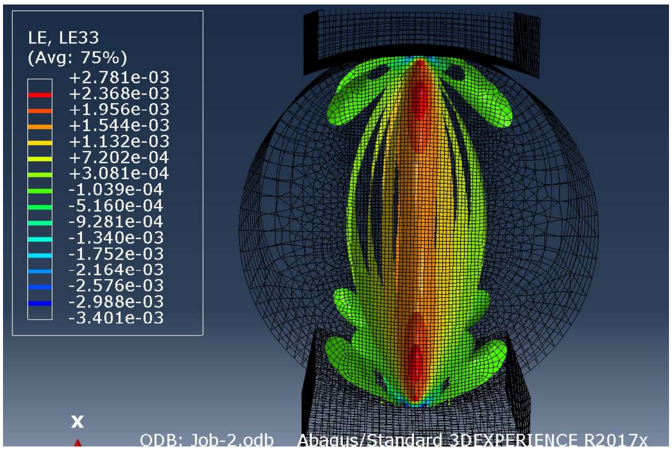


Figure 5. Tensile damage contours in CCL loading test
Figure 6. Tensile damage contours in SCL loading test
Figure 7. Principal plastic strains in the CCL loading model

At the examined plastic strain levels shown in Figures 7 and 8, the development of fatigue damage obtained in terms stiffness amount reduction appears to be mainly governed by the evolution of the plastic strain. Therefore, the measuring of CMOD values in

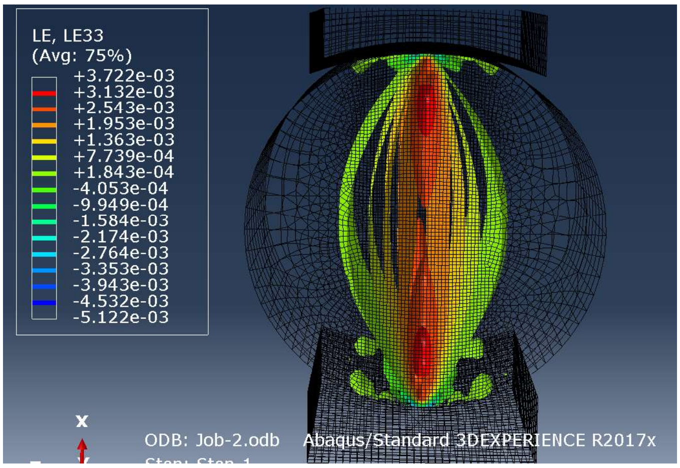


Figure 8. Principal plastic strains in the SCL loading model

the proposed loading models in this study can be utilized in practical cases. One practical conclusion is that to divide the fatigue behavior into two parts, namely into a "damage" and a "plastic" part, and to clarify values change during cyclic loading, where the proposed experimental and numerical approaches in this study can be applied for any other damage studies in rock fracture mechanics or fragmentation process.

4 CONCLUSIONS

Damage mechanism under static and various cyclic loading tests have been numerically and experimentally evaluated based on the strain-energy based model. The prediction of microfracture progress under static or cyclic loading is an important aspect of rock fracture mechanics, specifically for cyclic rock cutting technologies. CCNBD, UCS and BTS tests were carried out based on the ISRM suggested methods, and damage models created to simulate tensile and compressive damages by utilizing FEM software.

Presented damage values under static tests showed approximately 35% fewer values compared to the cyclic loading tests. Furthermore, damage determined by applying SCL test compared to the CCL loading test. This concept was confirmed in experimental and numerical assessments, which can improve understanding of cyclic disc cutter technology to be incorporated in hard rock excavation. Further numerical studies indicated extra plastic strains can be achieved by applying SCL mode compared to the CCL loading mode. Based on the strain energy model which have been evaluated in numerical models showed the current and previous experimental results are reasonably accurate and reliable and recommended to similar rock cutting problems.

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