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Stress distribution in multi-layered railway formation and subgrade

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ABSTRACT: On the Narrabri to North Star (N2NS) portion of the Inland Rail project, formation design is one of the key drivers of earthwork material volumes in the detailed design of the railway. Three-dimensional finite element analysis using software Plaxis 3D was carried out to assess stress distribution within saturated and partially saturated rail formation and subgrade due to train axle loads. The outputs of deviatoric stress from Finite Element Analysis using Plaxis 3D models inform the formation mechanistic based design. In this study, a Mohr-Coulomb constitutive model is used in the Plaxis 3D model to analyse multilayer track earthworks profiles comprising rail, sleepers, ballast, capping, structural fill and subgrade on the N2NS project. The results indicate that the elasticity model may not represent the behaviour of multi-layered railway earthworks due to the development of tension in the ballast layer. Change in deviatoric stress distributions due to train load within saturated and unsaturated formation and subgrade profiles assessed for the N2NS section of the Inland Rail project is presented and discussed in this paper.

1 INTRODUCTION

1.1 Narrabri to North Star Inland Rail project

The Narrabri to North Star (N2NS) Inland Rail Project is a brownfield section in northwest NSW, Australia, starting north of Narrabri Junction at kilometre 575.000 and terminating at North Star approximately 186km north at kilometre 760.500. The project comprises the upgrading of the N2NS existing railway line to allow for heavier train loads, increased train speeds frequency and tonnage. The alignment is generally low lying, crossing broad floodplains and expansive soils.

1.2 General practice in assessing stresses for rail formation design

Formation design of the project requires the formation (capping layer and structural fill) and subgrade to meet the project earthworks performance criteria. One of the functions of the formation layer is to reduce the induced stresses due to train loading in the subgrade preventing subgrade failure and excessive deformation, caused by repeated train loads. A conventional way of achieving this stress reduction is to increase the ballast and formation layer thickness to bear the cyclic load. The ballast thickness is based on practical construction and maintenance requirements. The primary variable in design of the formation is the structural fill and capping thickness, with some minor variation in ballast thickness permitted. The subgrade conditions are based on observations, laboratory data and parameters from project specific geotechnical site investigations. The level of stresses the subgrade can resist due to train loading can be established within a reasonable degree of confidence. The key to the formation design is to determine the appropriate thickness of the formation layer to reduce the stresses in the subgrade to the acceptable level. Where required geotechnical treatments are applied to ensure performance criteria are met. In the formation design, it is essential to assess the stress level applied in the formation and subgrade due to train axle loads.

AREMA (2012) presented four methods to calculate vertical stress due to axle loads: Talbot Equation, Japanese National Railways (JNR) Equation, Boussinesq Equation, and Love’s Formula. Both Talbot and JNR equations are empirical equations based on field or large-scale laboratory testing. The empirical equations and equations based on elastic theory represent a simplified situation for track under heavier axle loads, higher train speed, and multilayered track, formation and subgrade.

Software GEOTRACK and KENTRACK were developed to study the stresses within track and subgrade using multi elastic layer theory (Selig and Waters 1994). It is generally believed that Mohr-Coulomb (M-C) model is more representative of modelling soil material behaviour than the elastic model. One example of that is the stress level due to axle loads in formation/subgrade cannot pass its M-C failure line as it is capped by the M-C failure line. However, there is no such limitation to the predicted stress using an elastic model. Results indicate that elastic models predict higher vertical stress at shallow depths (i.e. ballast and formation) and lower stress within the underlying subgrade because axle loads can be distributed to a larger area on subgrade. The underestimated stress in subgrade due to elastic model limitations could result in thinner formation design and cause over stressing of the in-situ subgrade. This could potentially cause excessive cumulative deformation and strains.

The purpose of this study is to model the deviatoric stress distribution within formation and subgrade due to axle loads using M-C model. Finite element analysis implemented in the commercially available software Plaxis 3D was used to modelled the three-dimensional track/formation/subgrade profiles adopted
for the N2NS project. The results are used as part of the formation design methodology developed for the project reported in a separate paper in these conference proceedings (Tun and Blanchet et al. 2019).

2 GROUND CONDITIONS AND DESIGN REQUIREMENTS

2.1 N2NS ground conditions

Geotechnical site investigations undertaken for the detailed design of the N2NS project indicated evidence of cumulative plastic deformation due to the repeated train loads as illustrated on Figure 1 and Figure 2 showing a typical profile of existing rail track/formation/subgrade on N2NS.

![Figure 1. Existing rail formation and subgrade in the N2NS project. Cumulative plastic deformation can also be seen in the figure.](image)

![Figure 2. Evidence of cumulative subgrade plastic deformation observed during site investigations in the N2NS project.](image)

As shown in Figures 1 and 2, wavy or deformed material surface interfaces were observed from the test pit investigations. These observations are signs of formation and subgrade plastic deformation due to repeated train axle loads. These evidences of deformation were mainly observed under both ends of the sleeper at the base of ballast and underlying layers generally comprising ash, and granular fill over a relatively weak and close to saturation (85% to 90% degree of saturation) cohesive soil layer (compared to cohesive soil away from existing track formation).

2.2 Geotechnical design parameters

Rail track and formation/subgrade parameters used in this assessment are presented in Tables 1 and 2.

![Table 1. N2NS geotechnical design parameters.](image)

![Table 2. Rail and sleeper design parameters.](image)

2.3 Formation design requirements

The following was considered in the model as per N2NS project requirement:

- Ballast depth is 250 mm below bottom of sleeper, capping layer thickness is 200 mm and sleeper spacing is 600 mm.
- 30 TAL traffic load as specified in AS5100.2 (2017) is adopted. In the Plaxis 3D model, axle spacing of 1.8 m, 1.2 m and 1.8 m were adopted to model the train loads directly acting on the top sleepers.
- Effect of dynamic loading was applied with the introduction of a dynamic load factor. Dynamic wheel load is introduced by applying an impact factor (dynamic load factor) to the static wheel load as recommended by AREMA (2012). The adopted impact factor is 1.44 considering a train speed of 80 km/hr.
On this project, the design axle loads, track (including rail, sleeper and ballast) and minimum capping layer thickness are specified by ARTC for the project. One design criterion of formation design is to limit the deviatoric stress to an allowable stress level (Li et al 2016). Therefore, analysis of deviatoric stress with depth, drives the assessment of applied stress onto the subgrade.

3 METHODOLOGY

Plaxis 3D, a general three-dimensional finite element geotechnical software package, is used on this project to model a section of railway track, including rail, sleeper, ballast, capping layer, structural fill, existing fill, and in-situ subgrade. In the model, rail and sleeper are modelled as elastic material. The remaining layers are modelled as M-C material. Design axle loads are modelled as point loads on top of rail (8 wheels load to simulate a coupled bogie). Different mesh sizes are tested in the Plaxis 3D model to compare against the analyses to achieve consistency in the finite element analysis output.

The example reported in Li and Selig (1998) was reproduced using Plaxis 3D M-C model and are compared with the results reported in Section 4.

Two typical formation configurations (namely Formation Profile 1 and Formation Profile 2 are modelled (Figure 3). Formation Profile 1 has 500 mm thick structural fill layer overlying in-situ subgrade while Formation Profile 2 has 250 mm thick structural fill and 250 mm re-compacted subgrade on top of in-situ subgrade.

The purpose of the re-compacted layer is to assess the minimum strength required for the analysis results to meet the BoD performance criteria.

In each formation profile, two scenarios are investigated: (1) saturated subgrade – at some areas along the project alignment the railway track is subject to flooding and moisture is likely to increase over Design Life of 50 years; (2) unsaturated subgrade – the purpose of this scenario is to model behaviour of soil under unsaturated conditions. For each scenario two analyses types were carried out using finite element Plaxis 3D M-C model which is summarised below:

- Saturated subgrade (Drained analysis): Saturated subgrade uses a cohesion ($c'$) which could be obtained from conventional triaxial tests on saturated samples. The effective cohesion is generally low for saturated subgrade. For example, a cohesion of 4 kPa is adopted for saturated in-situ subgrade in Table 1;
- Saturated subgrade (Undrained analysis): In the model with saturated subgrade, when axle loads are applied it is expected the loads will cause pore pressure rise and M-C model Undrained B method is used;
- Unsaturated subgrade (Drained analysis): Following an extended M-C criterion developed by Fredlund et al. (1978) with a modified cohesion $c_m' = c' + (u - u_w) tan(\phi^b)$, where $(u - u_w)$ is matrix suction, 100 kPa from laboratory tests and $\phi^b$ is a friction angle based on the material property and assumed as $\phi / 2.5$. A modified cohesion of 20 kPa is adopted for the in-situ unsaturated subgrade as shown in Table 1;
- Unsaturated subgrade (Undrained analysis): In the model with unsaturated subgrade, as groundwater is deep and the subgrade is unsaturated, no pore pressure is expected but total stress analysis is still believed to be more applicable as the load duration is very short. M-C model Undrained C method is adopted.

The analyses carried out for the above scenario for each formation profile (Profile 1 and 2) are summarised in Table 3. Table 4 shows the construction sequence modelled in Plaxis-3D.

![Figure 3. Plaxis 3D model (Profile 2) of the N2NS railway track and earthworks.](image)

<table>
<thead>
<tr>
<th>Formation Profile</th>
<th>Saturated Subgrade</th>
<th>Unsaturated Subgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 2</td>
<td>M-C Drained</td>
<td>M-C Drained</td>
</tr>
<tr>
<td>1 and 2</td>
<td>M-C Undrained B</td>
<td>M-C Undrained C</td>
</tr>
</tbody>
</table>

Table 4. Staging modelling in Plaxis 3D

<table>
<thead>
<tr>
<th>Construction sequence</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Initial Phase</td>
<td>Generate initial stress in the subgrade</td>
</tr>
<tr>
<td>2) Railway Formation</td>
<td>Construct structural fill, capping, ballast, sleepers and rails on the top of subgrade</td>
</tr>
<tr>
<td>3) 30 TAL (214 kN/Wheel)</td>
<td>Apply 30 TAL train load (214 kN/Wheel) on the rails</td>
</tr>
</tbody>
</table>

4 COMPARISON BETWEEN ELASTIC AND MOHR-COULOMB SOIL MODELS

4.1 Railway track/subgrade model

Li and Selig (1998) used software GEOTRACK to set up several multitrack elastic models to investigate stresses in the track and subgrade. Design charts were
developed based on the analysis. In this study, Plaxis 3D is used to re-generate the example, where granular layer is equal to 0.46m, and the resilient moduli of granular layer and subgrade layer are respectively 280 MPa and 28 MPa. Both elastic and Mohr – Coulomb models are used in the Plaxis 3D model. The results are compared with the results in the reference.

4.2 Results and comparisons

75th percentiles of deviatoric stress increment, $\Delta(\sigma_1 - \sigma_3)$, due to train loading, on top of the subgrade is calculated for the comparison. It is found the Plaxis 3D elastic model gives similar deviatoric stress, $\Delta(\sigma_1 - \sigma_3) \sim 65$kPa, compared with the Li & Selig’s design charts using GEOTRACK.

However, in the Plaxis 3D elastic model, it is found tension (up to approximately 200 kPa) develops from the mid-depth to the bottom of the ballast layer (Figure 4). In the 3D multi-layered elastic model, when the relatively strong elastic ballast layer overlies the relatively weak formation/subgrade layers, tension occurs due to the sagging generated within the strong elastic layer, which is similar to the tension developed below neutral plane in a beam under bending. However, the granular layer is not expected to sustain such tension force. The existence of the tension force may further affect the accuracy of the calculated stress distribution in the subgrade.

Figure 4. Tension within ballast from the Plaxis-3D elastic model

M-C models of Drained and Undrained B methods are also used to analyse the above example. It is found that the change in deviatoric stress at the top of the subgrade, $\Delta(\sigma_1 - \sigma_3)$ is about 15 % to 40 % higher than presented in Li and Selig’s design charts. No tension (due to tension cut-off) is observed in the Plaxis M-C models, but plastic points and tension cut off points are developed in M-C models. Further study could be carried out to compare Li & Selig’s design charts with more representative soil constitutive models.

5 DEVIATORIC STRESS RESULTS OF PLAXIS 3D ANALYSIS ON N2NS PROJECT

The results of stress analysis using Plaxis 3D M-C constitutive model for all soil material to simulate the railway track and subgrade on N2NS project are presented.

5.1 Method to calculate deviatoric stress

A “Study Area” comprising a coupled bogie (8 wheels load) is defined as shown in Figures 6 and 7. Deviatoric stress at certain depths within formation or subgrade (for example at top of subgrade in Figure 7) is adopted to be the 75 percentiles of deviatoric stress (outputs from the model) at that depth within the “Study Area”. The selection of 75 percentile value is on the conservative side comparing with an average value. Furthermore, the 75-percentile value in the elastic model of the example can also match the Li and Selig design charts.

Figure 5. “Study Area” for 75 percentile calculations

Figure 6. $(\sigma_1 - \sigma_3)/2$ contour within “Study Area” at top of subgrade. Unit for x and y is in metre, and stress is in kN/m²

5.2 Saturated subgrade

At areas where saturated subgrade is expected, either Formation Profile 1 or 2 could be adopted at different locations depending on subgrade strength and deformation conditions. Plaxis 3D models are developed to simulate both formation profiles with saturated subgrade. Both drained behaviour and undrained behaviour (Undrained B) are investigated for each formation profile.

5.2.1 Drained behaviour

Results of deviatoric stress increments, $\Delta(\sigma_1 - \sigma_3)$, are plotted with depth for Formation Profile 1 and Profile 2 under drained behaviour in Figure 7. The results are also summarised in Table 5.
Fig. 7. Deviatoric stress increments with depth for saturated subgrade drained behaviour. Refer to section 3 for composition of Profile 1 & 2 and section 2.3 for ballast and capping thickness.

Table 5. Summary of deviatoric stress increments at top of layers under drained behaviour of saturated subgrade

<table>
<thead>
<tr>
<th>Formation / Subgrade</th>
<th>(\Delta (\sigma_1 - \sigma_3)) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of structural fill</td>
<td>64</td>
</tr>
<tr>
<td>Mid-depth of structural fill</td>
<td>-</td>
</tr>
<tr>
<td>Top of re-compacted subgrade</td>
<td>63**</td>
</tr>
<tr>
<td>Top of In-situ subgrade</td>
<td>51</td>
</tr>
</tbody>
</table>

* Formation profile 1, ** Formation profile 2

5.3 Unsaturated subgrade

At areas where unsaturated subgrade is expected, either Formation Profile 1 or 2 could also be adopted at different locations depending on subgrade strength and deformation conditions. Plaxis 3D models are also developed to simulated both formation profiles with unsaturated subgrade. Both drained behaviour and undrained behaviour (Undrained C) are investigated for each formation profile.

5.3.1 Drained behaviour

Unsaturated subgrade is modelled with a modified cohesion \(c_s'\) under drained behaviour to consider the influence of matrix suction. Results of deviatoric stress increments, \(\Delta (\sigma_1 - \sigma_3)\), are plotted with depth for Formation Profile 1 and Profile 2 under drained behaviour in Figure 9. The results are also summarised in Table 7.

Fig. 8. Deviatoric stress increments with depth under undrained behaviour of saturated subgrade. Refer to section 3 for composition of Profile 1 & 2 and section 2.3 for ballast and capping thickness.

Fig. 9. Deviatoric stress increments with depth under drained behaviour of unsaturated subgrade. Refer to section 3 for composition of Profile 1 & 2 and section 2.3 for ballast and capping thickness.

Table 6. Summary of deviatoric stress increments at top of layers under undrained behaviour of saturated subgrade

<table>
<thead>
<tr>
<th>Formation / Subgrade</th>
<th>(\Delta (\sigma_1 - \sigma_3)) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of structural fill</td>
<td>76</td>
</tr>
<tr>
<td>Mid-depth of structural fill</td>
<td>-*</td>
</tr>
<tr>
<td>Top of re-compacted subgrade</td>
<td>62**</td>
</tr>
<tr>
<td>Top of In-situ subgrade</td>
<td>53</td>
</tr>
</tbody>
</table>

* Formation profile 1, ** Formation profile 2

Table 7. Summary of deviatoric stress increments at top of layers under drained behaviour of unsaturated subgrade

<table>
<thead>
<tr>
<th>Formation / Subgrade</th>
<th>(\Delta (\sigma_1 - \sigma_3)) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of structural fill</td>
<td>108</td>
</tr>
<tr>
<td>Mid-depth of structural fill</td>
<td>-</td>
</tr>
<tr>
<td>Top of re-compacted subgrade</td>
<td>84**</td>
</tr>
<tr>
<td>Top of In-situ subgrade</td>
<td>66</td>
</tr>
</tbody>
</table>

* Formation profile 1, ** Formation profile 2
5.3.2 Undrained behaviour

Results of deviactor stress increments, $\Delta(\sigma_1 - \sigma_3)$, are plotted with depth for Formation Profile 1 and Profile 2 under undrained behaviour (Undrained C) in Figure 10. The results are also summarised in Table 8.

![Figure 10. Deviactor stress increments with depth for undrained behaviour of unsaturated subgrade. Refer to section 3 for composition of Profile 1 & 2, and section 2.3 for ballast and capping thickness.](image)

Table 8. Summary of deviactor stress increments at top of layers under undrained behaviour of unsaturated subgrade

<table>
<thead>
<tr>
<th>Formation / Subgrade</th>
<th>$\Delta(\sigma_1 - \sigma_3)$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
</tr>
<tr>
<td>Top of structural fill</td>
<td>108</td>
</tr>
<tr>
<td>Mid-depth of structural fill</td>
<td>-</td>
</tr>
<tr>
<td>Top of re-compacted subgrade</td>
<td>66**</td>
</tr>
<tr>
<td>Top of in-situ subgrade</td>
<td>55</td>
</tr>
</tbody>
</table>

* Formation profile 1, ** Formation profile 2

6 DISCUSSIONS

Similar deviactor stresses are predicted at same depths for both drained or undrained B analysis of both formation profiles. Deviactor stresses in Table 7 were adopted for the formation design.

For the soil in areas not affected by flooding where the subgrade is expected to be unsaturated, higher stresses are calculated using drained model as higher modified cohesion allows the drained material capable of resisting higher vertical stresses. Due to transient nature of the train loading, the results from the undrained model (Undrained C) as shown in Table 8 for the formation design were adopted.

At shallow depths the deviactor stress from M-C model is generally higher than that from Li and Selig’s design charts based on elastic theory as shown in Figure 7 to Figure 10. At about 3.5m depth the difference becomes negligible.

7 CONCLUSIONS

Mohr-Coulomb elastic-plastic constitutive soil model is preferred to the linear elastic model in the numerical modelling of vertical stress distribution within rail formation and subgrade under train axle loads. Significant tension force could develop in the ballast layer using the elastic model not considered as representative of the soil behaviour under train loading. A M-C conventional constitutive model was preferred to model stress-strain behaviour of soils.

When the subgrade is saturated, deviactor stress distribution outputs from Finite Element Analysis appears not sensitive to the adopted formation profiles and analysis types (Drained or Undrained B). This implies both drained and Undrained B analyses on unsaturated subgrade provides good agreement with the drained and undrained parameters adopted.

When the subgrade is unsaturated, deviactor stress distribution within structural fill is not affected whether drained or Undrained C models are used. However, the change in calculated deviactor stress within re-compacted subgrade or in-situ subgrade using the Drained analysis is considered to be over-estimated. Undrained C method is considered to be more representative to model unsaturated subgrade and transient train loading and was adopted.

The analysis results indicate the thicker and/or stronger the track/formation layers (ballast/capping and structural fill), the less deviactor stress applied onto the subgrade due to train axle loads is anticipated and generally slightly higher than Li & Selig’s Design Chart at shallow depth from the bottom of capping.

8 ACKNOWLEDGEMENT

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REFERENCES

Plaxis 3D Manual

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