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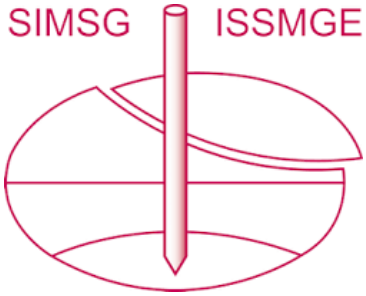

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Application of Rate Analysis to Settlement Problems Involving Creep

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SUMMARY - Although creep in foundation strata is not usually a prime consideration, it can, in some specialized structures, be of quite considerable significance. Creep estimation from laboratory tests is not easy, and the analysis of field behaviour is often complicated by variable loading, and other error sources. The paper examines the characteristics of a creeping foundation and shows that rate analysis methods may be applied with advantage for the estimation of future settlement.

SYMBOLS -

a, b	coefficients
k	Boltzmann's constant
m, n	indices
t	time
Q	activation energy
T	absolute temperature
U	degree of primary consolidation at time t
ε	strain
σ	stress

I INTRODUCTION

In general terms, creep describes continuing deformations that are observed in materials under constant (effective) stress. Since the studies of creep in metal wires by Phillips (Reference 14), the phenomenon has been shown to occur in a wide range of materials (Murrell and Misra, Reference 12) that includes soils and rocks.

For settlement calculations in clays, methods based on the classical theory of consolidation are considered to be not greatly in error (Barden, Reference 2). The primary component of total settlement is usually substantial (although a function of pressure increment ratio, Leonards and Girault, Reference 10), but, in soft rocks, the situation can be quite different, primary compression often being rapid and allowing long term settlement to be dominated by creep.

The analysis of creep is often vague and ambiguous. Accommodation for creep can sometimes be provided by routine servicing, but this is unlikely to be practicable for major foundations. In the event of a reasonably accurate settlement estimation being required, there are certain problems, and significant departures from familiar consolidation behaviour, which make normal extrapolation dangerous, and which this paper seeks to evaluate.

II THE ORIGIN AND THEORY OF CREEP

The mechanisms causing creep apparently vary, and some differences in the kinetic characteristics are possible. Nevertheless, the similarity in creep behaviour between quite diverse materials is striking (Cottrell, Reference 5), such that useful inferences may be made from unrelated materials. With soil materials, such analogies are valuable, since it is not easy to determine, with confidence, the analytical form of the creep-time curve.

In rocks, creep is thought to result from micro-fissuring and recrystallization (Robertson, Reference 16). In clays, the primary and secondary compressions have been associated with "diffusion" and "viscous" processes (Walker, Reference 19; also see Barden and Poskitt Reference 3). However, for most materials, an essential feature appears to be the absence of an equilibrium condition, as for primary consolidation as $U \to 1$, and thus most postulated creep equations have no strain asymptote for large time ($t \to \infty$)*. Further, whereas the rate of consolidation settlement is uniquely specified by U**, no such relationship is possible for creep, and direct interpretation of settlement - time data is unlikely to be possible.

It has been established from the theory of absolute reaction rates (Glasstone, Laidler and Eyring, Reference 8) that steady state creep rates vary with temperature and stress according to equation 1.

$$\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \propto \exp\left(\frac{Q}{kT}\right) \sinh\left(\frac{\sigma}{\sigma_o}\right) \qquad (1)$$

Transient creep, however, ($\dot{\varepsilon}$ variable with time) is normally described by empirical expressions only, although the logarithmic form (equation 3) can be derived from exhaustion theory (Wyatt, Reference 21).

In one of the earliest systematic studies of creep, Andrade (Reference 1) obtained the relation

$$\varepsilon = \varepsilon_o + at^{1/3} + bt \qquad (2)$$

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* Notable exceptions are the Glanville equation for concrete (see Murrell and Misra, Reference 12), and expressions resulting from the Kelvin-Voigt and Burgers models (Robertson, Reference 16).

** For example, when $U < 0.5$, $\dot{U} \propto \frac{1}{U}$

A term in t^3 was added later (Graham and Walles, Reference 9), the terms in t then being associated with the divisions of primary (transient), steady and tertiary creep * (e.g. Cottrell, Reference 5; Murrell and Misra, Reference 12) (Figure 1). Other expressions in common use include:-

$$\varepsilon = \varepsilon_o + a \log (t - t_o) \qquad (3)$$

(e.g. Phillips, Reference 14) and (Crussard, Reference 7).

$$\varepsilon = \varepsilon_o + a (t - t_o)^n \qquad (4)$$

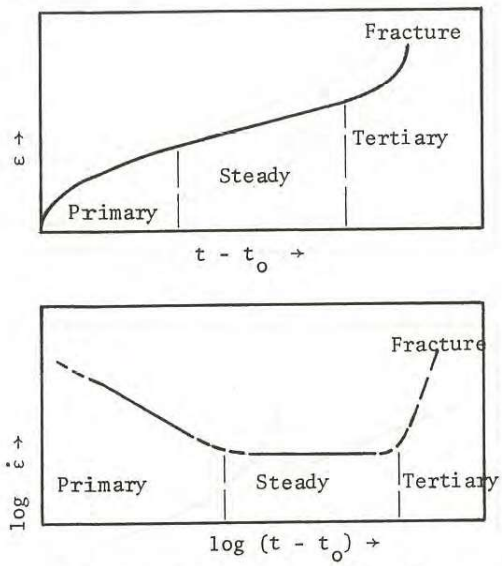

FIG. 1. GENERAL CHARACTERISTICS OF CREEP CURVES

Metals usually follow a third power low at high temperature, but show logarithmic behaviour at low temperatures. The situation is less clear for rocks, but logarithmic behaviour is often considered to apply (Robertson, Reference 16; Murrell and Misra, Reference 12). There is, however, no conflict in creep rates (equation 5) **

$$\dot{\varepsilon} = a (t - t_o)^{-m} \qquad (1 > m > 0) \qquad (5)$$

as shown by Crussard (Reference 7), and, by inference, the geometrical form of a creep equation, such as equation 3 and 4, is of little fundamental significance.

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* Some writers report an initial, or incubation, stage (Wachtman and Maxwell, Reference 20).

** Singh and Mitchell (Reference 17) have experimentally established a similar equation for clays, viz,

$$\dot{\varepsilon} = Ae \left(\frac{t_1}{t}\right)^m \qquad (5a)$$

where $1 > m > 0.75$. It is supported by earlier theoretical work of Mitchell (Reference 11).

III COMPRESSION RATE ANALYSIS

Successful extrapolation of future behaviour depends on a reasonable determination of the analytical nature of the creep rate variation. This however, must be done on minimal data, since creep performance cannot be monitored at an accelerated rate, as in the case of laboratory consolidation testing.

Some of the problems in establishing functional identity are discussed by Crussard (Reference 7), who showed that approximate linearity in a given range does not exclude the existence of a better relationship. The most powerful analytical techniques are based on differencing (a standard mathematical procedure, e.g. Pipes, Reference 15), and creep rates necessarily show a simpler mathematical structure.

A further advantage of equation (5) over equations (3) and (4) is that the initial condition, ε_o, is eliminated, which is fortunate, since the point of initiation of creep is often obscure. The datum of time t_o, however, is similarly obscure, but cannot be eliminated from any equations. This precludes the determination of unique values of a and m, except in so far as they approach their true values for t large in relation to t_o. Techniques for the estimation of t_o do not appear to have been considered in literature.

Although equation 5 remains sensitive to t_o, an estimate, at least, of its value may be made by writing the equation as

$$(t - t_o)^m = \frac{a}{\dot{\varepsilon}} \qquad (6)$$

Since $m > 0$, a graph of t against $\frac{1}{\dot{\varepsilon}}$ must then indicate a value of t_o as the intercept where $\frac{1}{\dot{\varepsilon}}$ vanishes regardless of the values of a and m. A conventional graphical analysis will then yield values of a and m with minimum ambiguity (Figure 2).

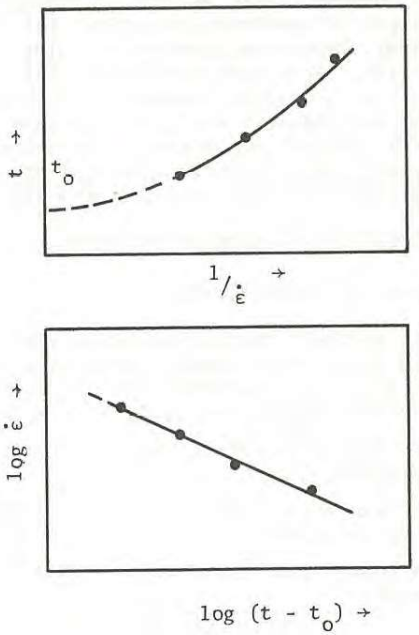

FIG. 2 CREEP RATE ANALYSIS (Diagrammatic)

IV SOME COMMENTS ON DIFFERENTIAL METHOD

The process of differentiation, on the derivation of a gradient function from a set of data, causes changes of character that are not easily visualized, particularly where the results are presented on logarithmic scales. Several points, therefore, will be noted.

Firstly, differentiation is a fully reversible process, without prejudice to the accuracy of the initial data, which may be recovered in toto by summation. It is merely a sorting process, and no new information is created.

Secondly, differentiation is a process which may be accompanied by diminished convergence. * In relation to a sequence of experimental data, differencing will amplify imperfections initially present, whereas summation will smooth and obscure any variations. The same imperfections, however, are present throughout.

Thirdly, because constant terms are eliminated in differentiation, short term factors (such as elastic strain), which may have a permanent effect on cumulative strain, have only a transient effect on strain rate. Thus, whereas a strain record does not readily permit differentiation between its various components, a strain rate record allows the creep component to be isolated, subject only to transient aberrations.

Such aberrations, which may interrupt or alter the basic creep pattern, are herein termed "secondary features", and arise due to environmental changes, such as variable loading. Some typical secondary features of creep are discussed below, as an understanding of these matters is pre-requisite to any reasonable assessment of creep behaviour.

Consolidation analysis is also subject to some secondary features, such as that resulting from the finite length of construction period, the subject of an empirical correction proposed by Gilboy (Taylor, Reference 18). The correction is possible because the effects of a loading sequence on cumulative strain are more or less additive. Whether this also applies for creep is not known, since there is no certainty that an equilibrium state exists. Only the creep rate shows any evidence of a limiting state.

V SOME SECONDARY FEATURES OF CREEP

(a) Effects of Stress Changes

It is unlikely that a foundation will not be subject to some stress changes during its early life, and these are likely to affect the deformation of a creep sensitive foundation in three ways, namely,

(i) An immediate effect, due to elastic compression

(ii) A short term effect, due to Terzaghi consolidation

(iii) A long term effect, due to a modified creep rate.

* For example, whilst a power series can always be integrated, there is no assurance that a derivative can be found (Pipes, Reference 15).

Often, it is not clear what stress changes have taken place, except by inference from their effect, which may show as a cusp on the settlement curve (Figure 3). If the curves are obtained as a sequence of experimental data, subject to reading error, it may be difficult to do other than draw a single smooth curve, which, of necessity, will over-estimate the creep rate and the deformation at large time.

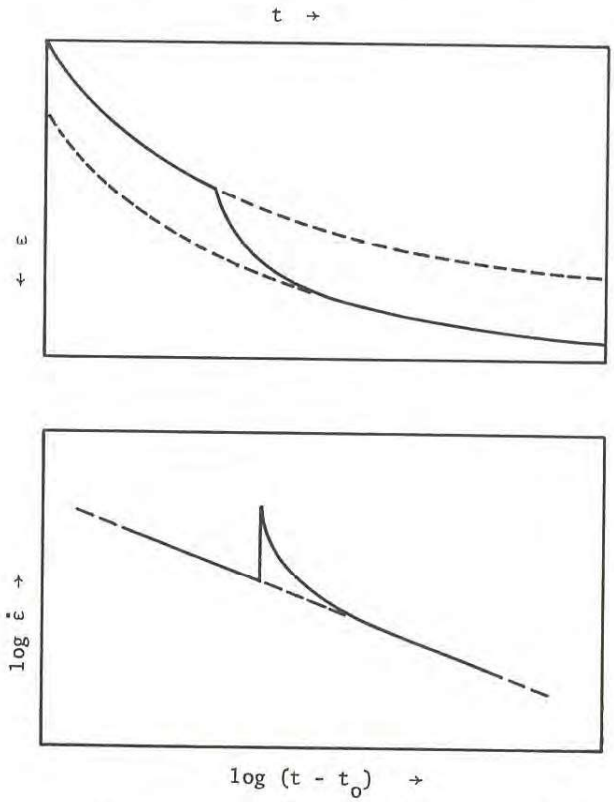

FIG. 3. SECONDARY FEATURE DUE TO CONSOLIDATION

Whether or not the creep mechanism remains operative through such interruptions is a matter of conjecture. By definition, creep is a component of strain at constant effective stress, and it is only studied under such conditions. However, the possibility of its continuance under varying effective stress is not excluded. In fact, Crawford (Reference 6) regards it as part of a continuous consolidation process, and Barden (Reference 2) acknowledges the probable initiation of creep during the primary compression stage. It is not unreasonable to assume, therefore, that creep acts continuously through any stress change and is only temporarily masked by other deformations. This being so, the two segments of a creep rate curve are likely to be approximately co-linear (Figure 3), and in practice this appears to be the case. Thus the creep rate curve has sufficient continuity to permit analysis whereas the creep curve has none.

In Figure 4, a secondary feature resulting from a temporary surcharge load is illustrated. Here, elastic recovery may cause the net strain rate to be temporarily arrested, prior to converging onto a parallel creep curve.

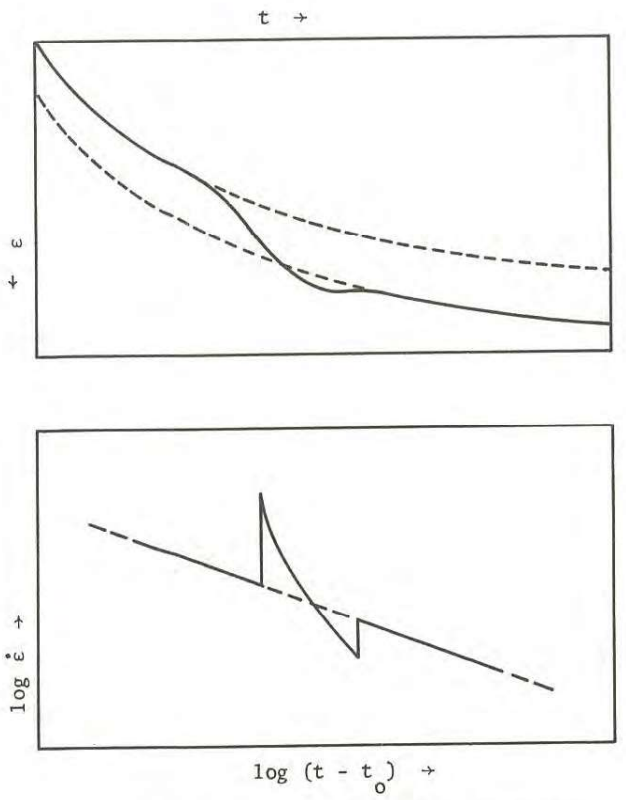

FIG. 4. DISTORTION DUE TO TEMPORARY SURCHARGE LOAD
Causing Elastic and Consolidation Settlement

(b) Effect of Stress Changes on Creep Rate

During small stress changes, some modification of the creep rate is possible. For steady state creep, stress dependence is usually expressed in the form*

$$\dot{\varepsilon} \propto \sigma^n \qquad (7)$$

The value of n varies considerably in rocks, from 1 at low stress up to 4 or 5 at elevated stress (e.g. Robertson, Reference 16), but the lower limit is more likely at engineering stress levels. Hence small changes in stress may be expected to produce relatively small changes in the steady state creep rate, and, in some instances, such as granodiorite and gabbro (Murrell and Misra, Reference 12), very little change.

Little is known of the stress dependence of primary creep, but the situation is unlikely to differ much from the preceeding. If, further, it is assumed that the time decay terms for transient creep are not stress dependent, then the effects of a stress increment would appear as a change of time scale, or, as a parallel displacement of creep rate curve on log-log scales (Figure 5). Once again, such behaviour is not readily discernible on a graph of creep strain, and any attempt to draw a single creep curve will result in an over-estimation of final strain.

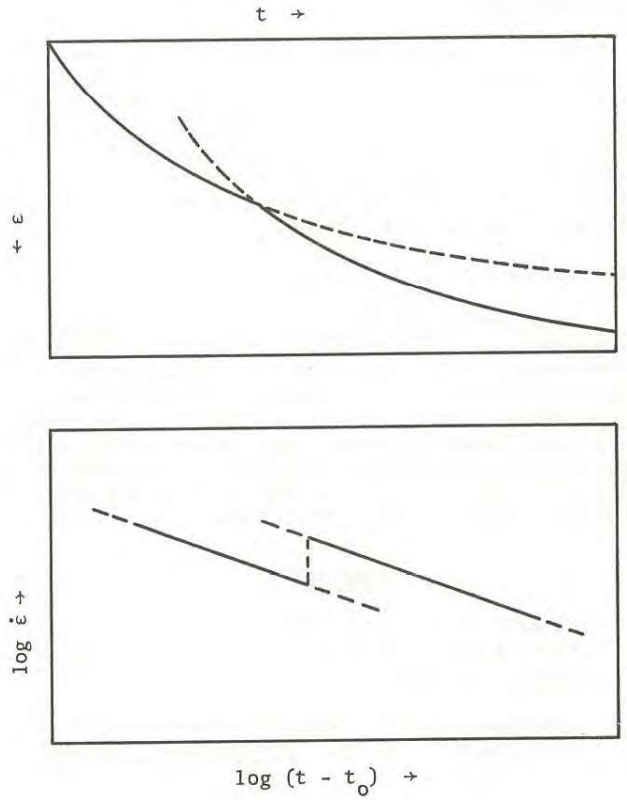

FIG. 5. EFFECT OF STRESS CHANGE ON CREEP RATE

The assumption above draws some support from the fact that it renders transient creep subject to the Boltzmann Superposition Principle, which is considered to apply for the non-constant loading of metals in steady state creep (e.g. Onaran and Findley, Reference 13). This states that, under various loads P_1, P_2.... applied at times t_1, t_2....previous to the final loading being considered, the final deformation can be computed by simple superposition of each of the loads acting independently. Evidently, the Principle operates best at lower stresses when n is close to 1.

(c) Effects of Fracture

Most structural foundations possess a high degree of rigidity in relation to the strata on which they are founded, and this results in stress concentrations at the periphery, in accordance with the effective rigidity factor (Borowicka, Reference 4). Although these are not normally of any consequence in plastic soils, the situation can be rather different in soft rock, with a brittle failure characteristic.

* Equation 7 is similar to the Ostwald Equation, on which Barden (Reference 2) has based his non-linear analysis of clay deformation. He indicates that n = 1 leads to Taylor's Consolidation Theory B. Equation 7 is evidently an approximation to Equation 1 for a restricted stress range.

It is not unlikely, therefore, that a foundation on creep sensitive rock may incur local peripheral fractures. Whilst there may still be a substantial factor of safety on general shear failure, local failures will result in a certain redistribution of stress, and may have a significant bearing on settlement behaviour.

The secondary features resulting from local shear failures will evidently be similar to those for stress changes, except in so far that their influence extends over a rather greater time. It is probable that fracture initiation involves a period of tertiary creep (Figure 1), which may be reflected in the overall deformation rate (Figure 6) during the period of fracture initiation.

VI CONCLUSION

Some aspects of the behaviour of a creeping foundation have been examined analytically, and, in particular, those factors which interrupt or obscure ideal behaviour. In each case, it is shown that a single interpolated curve on a strain v. time graph will significantly over-estimate future settlements. Such complications make the analysis of a creep-time graph virtually impossible, and any fruitful analysis should be made by way of creep rates.

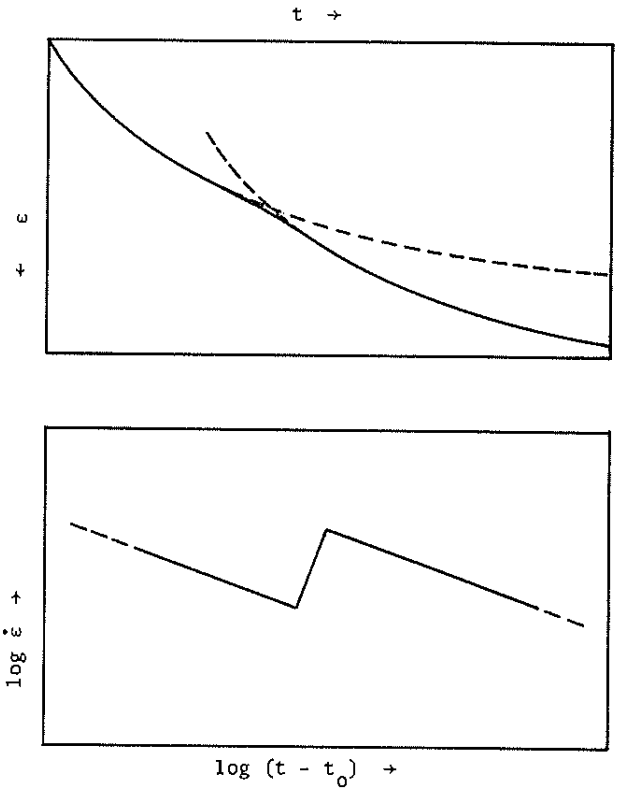

FIG. 6 SECONDARY FEATURE DUE TO FRACTURE

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