A Statistical Method for the Design of Rock Slopes

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SUMMARY.- Procedures are introduced for the design of rock slopes by determination of the probability that the slope will be undercut by joints, or combinations of joints, in unstable orientations. The procedures include graphic methods for the statistical analysis of joints, graphic tests for kinematic and kinetic stability, and procedures for the economic analysis of rock slopes.

I. INTRODUCTION

The principal sources of uncertainty in the design of a rock slope are usually the locations and orientations of the joints and other rock defects in the rock mass behind the slope. Steep rock slopes have a higher risk of failure than flatter ones mainly because of the greater probability of undercutting joints, or combinations of joints, in unstable orientations.

This paper introduces graphic procedures for determining the probability of failure of a rock slope by statistical analysis of joint orientations in conjunction with graphic kinematic and kinetic analyses of stability over a range of possible slope angles. The design slope is then selected on the basis of one of the following criteria:

1. Where safety is the overriding consideration; the design slope is the slope angle at which the probability of failure approaches zero.
2. Where partial failure of the slope can be allowed; the design slope is the slope with the minimum total cost consisting of the initial cost of excavation plus the costs resulting from partial slope failure.
3. Where the slope with minimum total cost has an unacceptably high probability of failure, the slope angle may be decreased to an acceptable compromise between the above two criteria.

In the following sections the design procedure is first demonstrated by reference to a simple example involving sliding along a single joint. Extension of the procedure towards the design of slopes where the failure occurs along combinations of joints is then discussed.

II NOTATION

A_u Area of Schmidt Projection covered by unsafe orientations of joints.

A_t Total area of a Schmidt Projection.

c Cohesion

C_a Portion of costs of slope failure that are independant of volume of slide material.

C_b Unit cost of slope failure where costs are related to volume of slide material.

Co Cost of initial excavation.

co Unit cost of bulk excavation.

C_f Cost of slope failure.

C_t Total cost of a rock slope.

i Angle of roughness.

M Multiple of the standard deviation being contoured.

N_a Average number of joint directions observed in a small element of the rock mass.

N_u Number of joint poles in unsafe orientations.

N_t Total number of joints in sample.

n Number of small elements in the rock mass behind the slope.

P_c Percentage of joints in a set that are judged to be sufficiently extensive and continuous to result in a slope failure.

P_f Probability of slope failure.

P_i Probability that the ith element will be undercut by joints in unstable orientations.

P_j Probability of a joint occurring in an unsafe orientation.

$\emptyset$ Angle of surface friction.

$\emptyset a$ Apparent angle of friction.

S_x Sample standard deviation in x-direction.

S_y Sample standard deviation in y-direction.

σ Normal stress at failure.

τ Shearing stress at failure.

u Hydrostatic uplift

V_o Volume of initial excavation.

V_f Expected volume of material involved in slope failure.

V_i Volume of the ith element.

x X-co-ordinate.

y Y-co-ordinate.

III THE EXAMPLE

The following example will be used to illustrate the design procedure:-

A rock slope 100 feet high and facing due West is to be excavated for a highway running N-S. The natural ground surface slopes at 27° and the rock consists of a gneissic granite, with a well-developed foliation. Outcrops are scattered and the foliation is observed to change direction in complex fashion between outcrops. Joints parallel to the foliation extend several hundred feet and are termed joint set 1. Other joints extend less than 10 feet. The groundwater table is below the base of the slope.

Observations on several existing slope failures in the area indicate that failure invariably occurs whenever the true dip of foliation is undercut by a slope and exceeds 34 degrees. It is also observed that sliding on a serious scale only occurs along the foliation joints but that tensile failure can occur in almost any direction in the rock mass, due to separation along combinations of minor joints, whenever necessary to permit sliding on the major joints.

The slope will be allowed to stand for 18 months after excavation before the highway is opened for traffic. Initial bulk excavation is estimated to be $1.00 per cubic yard (in-situ). Clean-up of partial slope failure is estimated to be $5.00 per cubic yard including allowances for scaling down, repair of pavement, and general inconvenience.

IV DESIGN PROCEDURE

The following steps are necessary for the design of a rock slope by the proposed method:

(a) Geological Mapping

Regardless of whether the problem is to design a highway slope, or the slope of an open-pit mine, the first step must be geological mapping to divide the area into structural regions that are statistically homogeneous with respect to jointing and other rock defects (Ref. 1). This mapping also serves to locate major features such as wide fault zones which must be treated as individuals rather than as statistics for the purpose of rock slope design.

(b) Statistical Sampling

An adequately representative sample of joint orientation must be obtained from each structural region included in the slope. Since only joints exposed on rock outcrops, excavated surfaces or a limited number of bore holes are available for measurement, a perfectly random sample, which would imply that every individual in the population has an equal chance of being chosen for the sample, is clearly impossible. When extensive outcrops are available, formal sampling schemes, which are carefully designed to minimise observer bias, may be applied (Ref. 2). Where only scattered exposures are available, an adequate sample can often be obtained by measuring the three to five most prominent joint directions at each outcrop visited (Ref. 3).

(c) Statistical Analysis

The purpose of taking any sample is to reach conclusions regarding the properties of the population (i.e. the totality of all things that the sample is designed to represent).

Graphic statistical analysis of the joint orientation data in three dimensions is made possible by the Schmidt method of representing the orientation of each joint as a point (pole) on a Lambert Equal-Area Projection* of a sphere, as shown in Fig. 1. Detailed procedures for working with this projection are given in Refs. 1 and 4.

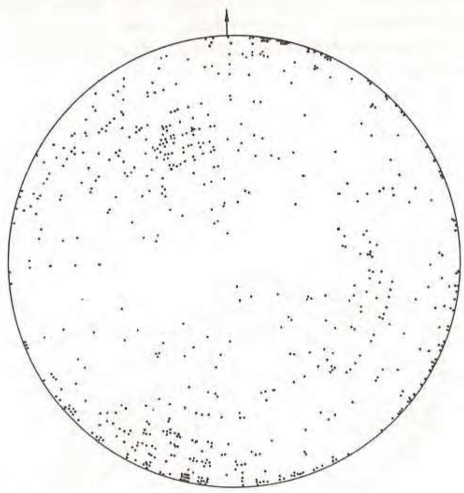

Fig. 1: Representation of joint orientations as poles on a Lambert Equal-Area Projection (631 poles to joints from Geehi Gorge, N.S.W.)

The traditional geologic method of contouring Schmidt Diagrams in terms of the density of poles as shown in Fig. 2 has statistical usefulness only when the underlying population is suspected of being uniform as demonstrated by Winchell (Ref. 5), and Flynn (Ref. 6). In most actual cases however, preferred orientations in geologic data are self-evident and the poles are seen to be clustered into groups which, in the case of joints, are called sets.

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* Throughout this paper the lower hemisphere of the Lambert Equal-Area Projection is used.

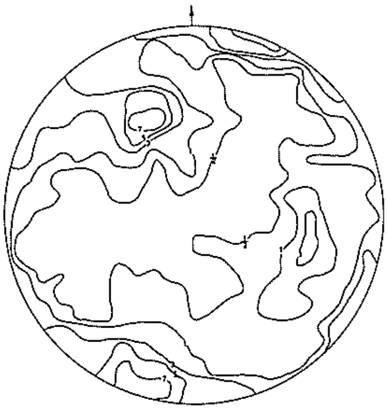

Fig. 2: Conventional contours of the density of poles shown in Fig. 1. Contours ½, 1, 3, 5, 7% per 1% area.

If it can be considered that the centre of the set represents the ideal orientation of the joints and that the scatter of joints around this ideal orientation is due to chance causes then, as shown by Vistelius (Ref. 7) and subject to some additional limitations discussed by him, the population distribution along any cross section through the joint set may be approximated by the Normal Distribution.

Extending this concept to three-dimensions, if the probability distributions along any two cross-sections at right angles through the population are normal and mutually independant, the population follows a two-dimensional normal distribution as shown in Fig. 3.

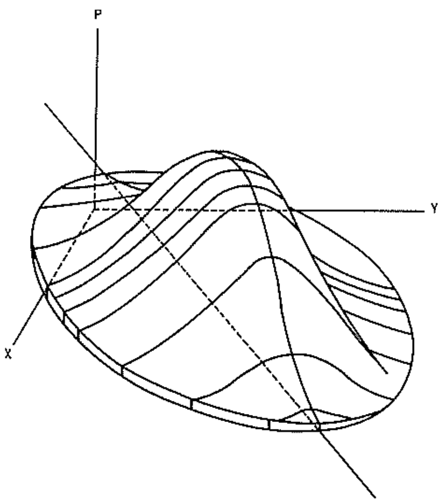

Fig. 3: Representation of a Two-dimensional Normal Distribution (From Crow, Davis and Maxfield Ref.8)

Fitting two-dimensional normal distributions to joint data is complicated by the presence of several joint sets, of which some may be overlapping. The following graphic procedure makes use of the powerful human ability for pattern recognition and, therefore in the writer's opinion, has considerable practical advantages over methods based on vector procedures which are purely mathematical and difficult to envisage.

The first step is to divide the joint system into joint sets using the density contours as a guide. Two great circles, mutually at right angles are then drawn along the directions of near-symmetry through the centre of each joint set. Fitting great circles in this manner is a standard procedure in structural geology (Ref. 1). Where the joint sets are elongated, the correct position of the great circles is obvious. Where the joint sets are near equant, the great circles may be placed arbitrarily.

The standard deviation of the angular distances of the joint poles is then determined along each great circle in turn. Where two joint sets overlap the standard deviations are computed from the non-overlapping portions.

Elliptical contours, distorted by the properties of the projection, are then drawn with the great circles as principal axes for various multiples of the standard deviation as shown in Fig. 4. These contours define the estimated probability distribution of the population represented by the joint set. They are smooth in contrast to the density contours whose minor irregularities are usually not reproducible for different samples, and are therefore meaningless in terms of the population.

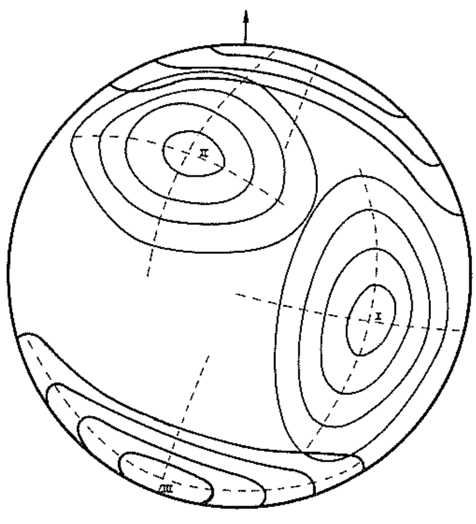

Fig. 4: Contours of 2-dimensional normal distributions fitted to three joint sets from the data shown in Figs. 1 and 2. Contours are ½, 1, 1½, 2 standard deviations from the mean.

The equation of the elliptical contours, may be derived from the standard formula for an ellipse. With respect to the two great circle axes labelled X and Y, the equation is:

$$y = \pm \; M^2 \, Sy^2 \left(1 - \frac{x^2}{M^2 \, Sx^2} \right)^{\frac{1}{2}} \quad \text{-----} \quad (1)$$

Using the properties of the two-dimensional normal distribution given by Crow, Davis and Maxfield (Ref. 8), the theoretical probability of a joint falling within any part of the distribution can be computed. The goodness-of-fit of the distribution can therefore be checked by means of the Chi-Square Test (Ref. 8).

Although the normal distribution has the practical advantages that its properties are well known and easily accessible in tabulated form, its theoretical development assumes distribution on a plane and it is therefore an approximation when used to represent the distribution of points on a projection of a sphere. The approximation is best when the dispersion is small. If required, more precise representation such as Fisher's distribution on a sphere could be used (Ref. 15).

Although the writer believes that the two-dimensional normal distribution is probably the best estimation of the probability distribution of most natural joint sets it is emphasised that the proposed design procedure does not necessarily depend on this assumption. For example, in highly irregular joint systems it may be suspected that the underlying probability distribution is uniform. In this case the area enclosed by any density contour may be predicted from the Poisson Distribution (Refs. 3 & 5), and the goodness-of-fit determined by means of a chi-square test.

Where no theoretical distribution appears applicable, the sample frequency distribution may be taken as the best estimate of the population probability distribution. However, in this case, a much larger sample is normally required.

(d) Kinematic Testing

The purpose of kinematic testing is to determine the range of joint orientations for which movement is possible. If the action of the resultant force is to move the block into the slope then failure is clearly impossible and further consideration of the shear strength of the joint is unnecessary. Extensive discussions on kinematic tests for blocks bounded by two or three joints are given in Refs. 4,9,10 and 11. The simple tests given by Panet (Ref. 9) are particularly useful.

Panet's tests distinguish between wedge type failure, where the sliding occurs along the intersection of two joints, and block type failure where sliding occurs along a single joint. Wedge type failure is kinematically possible only if the line of joint intersection is undercut by the slope. Block type failure can occur only if the line of the direction of movement on the joint is undercut by the slope. Panet examines only the case where the direction of movement is along the true dip of the sliding joint which is the case where gravity and hydrostatic uplift are the only active forces. However the method can be generalised to include the effects of earthquakes and any external forces by using the methods described by John (Ref. 4) to determine the direction of the resultant force. The direction of movement is then given on the equal-area projection by the intersection of the great circle representing the joint plane and the vertical great circle containing the direction of the resultant force.

The problem of kinematic analysis is reduced to its simplest form in the situation where gravity and hydrostatic uplift are the only forces involved, sliding can occur only along one set of joints, and tensile separation can occur in many directions in the rock mass. Failure in this case is kinematically possible only when the true dip of a foliation joint is undercut by the slope.

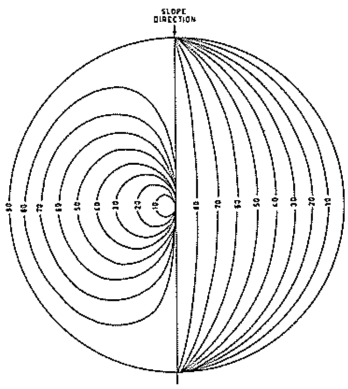

Fig. 5: Kinematic Test for sliding along a single joint. Use is explained in text.

On Fig. 5 the great circles on the right hand side of the diagram represent various possibilities for the design rock slope. The closed curves on the left hand side are the loci of the poles of the joints whose true dips lie on the corresponding rock slope (e.g. joint 3 in Fig. 4). These curves enclose the poles of all joints whose true dips are undercut by the indicated slope; that is, all joints that are kinematically unstable with respect to the slope.

(e) Limiting Equilibrium Analysis

A review of limiting equilibrium methods by Hoek (Ref. 12) shows that the relationship between shear stress τ and normal stress σ at failure can be adequately approximated by a straight line relationship:

$$\tau = c + (\sigma - u) \tan (\emptyset + i) \quad ---- \quad (2)$$

In rock slopes the cohesion c is primarily due to the presence of rock bridges between joints as discussed by Jennings (Ref. 13) and Robertson (Ref.2). The surface friction $\emptyset$ is determined by the nature of the rock material and probably by the effects of minor undulations on the joint surface.

The angle of roughness (i) is defined by Patton (Ref. 12) as the average angle between the major undulations on the joint surface and the direction of sliding along the joint.

In the writer's opinion, the best estimates of the shear strength parameters are those obtained by back-calculation of existing slope failures using the same analytical method that will be used to design the planned slopes. Assuming that the sliding blocks can be regarded as rigid and as long as the old failures are in slopes of approximately the same height as the planned slopes and have similarly continuous joints the cohesion and friction can be treated as a single angular quantity (Øa), computed as follows:

$$\phi_a = \tan^{-1}\left(\frac{\tau}{\sigma}\right) \quad \text{---- (3)}$$

Usually the least satisfactory aspect of back-calculating old failures is that the groundwater conditions at failure are poorly known. However, a realistic approach is to use the same assumptions regarding the height of the water table in both the back calculation and the design analysis. A conservative approach is to assume a higher level for the water table than that used in the back calculation.

In areas where old landslides are not available, the shear strength parameters must be estimated from laboratory and field tests and by judgment based on first principles. In this case, particularly, careful observation of the slope behaviour in the early stages of construction, followed by re-design if necessary, should be considered an integral part of the design process.

The designer has the option of applying a safety factor during the limiting equilibrium analysis (Ref. 4). The value assigned will naturally depend on the purpose of the slope and the possible consequences of failure.

In most cases however, where partial failures are considered acceptable, the writer prefers an approach whereby the limiting equilibrium analysis is carried out for a safety factor of one and judgement applied to the consideration of an acceptable level for the probability of failure.

In the case of a simple block slide an area of kinetic stability is defined as a small circle of angular radius Øa centred on the piercement point of the resultant force. For a dry rock slope affected only by gravity this small circle is centred on the centre of the projection as shown in Fig. 6 for the example where Øa is 34°.

Superposition of the small circle defining the area of kinematic instability and the curves defining areas of kinematic instability for various design rock slope angles on the probability distribution of the joint set 1 (from Fig. 4) is shown in Fig. 6. Joints with poles lying inside the area of kinematic instability and outside the area of kinetic stability are in unsafe orientations with respect to the indicated slopes.

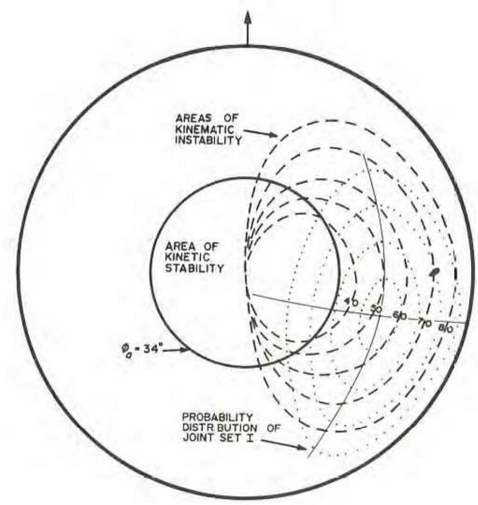

Fig. 6: Superposition of area of kinetic stability and areas of kinematic instability on the probability distribution of a joint set. Contours of the joint set are ½, 1, 1½, 2 standard deviations from the mean.

(f) Determination of the Probability of Joints Occurring in Unsafe Orientations

In any small portion of a rock mass joints are usually found to occur in between 3 and 6 directions. Where the joint system can be divided into sets it can be assumed for the purposes of slope design that one member of each set is present in any small element of the rock mass.

For any specific rock slope, it is usually found that sliding can occur only along joints belonging to one or two of the joint sets. These joint sets and the safe and unsafe orientations within them are determined by superimposing the results of the kinematic and limiting equilibrium analyses on the contours of the joint set. The probability that a joint will occur in an unsafe orientation behind the slope is obtained in one of the following three ways:

(1) If the joint set fits a two-dimensional normal distribution the probability that the joint will occur in an unstable orientation can be determined by the graphic transformation (for example Fig. 7 shows a graphic transformation of Fig. 6) which is superimposed one quadrant at a time over the Rectangular Normal Probability Chart published by Crow, Davis & Maxfield (Ref. 8) and reproduced in Fig. 8. The probability of an event occurring in any portion of this diagram can then be determined by counting rectangles.

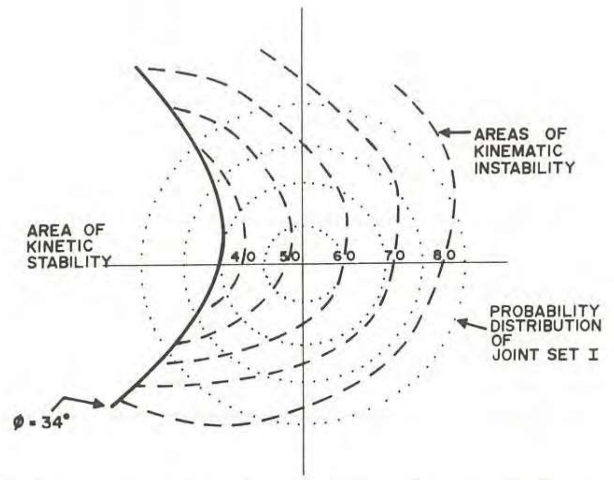

Fig. 7 : Graphic transformation of the data on Fig. 6. The elliptical contours defining the probability distribution of the joints in Fig. 6 are transformed to circles of radius suitable for superposition on Fig. 8.

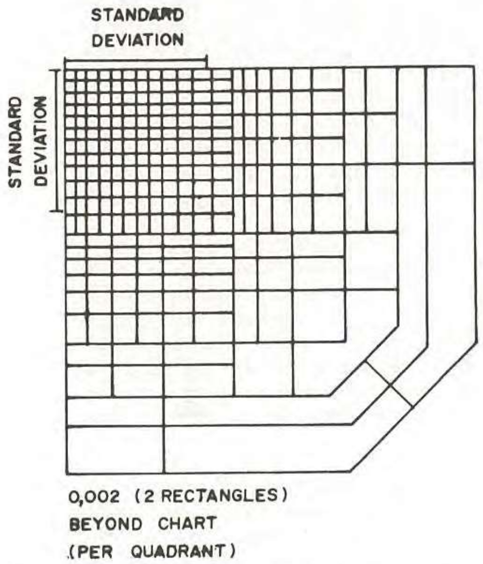

Fig. 8: Rectangular Normal Probability Chart Reproduced from Crow, Davis and Maxfield (Ref. 8) and attributed by them to A.D. Sprague. Each rectangular area represents a probability of 0.001.

(2) If the joint system has a uniform population distribution the probability Pj that a joint will occur in an unstable orientation is given by:

$$Pj = N_a (Au/At) \text{ ---- } (4)$$

(3) If the assumption is made that the sample distribution represents the best estimate of the population probability distribution the probability that a joint will occur in an unstable orientation is given by:

$$Pj = N_a (Nu/Nt) \text{ ---- } (5)$$

(g) Determination of the Probability of Failure for a Range of Slope Angles.

In simple cases where the joints are uniformly continuous for distances that are large compared to the size of the slope, the probability that a joint occurs in an unstable orientation, Pj, can be taken as the probability of failure. Where only a proportion, Pc, of the joints of any set are sufficiently continuous to result in a slope failure the probability of failure, Pf, is given by:

$$Pf = Pj.Pc \text{ ----(6)}$$

The probability of failure is then computed for a range of slope angles as shown in Fig. 9.

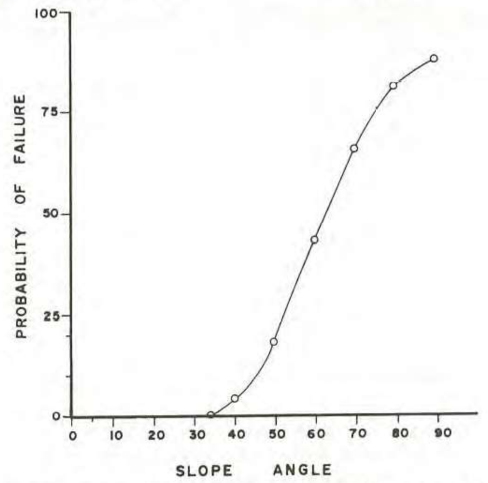

Fig. 9: Curve of Probability of Failure vs. Slope Angle for the example shown in Figs. 6 and 7. (In this case Pc = 1).

Where a slope is to be designed with safety as the over-riding consideration, the slope angle at which the curve becomes asymptotic to zero would be the appropriate design angle. Where partial slope failure can be contemplated the design slope angle is determined as described in the following section.

(h) Determination of the Most Economic Slope Angles

The total cost of a rock slope is given by:

$$C_t = C_o + C_f \text{ ---- } (7)$$

where:

C_t = the total cost of the slope

C_o = the initial cost of excavation

C_f = the cost of failure

For any rock slope the initial cost of excavation can be considered to be the cost of excavation of the volume of rock in excess of that required for a vertical rock slope; therefore:

$$C_o = co\ Vo \text{ ---- } (8)$$

where:

co = the unit cost of bulk rock excavation

Vo = the volume of rock excavated in excess of that required for a vertical slope

The cost of slope failure is given by:

$$C_f = P_f (C_a + C_b V_f) \text{ ---- } (9)$$

Where C_a represents those costs that are independant of the volume of material in the slide, such as part of the costs for loss of production, mobilization of equipment, and the costs of slope monitoring.

Determination of the portion of the cost of failure that is proportional to slide volume is complicated by the fact that the probability of failure and the expected volume of failure are related quantities and are both determined by the probability distribution of the joints behind the slope. A steep slope will fail back to the plane of the undercutting joint which can lie anywhere between the plane of zero probability of failure and the excavated slope surface. The product Vf Pf is therefore best determined by an incremental procedure as follows:

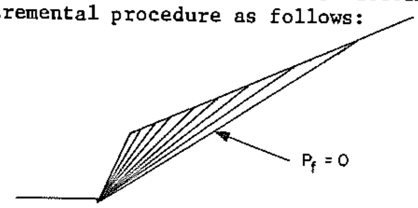

Fig. 10: Slope divided into a series of n elements.

If the slope is considered to be divided into a series of n elements as shown in Fig. 10 such that the base of the 1st element is along the plane with zero probability of failure, the top of the nth element is along the proposed slope surface, the volume of the ith element is Vi and the probability that a joint undercuts the base of the ith element is Pi then:

$$P_f V_f = \sum_{i=1,n} P_i V_i \text{ ----- } (10)$$

Fig. 11 shows the curve of total cost per foot of slope vs. slope angle for the example, assuming $C_a = 0$ and the slope direction and probability distribution of the joints are as shown in Fig. 6 and the probability of failure is as shown in Fig. 9.

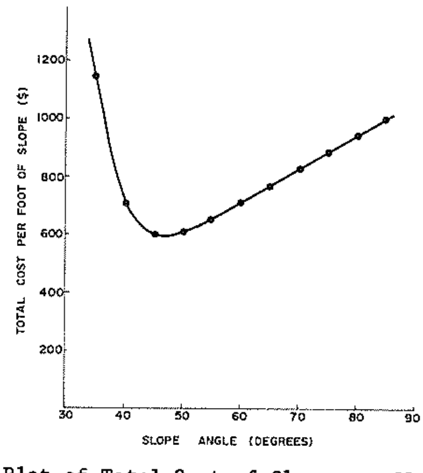

Fig. 11: Plot of Total Cost of Slope vs. Slope Angle.

The following conclusions may be drawn from Figs. 9 and 11:

(1) The slope with the least total cost ($600 per foot of slope length) is between 45 and 50°. The probability of failure for a slope of 45° is 10% compared to 19% for a slope of 50°.

(2) The slope with zero probability of failure is 34°. This slope has a total cost of $1,130 per foot.

It is interesting to note that a safety factor of 1.5, applied to the slope of zero probability of failure in this case would result in a cut slope of 24° which would be 3° flatter than the natural slope. This illustrates the absurdity of applying arbitrary safety factors to slope designs already based on the worst likely conditions.

V EXTENSION OF THE PROCEDURE TO SLIDING ON COMBINATIONS OF JOINTS

When more than one joint set is involved in the formation of the sliding surface the problem is complicated by the fact that the joint sets are rarely independant in their orientations. A conservative method is to assume that the joint sets are completely dependant so that any joint in the first set will be combined with a joint in the second set that is orientated to form the least stable combination. The methods of John (Ref 4) can then be used to define the areas of kinematically and kinetically unstable orientations for joint set 1. The slope design can then be completed as described in the previous section.

The least conservative approach is to consider that the joint sets are independant so that the probability of finding any combination of orientations is the product of the probabilities of finding each orientation alone. An intermediate approach, which the writer feels is probably the most realistic in many cases is to assume that any joint in one set is combined with the mean joint orientation in the other set.

In open pit mines large scale failures occur along complex failure surfaces which may consist partly of major weakness such as faults and partly of step-like surfaces composed of many joints. If as shown in Fig. 12, the failure is rotational it may be analysed by the methods of soil mechanics as suggested by Wilson (Ref. 14) using the failure criterion given in equation 2. In this case one of the principal variables is the roughness of the step-like series of joints. The probability distribution of the roughness can be determined for any small portion of the failure surface by the following method:

(1) Determine the probability distribution of the joint set forming the flatter portion of the step.

(2) Plot the direction of the average failure surface as a pole. The angle between this pole and a given joint orientation is the angle of roughness where this joint direction prevails.

(3) Determine the probability that a joint pole lies within any required radius of the pole to the failure surface by constructing inclined small circles around the pole and analysing by the graphic procedure described in the preceding section.

(4) The probability of failure is then given by the probability that the roughness is less than that required to provide the required safety factor.

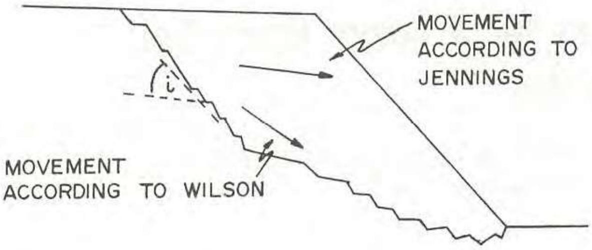

Fig. 12: Rotational Failure along step-like patterns of joints.

If as shown in Fig. 12, the failure takes place by sliding along the flatter joints and separation along the steeper joints as suggested by Jennings (Ref. 14) the problem is simply an extension of the problem of sliding along a single joint as described in the preceding example.

In actual slopes, the sliding mass is of course non rigid and the initial movement is probably partly rotational and partly transitional.

If the step-like failure surface is considered to be very large with respect to the joint spacing so that it contains a large number of joints then the mean direction of the joints, in the direction of sliding, can be used in the analysis as suggested by Jennings (Ref. 14). In this case the probability that the joint continuity is greater than the value required for stability may become the principal basis for design.

For very large slopes the probability that the slope will fail can be interpreted as the percentage of the slope that can be expected to fail. Estimation of this percentage is the principal problem in open-pit design.

CONCLUSIONS

The probability of failure provides a realistic basis for the design of rock slopes where the locations and orientations of the rock defects behind the slope are not known in detail.

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