# An Investigation of an Earth Pressure Problem using a Rod Model Analogue

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SUMMARY.- The paper presents results obtained from a small rigid retaining wall rotating about its top into brass-rod fill material used to model a granular medium deforming in plane strain. Detailed measurements were made of the variation of the wall pressure distribution with wall rotation and the complete displacement and strain fields produced in the model. These are discussed and compared with published data obtained from similar walls rotating into sand backfill. Whereas similarity between both measured and predicted results suggests that in these circumstances the rod material is a viable analogue for a loosely packed granular medium their kinematic behaviour is disimilar. In contrast to a loose granular medium distinct velocity discontinuities develop in the rod material around which volumetric expansion occurs and changes of mean stress produce negligible volumetric strains.

## I.- INTRODUCTION

In the investigation of plane strain models incorporating granular materials an unavoidable problem is introduced by the restraining forces which arise from the relative motion between the deforming material and the structure introduced to impose the plane strain boundary condition. There are two basic alternative approaches to this problem, either to use relatively wide models (Ref. 1,2) and by taking all measurements as nearly as possible on the centre plane of the model minimise the effect of the extraneous plane boundary forces or to use much narrower models (Refs. 3,4) across which any additional forces will be essentially uniformly distributed and accept the inherent errors that these will produce. Whereas measurements of stresses on the centre section of a long retaining wall (Ref.1), or foundation (Ref.2), is feasible detailed measurements of displacement and strain fields can, at present, only be made on the boundary planes of such models (Ref. 5,6). The two sets of data obtained this way will not therefore be strictly comparable. Use of a narrow model can ensure that the measured stress and strain fields do refer to the same plane and since both of these will be practically uniform across the width of the model the displacement field may be determined equally well either in the centre plane using X-ray techniques (Refs. 3,4) or on the boundary planes (Refs. 2,5,6). Since neither of the models mentioned is entirely satisfactory a number of workers have used a two dimensional rod-medium as an analogue of the granular material (Ref. 7 to 12). Such an analogue has immediate attractions in that the plane strain boundary condition is automatically and simply satisfied without using any confining structure and also, by using small diameter high density metal rods, relatively large forces can be generated in small models. The model size is then mainly governed by the requirement that the mean rod diameter must be statistically small in relation to the dimensions of any load cells or displacement measuring grids used. However any use of a "rod-analogue" makes the fundamental assumption that it is indeed a valid representation of a granular material deforming plane strain. This is an unavoidable requirement whether the analogue is to be used for obtaining data on either detailed or overall kinematic behaviour or detailed or overall stress distributions. This paper attempts to investigate the validity of a rod-analogue used in conjunction with a rigid model retaining wall rotating about its top into the fill by comparing in some detail the experimental results obtained with those obtained from similar models using sand. (Refs. 3,4,15)

## II.- THE EXPERIMENTS

### (a) The Rod Material

Brass rods of three sizes were used, (2.62, 1.56 and 0.71 mm diameter, by 73 mm long) combined in the ratio of 20% : 50% : 30% by weight respectively, which from detailed visual inspection appears to overcome the very strong anisotropy generated when single sized rods are used alone. It was found that hand packed rods, lightly tamped in place in the model, gave an equivalent specific gravity of 6.50 readily reproducible to within ± 2%. The converse of this attractive feature is that the achievable range of specific gravities is quite small being from 6.0 to 6.75. In order to satisfy the statistical requirements mentioned above minimum load cell dimensions of 25 mm and displacement grid dimensions of 15 mm were adopted. Biaxial tests on this material (Fig.1) using lubricated end plattens suggested that its Coulomb $\phi \approx 32^{\circ}$ at the low mean stress levels developed in the model, that it has a brittleness index = 0.9, that it is essentially incompressible under small spherical stress increments and that its friction angle on the mild steel wall = $7^{\circ}$.

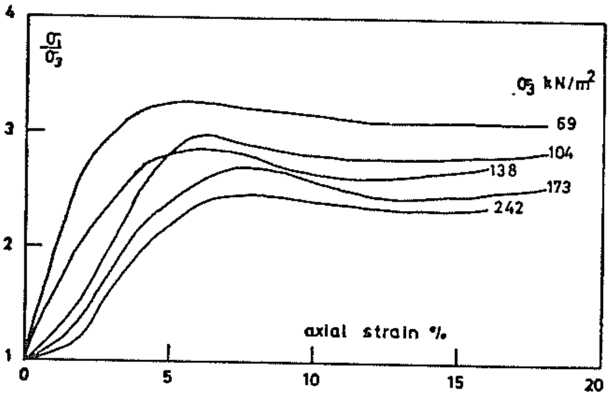

FIG.1

### (b) The Retaining Wall Model

This is shown schematically in Fig. 2 the most important feature of which is the rigid model wall, 190 mm high (H), built entirely from seven small full-width (73 mm) load cells. They each had an active face height of 25 mm, were very stiff (1.5 KN/mm) and measured normal forces only up to 100 N on this face with a sensitivity of 0.4 N irrespective of the eccentricity of the force. The wall could be rotated in displacement controlled increments about an axle, 25 mm above top fill level, supported in a very stiff (10KN/mm) instrumented bearing designed to measure the vertical and horizontal reaction forces on the axle. The force in the cable rotating the wall via the quadrant at its top (Fig. 2) was also measured. These three forces therefore provided an independent check of the resultant forces acting on the wall.

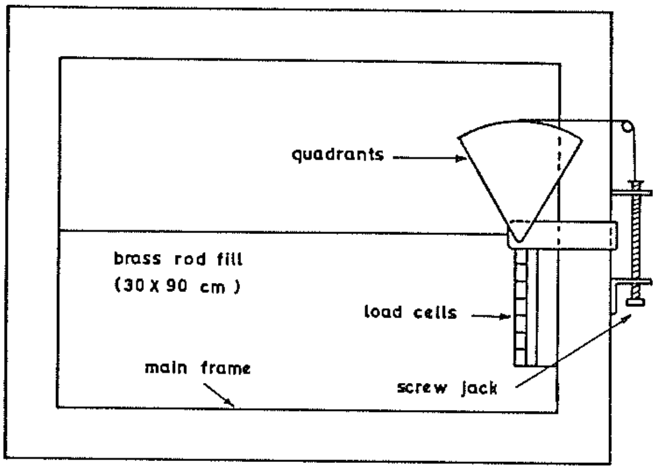

FIG. 2

## III.-RESULTS

### (a) Wall Pressures

Fig. 3 shows as full lines typical dimensionless wall pressure distribution $(\sigma/\gamma H)$ diagrams obtained as the wall was rotated into the fill progressively through angles of 0.2°, 0.4°, 0.8°, 1.5°, 3° and 6°, where $1\frac{1}{2}°$ rotation corresponds to a wall toe movement of about 5 mm. The total normal force on the wall increased rapidly with wall rotation reaching a maximum at between 3° and 6° and only decreased by typically 10% as the rotation was increased to 10°. Its maximum value was 270 N ± 10 N acting at 0.28 H above the wall toe which is quite close to the total normal 'Rankine' force of 284 N calculated for a smooth wall and $\phi = 32°$.

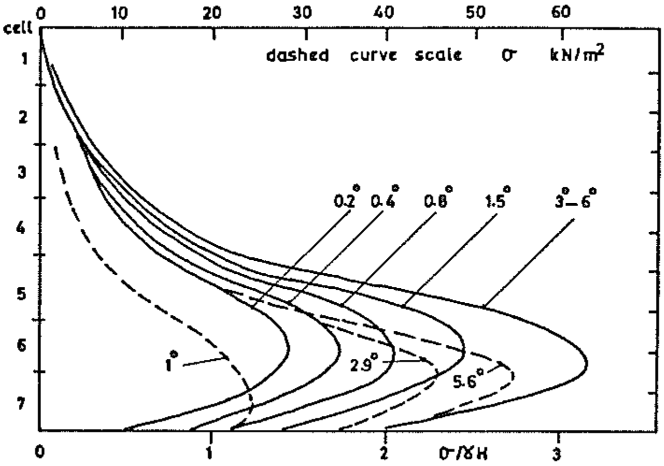

FIG.3

Also shown in Fig. 3 (dashed lines) are actual pressure distributions taken from Ref. 4, Fig. 19 for a similar model wall rotating into a loose sand ($e \simeq 0.70$, $\phi \approx 35°$) through angles of 1°, 2.9° and 5.6° which are of a generally similar shape to those obtained from the rod model and also indicate a maximum total force occurring at about 6° wall rotation although the centre of pressure is rather lower on the wall at 0.21 H above the base. In this case the maximum wall force measured was 720 N whereas a smooth wall $\phi = 35°$ calculation gives 690 N and 720 N requires a wall friction angle of 2°. The similarity between the wall pressure distributions, the rotation to peak normal force on the wall and its small decrease with additional rotation in both the rod model and the loose sand model suggests that their behaviour is closely analogous at the low stress levels used. It should be noted that identical tests using a dense sand ($e = 0.53$) gave a peak normal wall force at 1.6° rotation which had dropped by more than two thirds at a rotation of 6° (Fig. 17a, Ref.4). Measurements made on a long wedge shaped body (60° apex angle) driven vertically into both sand and the same rod material (Ref.2) showed that again the penetration resistance varied similarly in the rod model and loose sand. Other experimental evidence (e.g. Refs. 10,11) also supports this conclusion.

Unfortunately the overall vertical friction forces acting on the walls could not be compared since the vertical force sensitivity of the axle support bracket in the rod model, proved to be inadequate although the summation of the load cell readings agreed almost exactly with the resultant horizontal force measurements.

## (b) Displacement and Strain Fields

Photographs of the deforming model were taken with the intention of measuring the displacement of a marker grid (Figs. 4) (in this case the end point of a shadow cast by protruding rods) from which the incremental strain field could be obtained. This was attempted for a cumulative wall rotation from $0^{\circ}$ to $6^{\circ}$ by taking hand measurements from an enlarged projection of the photographic negatives, a method which proved to be not only very laborious but also insufficiently sensitive. At a later date we developed a stereo-photogrammetric displacement field measuring technique (Refs. 5,6) which does not require the use of a special marker grid and is equally applicable to either granular materials or rod models. This was used with the negatives of the two photographs shown in Figs. 4 (a,b) which refer to one small wall rotation increment of from $4\frac{1}{2}^{\circ}$ to $6^{\circ}$. If these plates are viewed in an ordinary mirror stereoscope the three dimensional relief which was contoured to obtain the complete horizontal and vertical displacement field components shown in Figs. 5a, b will be clearly seen. In particular the major velocity discontinuity occurring along the bounding slip line is very evident.

The complete displacement contours were drawn very simply in about two hours using a Galileo-Santoni Mark II plotting machine as described in Refs. 5,6. Subsequently more precise "spot-height" measurements were made on the stereo model at points spaced 16 mm apart in the real model from which the incremental strain fields shown in Figs 6(a,b) were calculated. For this purpose the displacement components were measured to 10-micron sensitivity using the original 58 mm square roll-film negatives which were approximately a 1:5 scale reduction on the actual model size. Obviously larger scale photographs on plates can potentially produce much more precise information. The rotation increment of $1\frac{1}{2}^{\circ}$ (.025 radians) is almost exactly that used (.023 rads.) by workers at Cambridge to obtain the maximum shear strain contours published as Fig. 6 in Ref. 3, Fig. 25a in Ref. 4 and Fig. 19a in Ref. 15 for a wall rotating about its top into a dense Leighton Buzzard sand (e= 0.52 - 0.55).

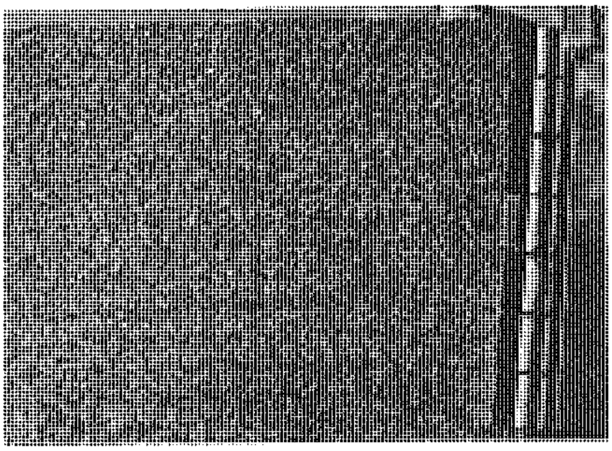

FIG. 4a

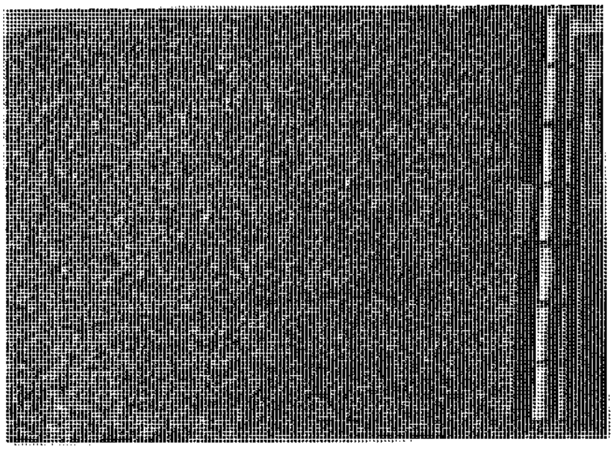

FIG. 4b

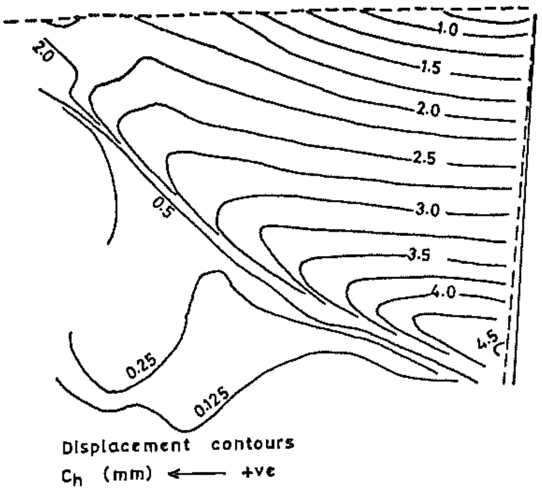

FIG. 5a

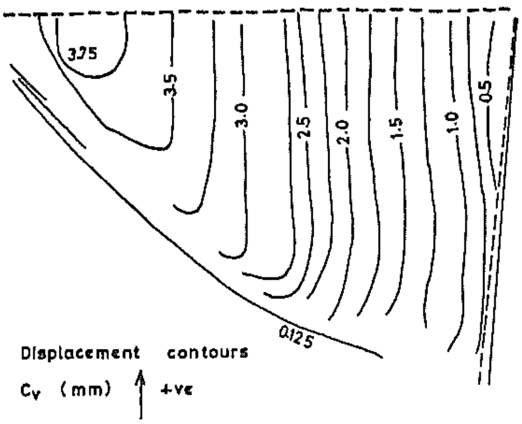

FIG. 5b

Comparison of the maximum shear strain increments shown in Fig. 6 (a) with any of the Cambridge figures illustrates two main features. Firstly that the sensitivity of the photogrammetric technique is comparable with that of the highly complex'X-ray' method used at Cambridge and secondly that the zones of intense shear distortion which occur around the major velocity discontinuity are rather more pronounced in the rod model than in the dense sand. By contrast measurements in loose sand, for example Figs 17 (a) Ref. 15 and Fig. 26(a), Ref. 4, show that no such zones are developed.

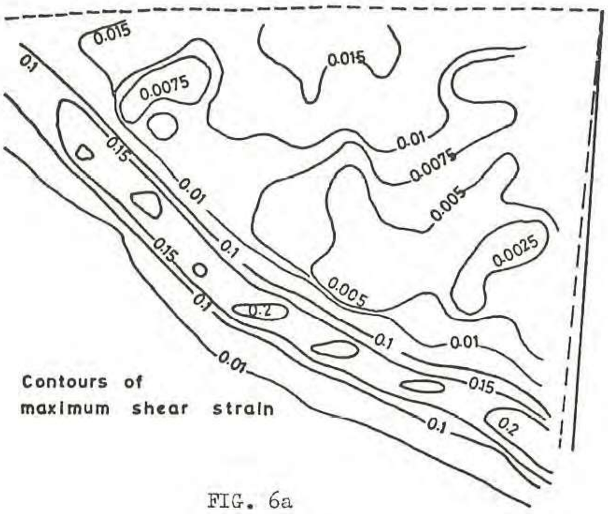

FIG. 6a

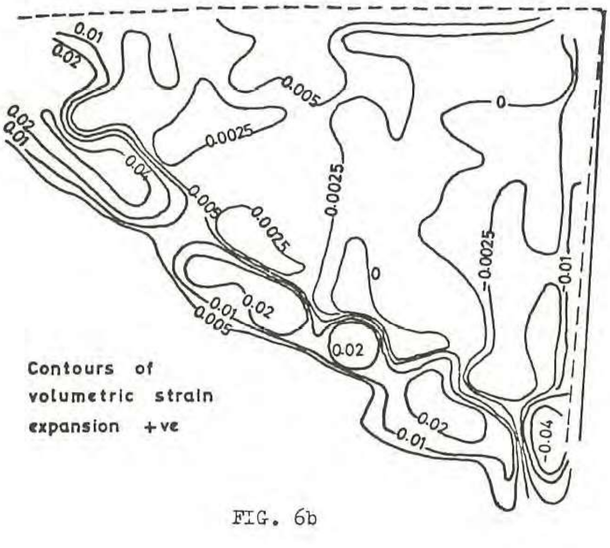

FIG. 6b

Fig. 6(b) is a plot of the volumetric strain increments which occur simultaneously with the maximum shear strain increments of Fig. 6(a). The former are seen to be confined exclusively to the zones of high shear strain and are obviously inter-related phenomena. The only comparable published data appears in Figs. 26 (c,d) Ref.4 which gives much less detailed information on a loose sand test. Here there was no zone of intense volume change although interestingly some decrease in volume was measured near the toe of the wall as shown in Fig. 6(b). However results for a dense sand Figs. 22, Ref.4, although related to a wall rotating about its toe, do indicate the development of volumetric strains associated with a velocity discontinuity much more closely analagous to the rod material behaviour.

These comparisons demonstrate quite conclusively that the rod analogue does not deform as a loose granular medium but that its kinematic behaviour is much more analogous to that of a densely packed granular material.

This statement is again supported by kinematic measurements made on the previously mentioned wedge penetration tests into rod material and sands (Ref.2)

(c) The Displacement Field

Although almost invariably in soil models displacement fields are measured and subsequently differentiated to obtain strain fields the displacement fields themselves have until recently (Refs. 2,13,14) received very little attention. Displacement (or velocity) field data is best summarised in a velocity hodograph. This is merely a diagram in which the radius vector to any point in it, measured from an origin in the diagram, uniquely represents the actual displacement (or velocity) vector of the corresponding point in the deforming field (Refs. 13, 14). In particular the hodograph of any field which is rotating as a rigid body will merely reproduce the field rotated through a right angle. Furthermore equal interval displacement contours of such a field will appear as equi-spaced orthogonal straight lines in diagrams such as Figs.5 (a,b.) A large area of these figures does appear to be approximately of this form and therefore a hodograph of the network of points shown in Fig.7(a) has been constructed (Fig. 7(b)) using the displacement components obtained from Figs. 5(a,b)

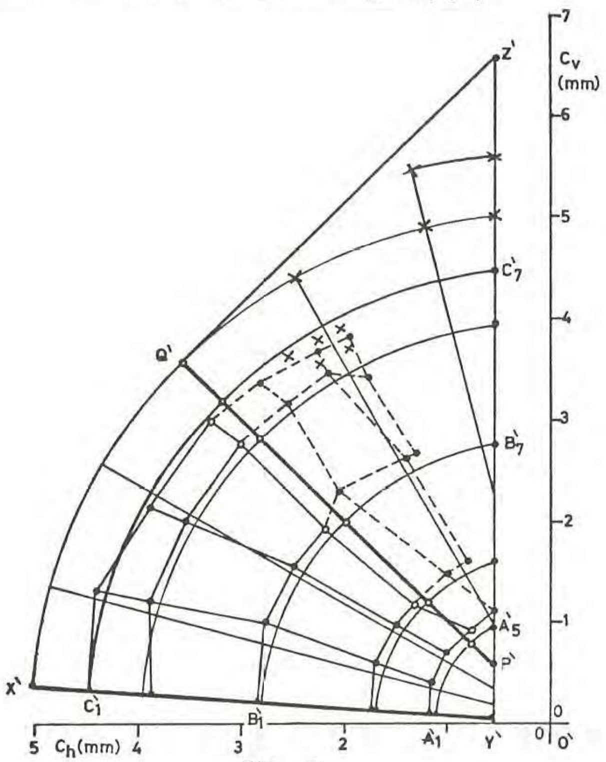

FIG. 7b

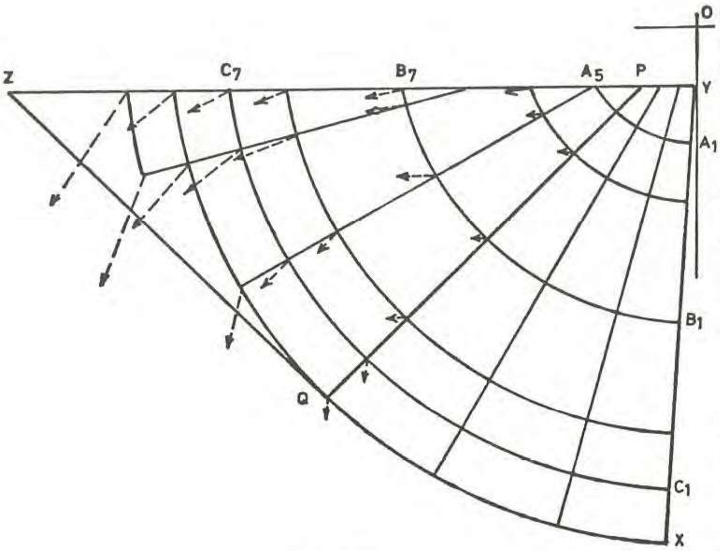

FIG. 7a

More specifically if the whole region XYZ in Fig 7(a) were rotating as a rigid body about the wall rotation centre O all points in XYZ (for example $A_1$-$A_5$, $B_1$-$B_7$, $C_1$-$C_7$ etc.) would map into the hodograph Fig. 7(b) as X'Y'Z' and the specific points mentioned above would appear at $A_1'$ -$A_5'$ , $B_1'$ - $B_7'$ , $C_1'$ - $C_7'$ etc. It should be noted that the displacement scale in the hodograph refers to the absolute displacement increments of the "soil" particles in mm and that the theoretical figure is plotted for a general rotation of the model of $1\frac{1}{2}^\circ$ (from $4\frac{1}{2}^\circ$ to $6^\circ$ away from the vertical). The comparisons being made therefore relate to predictions of extremely small displacement increments of individual "soil" grains. Fig. 7(b) also shows the less regular hodograph obtained by plotting the actual displacements of points $A_1$ ... $C_7$ etc. from the experimental measurements shown in Figs. 5(a,b). Within the zone corresponding to XYPQ (mapped at X'Y'P'Q' where PQ is at $45^\circ$ to the horizontal) the agreement between the two hodographs is quite remarkable which demonstrates that this region did indeed rotate essentially as a rigid body during the rotation increment. This is, of course, further substantiated by the fact that within this region in Figs 6(a,b) both the volumetric and maximum shear strain increments are very nearly zero.

In the regions PZQ and P'Z'Q' the two hodographs are of quite different form and it is of interest to discuss this briefly. The boundary conditions along the wall XY are kinematically determined as explained in Ref. 14 and therefore one limiting displacement field could be a simple rigid body rotation about O of the whole failing zone XYZ as discussed above leading to the X'Y'Z' hodograph. Alternatively, and more probably, the stress governed boundary conditions along YZ will govern the behaviour near this boundary and a further plausible kinematic solution allowing for this would postulate that within PQZ the velocity characteristics may form an orthogonal straight line network out-cropping at the consistent angle of $45^\circ$ along PZ and marrying up with the circular arc fan along PQ. Interestingly the bounding velocity discontinuities in Figs. 5(a,b) do outcrop at $45^\circ$ and conform very closely to the outer boundary of the figure just described. If now this whole region (XYZ) started to rotate about O the triangular zone (PQZ) could slip consistently in an "active" sense along planes parallel to PQ to maintain continuity of the body along ZQ. The resultant displacement field produced has much in common with those discussed in relation to retaining walls by James and Bransby (Ref.15). In particular the velocity hodograph for such kinematic behaviour would comprise only the region X'Y'P'Q' in Fig. 7(b) with the whole of the region PQZ "collapsing" onto P'Q' (Refs 13, 14).

The two alternative mechanisms described probably represent reasonable approximations to the extremes of kinematically admissable solutions and it is intriguing to note that the measured behaviour falls very neatly between them.

In Fig. 7(a) relative displacement vectors have been added (measured directly from Fig. 7(b)) attached to the points within PQZ. These show in magnitude and direction the measured displacement of each point relative to a basic rigid body rotation of the whole field of $1\frac{1}{2}^\circ$ about O.

## IV.- CONCLUSIONS

These relate solely to the preceeding comparisons of the measured behaviour of rod analogue material, deforming at low stress levels behind a fairly smooth rigid model retaining wall rotating about its top into the rod fill, with that of a similar model using sand backfill deforming in plane strain.

(a) Wall Pressures.

The rod material appears to be a reasonable analogue of a loose sand. This is supported by the facts that they both

- (i) do not exhibit either peaks in their stress-strain behaviour or any appreciable decrease in apparent friction angle at high strains.
- (ii) the total forces on both walls reached their maximum value at rotations of $5^\circ$-$6^\circ$ and decreased very little with further rotation of the wall.
- (iii) In each case the total force could be approximated reasonably by conventional smooth wall passive earth pressure calculations using experimentally measured $\phi$ values.

(b) Kinematic behaviour of the backfill.

Here the rod analogue material behaves quite differently from a loose sand and more closely reproduces the behaviour of a densely compacted sand in that

- (i) their volumetric compressibility under spherical effective stresses is relatively low
- (ii) they can both form distinct velocity discontinuities which produce zones of intense shear distortion within which high

dilatancy rates can be measured.

(iii) detailed measurement and analysis of both strain and displacement increment fields shows that rigid body motions develop in the rod material which do not do so in a loose sand.

(c) The behaviour of the rod-analogue is therefore somewhat ambiguous and it does not appear to be suited to providing information on either the kinematic behaviour of loose granular materials or the stress distributions associated with dense ones.

The measurements made directly on the rod analogue model were obtained by Messrs. R.J. Reynolds and M.K. Chatterjee (1967) in connection with their student research projects.

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