Patterns of Strain in Strength Test Samples

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SUMMARY. - The paper presents internal strain data obtained using an X-ray Method from rectangular prismatic plane strain compression samples and on cube and cylindrical samples tested in triaxial extension and compression. The results show that homogeneous strain conditions develop in these samples as long as tests are performed using lubricated platens. The use of rough platens should be avoided since the severe distortions which are produced make the strain data of questionable value.

I.- INTRODUCTION

Reports by Kirkpatrick & Belshaw (ref.2) and Kirkpatrick & Younger (ref.3) describe investigations into the distributions of strain within cylindrical triaxial compression test samples in an attempt to establish the value of this test as a means of providing reliable stress-strain data. It was concluded in these reports that for drained sands at least, relatively uniform distribution of strain and hence homogeneous strain states are developed within the compression cylinder as long as adequate precautions are taken to reduce end restraints by providing satisfactory lubrication at the end platens. This condition is important since the usual practice of estimating strains from displacements measured at the sample boundaries is thus valid. Such practices cannot however be considered valid in cases where platen lubrication is not provided since the severe distortions induced in these cases result in large gradients of strain throughout the sample.

Since the publication of these reports the work has been extended and the present paper describes investigations into the conditions within cylindrical extension samples, cubical samples under direct extension or compression, and rectangular prismatic samples tested under plane strain deformation.

II.- EXPERIMENTAL WORK

Internal strains are determined from displacements of lead shot placed within the samples on a square grid at roughly 25mm spacing observed on radiographs exposed to an X-ray beam directed through the sample. The principles of the method and the experimental procedures have been described previously by Kirkpatrick and Belshaw (ref.2).

a) Cylindrical Samples

Cylindrical Samples where both rough and lubricated end platens were used were tested under conditions of triaxial extension. Rough sand paper discs were inserted at the sample ends to produce rough platens while end lubrication was achieved by placing two layers of greased latex over polished glass plates placed on top of heavy duralumin platens.

Samples in all cases had a diameter of the order of 220mm and a length to diameter ratio of 1. Conventional tests with rough platens are performed on samples with length to diameter ratios of the order of 2. For the convenience of testing in the present tests it was presumed that the strain conditions in a symmetrical half of such a sample could be reproduced in a sample with a length to diameter ratio of 1 tested with a lubricated platen at one end and a rough platen at the other. The adequacy of this simulation in which the lubricated platen is taken to represent the mid plane of the conventional rough platened sample has been shown for the compression test by Kirkpatrick and Younger (ref.3).

b) Cube Samples

Both compression and extension tests were performed on samples of cubical form. These samples had side lengths of approximately 190mm and were tested in all cases between lubricated end platens. A cube sample set up and ready for a compression test is shown in Fig. 1. The symmetry of deformation in cube tests was checked by comparing the lateral displacements measured across opposite faces of the cube with those measured on the diagonal between opposite corners by taking X-ray observations of lead shot placed in both of these planes. This was done in two ways. In the first case the lead shot was placed in both planes within the same sample and X-ray exposures were taken normal to each plane. This entailed unloading and rotating the sample for each observation. In the second case the lead shot grids were set up separately in pairs of samples, a diagonal grid being formed in one while the grid across the faces was set up in the other. Both samples in the pair were prepared to have closely comparable initial porosities and the X-ray observations were taken at the same boundary strain values in each test. The symmetry checks were thus made without needing to unload the samples. Symmetry checks were not made in the tests on cylindrical samples since previous observations in this test (Kirkpatrick and Belshaw ref.2) indicated that a high degree of symmetry can be expected. X-ray observations in the cylindrical tests were only made therefore while the sample was

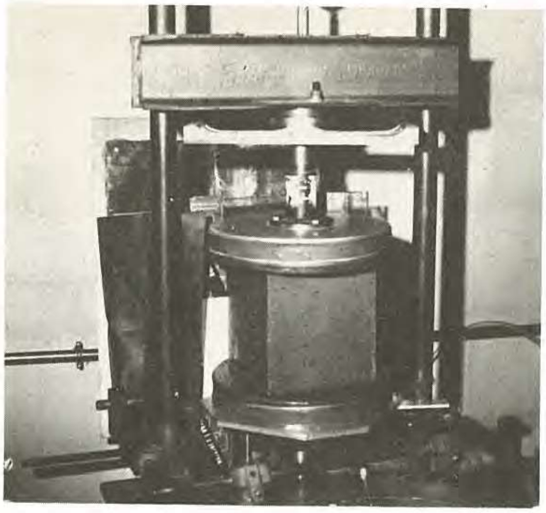

FIG. 1 CUBE COMPRESSION SAMPLE

in the loaded state.

c) Plane Strain Samples

These samples had breadths of approximately 200mm and length to breadth ratios of 1 or 2 depending whether they were tested with lubricated or rough end platens. The plane strain condition $\varepsilon_2 = 0$ was obtained by testing the samples between rigid perspex plates giving a thickness of sample in the 2 direction of about 75mm. This thickness was chosen to allow clear radiographs to be obtained with X-ray equipment available at that time although it was not considered ideal from the point of view of stress measurement. A double layer of greased latex was provided between the sample membrane and the perspex plate to cut down side friction. Although the side friction cannot be entirely eliminated it is presumed that the displacements at the centre of the sample will not be influenced by the side restraint. Allowance was made for friction for the calculation of the stresses by assuming that $\sigma_2 = \frac{1}{2} \times (\sigma_1 + \sigma_3)$ and taking the value of 0.1 for the coefficient of friction between the greased latex and the perspex plate.

III.- SOIL PROPERTIES AND TEST CONDITIONS

The material used in the tests was the proportion of Leighton Buzzard sand passing No. 14 B.S. sieve and retained on No. 25 B.S. sieve. The limiting porosity range for the material is 0.33 to 0.44 and the method of placement as described in Belshaw (ref.1) gave porosities in the order of 0.36.

The sand was tested in a dry condition and confining pressures were induced by vacuum applied to the inside of the samples. This lead to the most efficient use of X-rays since the outer casing and cell water used in normal procedures were no longer required. The effective confining pressure on all the samples was of the order of 77 kN/m^2 (11 p.s.i.) Vertical axial loads were applied through non tilting platens. The confining pressure was kept constant during each test, the axial stress being increased in a compression test and decreased in the extension test. The procedures used initially in preparing the extension test samples was identical to that used for compression samples. It was found however that the use of the enlarged platens, required in the case of the compression tests to allow the lubricated samples to spread latterally over their ends, were not satisfactory for the extension tests. Difficulty was found in preparing the tops of the samples when these enlarged platens were used since it was not possible to ensure that gaps did not develop between the top of the sand and the base of the top platen. This apparently did not adversely affect the samples tested in compression but in the extension test the presence of these gaps seemed to allow the sample to loosen non-uniformly at the top so that failure occurred prematurely due to localised necking which developed in the sample immediately below the top platen. In the extension tests the diameter reduces as the strains increase so that oversized platens are not necessary. The difficulty with the gaps was overcome therefore by using slightly undersized platens which fitted inside the former supporting the sample during preparation so that the platen could be made to rest positively on top of the sand.

IV.- RESULTS

a) Cylinder Extension Tests

Figure Nos. 2 and 3 relate to a cylindrical extension test in which lubricated platens were used at both ends of the sample. In this test the peak stresses occurred at a boundary axial strain of about 8% so that the data presented is for prepeak conditions. In Figure 2 the radial displacements u, for each row of lead shot are plotted on bases corresponding to the initial vertical position of the row in the sample as illustrated. A high degree of symmetry is observed in the upper two thirds of the sample while in the lower third there is a tendency for the left side to deform more rapidly than the right. Also the lateral displacements are slightly greater at the top than near the base. Despite this slight non uniformity it can be said for practical purposes that a homogeneous state of lateral strain existed over most of the sample and in view of the linearity of the relationship between radial displacement and radial distance, r, from the centre of the sample the condition of equality between the radial and tangential strains ($\varepsilon_r (= \partial u / \partial r) = \varepsilon_\theta$ (= u/r) generally assumed in the interpretation of such tests held true.

Fig. 3 shows the axial displacements of the columns of shot measured in the same test. Only slight differences in behaviour were observed between the left side and the right side of the sample and there was little to distinguish between the individual columns on respective sides. For the sake of clarity therefore only the right side data is shown. There was a tendency for the axial strain, defined as $\varepsilon_z = \partial w / \partial z$, represented by the slope of the displacement curves, to be smaller at the base and larger at the top than the average strain measured over the platens. The strain over the centre half to two thirds of this sample was however, approximately equal to the average strain.

Data for a simulated half of a rough ended sample are shown in figs. 4 and 5 where displacements from an extension test in which a lubricated top platen

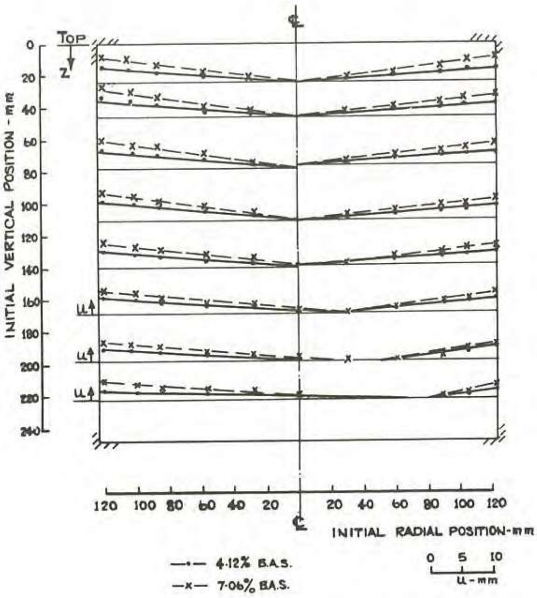

FIG. 2 CYLINDER EXTENSION LUBRICATED ENDS 1 : 1 RADIAL DISPLACEMENTS TEST 101.

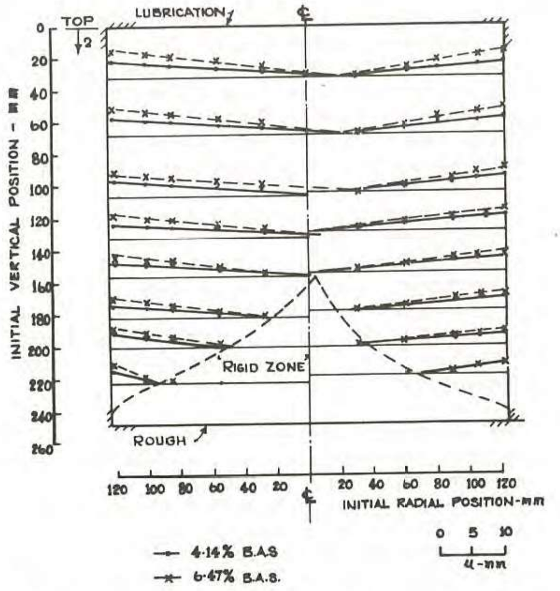

FIG. 4 CYLINDER EXTENSION - LUBRICATED ROUGH 1 : 1 RADIAL DISPLACEMENTS TEST 102.

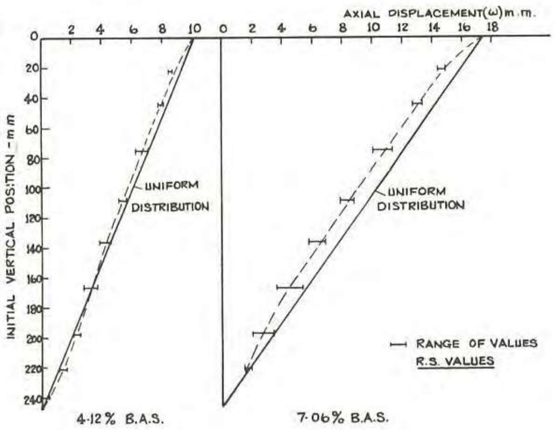

FIG. 3 CYLINDER EXTENSION LUBRICATED ENDS 1 : 1 AXIAL DISPLACEMENTS TEST 101.

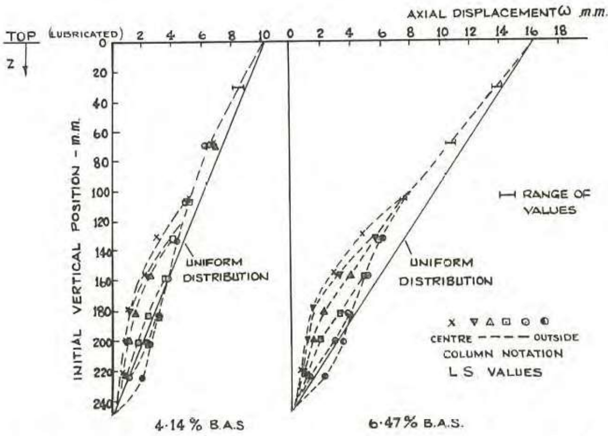

FIG. 5 CYLINDER EXTENSION LUBRICATED/ROUGH 1 : 1 AXIAL DISPLACEMENTS TEST 102.

was used in conjunction with a rough platen at the base. Observations were taken at two boundary axial strain (B.A.S.) values the latter of which was close to the peak strain which was somewhat smaller than the peak strain in lubricated samples.

The radial displacements shown in Fig. 4 are fairly symmetrical about the geometric centre line of the sample. A rigid zone in which the displacements are zero or too small to be measured is seen to have developed near the rough base and as a result of this a homogeneous state of strain cannot be claimed to exist in the sample. This conical shaped rigid zone has a height equal to about 0.35 x diameter and is appreciably smaller than that (=0.5 x dia.) developing near rough platens in the compression test. Although the base angle of the circumscribing straight sided cone at 35° is on the correct side of the 45° line compared with the compression test it is appreciably greater than the angle of 27° (45° - $\phi/2$) predicted for the failure plane by the Mohr Coulomb theory. Within the height of the sample containing the cones the lateral tangential and radial strains are not equal since $u/r \neq \partial u/\partial r$.

Lack of uniformity is also evident in the axial displacements plotted in Fig. 5. This non-uniformity is most pronounced near the rough base where the data indicate appreciably larger axial strains for the columns of lead shot near the outside of the sample than for those near the centre.

The distortions within samples with rough platens in the extension test are much less severe than those of the compression test over much of the stress strain curve. Another disadvantage of the rough platened extension test under normal conditions lies in its tendency to form local necking in the sample at stresses close to peak. This clouds the accuracy of the stress determinations at these stages. Necking was not detected in the present tests but rather a reduction in diameter which was greatest at the lubricated platen. This difference in behaviour compared with that more generally reported for conventional rough platened extension tests with length to diameter ratios nearer 2 may be due to the inexact simulation of the mid height conditions provided by the lubricated platen in the present tests. The conditions at the rough platens should however be representative.

The strain conditions in the lubricated cylindrical extension test are in contrast reasonably uniform and only slightly less ideal than those already proved for the lubricated compression cylinder. These conditions are appreciably better than those provided by samples tested with rough platens and the benefits of using platen lubrication is therefore readily apparent.

b) Cube Extension Tests

The stress-strain behaviour of the cube samples in extension was very similar to that of the cylinders. In Fig. 6 the axial displacement data

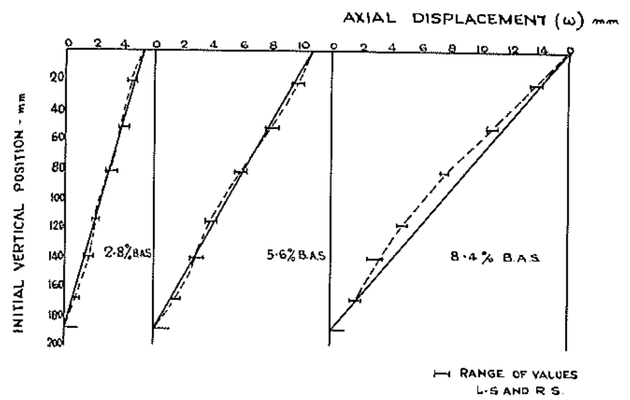

FIG. 6 CUBE EXTENSION AXIAL DISPLACEMENTS TEST 201

for an extension test shows that uniform strains develop throughout the sample up to high strain values. Lateral displacements measured across a diagonal plane between opposite corners are shown in Fig. 7. These indicate a high degree of symmetry about the geometric centre line of the sample. The distributions of lateral displacement are linear from the centre line outwards. A high degree of homogeneity is thus seen to exist up to strains beyond the peak stress conditions which occurred at boundary axial strains (B.A.S.) slightly below 8%.

The shape of the sides of the sample at various axial strain values are also shown in Fig. 7. The sides are seen to have moved inward uniformly up to

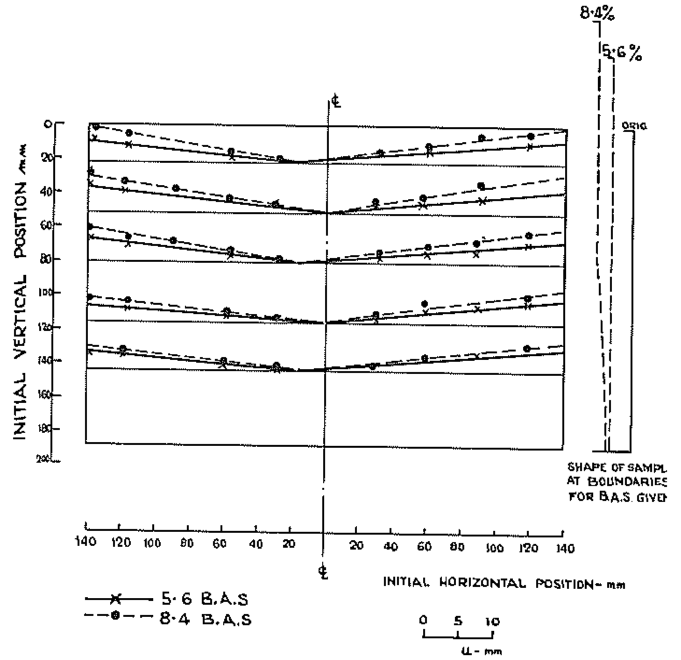

FIG. 7 CUBE EXTENSION LUBRICATED LATERAL DISPLACEMENTS ACROSS OPPOSITE CORNERS TEST 203

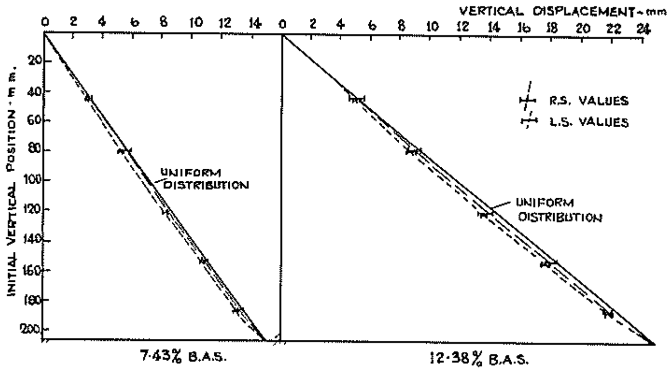

FIG. 8 CUBE COMPRESSION LUBRICATED AXIAL DISPLACEMENTS BETWEEN FACES TEST 107

boundary axial strains of about 6%. Beyond this stage the base of the sample shows no further movement while movement has continued at the top. Shear planes developed at about 10% axial strain but no localised necking occurred in any of the samples. This slight lack of uniformity also occurred in the cylinder extension tests as indicated previously. It is suspected that this behaviour has been caused by the sample having a tendency to come to failure first in the zone near the top where under the slight gradient of body stress due to the weight of sand the stress deviator ($\sigma_1 - \sigma_3$) will be slightly greater than at the base. The effect which is exaggerated in the present tests involving large samples at relatively low confining pressures was not so noticeable in the more normal type of extension test performed on 100 mm x 100 mm sized samples at confining pressures of the order of 420 kN/m^2 (60 psi). It is claimed however that the present tests show highly homogeneous strain states although tests performed under more normal practice would be even more ideal. Since failures by localised necking did not occur estimates of sample cross section areas to be used in the calculation of axial stresses should be relatively accurate.

A comparison of the lateral displacement data measured in diagonal planes between opposite corners with that measured in the planes normal to opposing faces allows a check on the symmetry of lateral deformation to be made. In all cases the strains, (measured as the slope of the displacement curves) were uniform and equal irrespective of the direction in which the strain was measured.

The equality of the lateral strains and the general homogeneity of the strain state development in the cube extension test leads to the acceptance of the method at least from the point of view of strain conditions as being equally as good as the cylindrical extension test. Non-uniform strain conditions due to "corner effects" have not been detected in the cube extension tests and hence such effects cannot be the reason, as suggested by Procter & Barden (ref. 4) for the different strengths measured by some investigators in the cube and cylindrical extension test.

c) Cube Compression Tests

In cubical samples tested in compression highly homogeneous strain conditions developed throughout the sample. Axial deformations are plotted in Fig.8

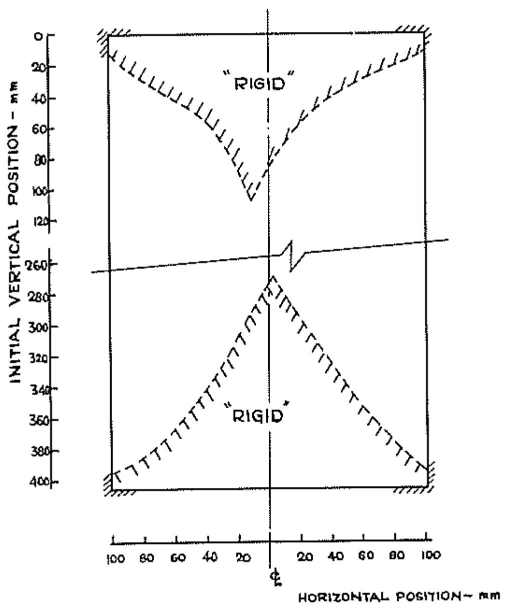

FIG. 9 PLANE STRAIN COMPRESSION 'RIGID' END ZONES ROUGH PLATENS TEST 150

and show that the displacements measured throughout the entire sample lie close to a uniform distribution. As in the case of the cube extension sample lateral strains were found to be equal irrespective of the direction of measurement and corner effects were again absent. It is again seen therefore that accurate estimates of strain can be obtained from displacements measured at the boundaries and that the lubricated compression cube can be accepted along with the other lubricated cubical and cylindrical extension and compression triaxial samples as relatively ideal test forms.

d) Plane Strain Compression Tests

The influence of the end restraint conditions provided by rough and lubricated platens observed on cylindrical triaxial samples was also noted in the plane strain compression tests. Fig. 9 shows the 'rigid' wedge shaped zones identified from lateral displacement measurements, which developed in a plane strain sample tested with rough platens. These zones extend into the sample to distances of about 0.6 x the width. The wedges thus occupy 60% of the height of the 2:1 length to width ratio sample tested and are somewhat more extensive than the rigid cones developing in the 2:1 length to diameter ratio cylindrical rough ended compression samples. These "rigid" zones develop early in the test and do not appear to vary as the strain increases.

The severely non homogeneous strain conditions noted from the lateral deformations are also reflected in the data shown in Fig. 10. In this figure the

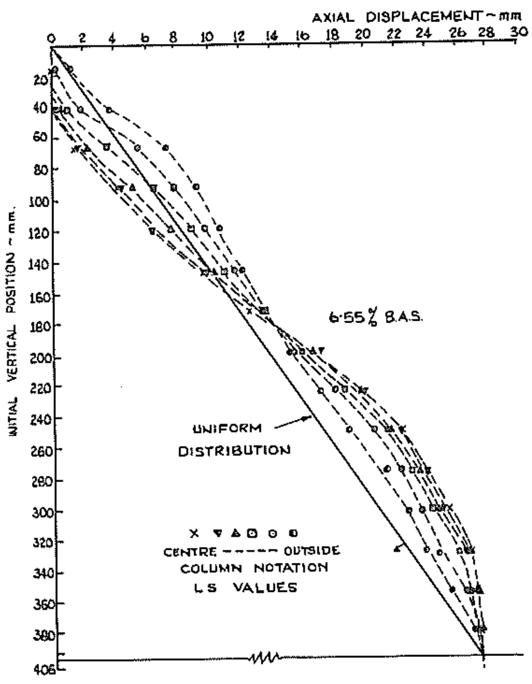

FIG. 10 PLANE STRAIN COMPRESSION - AXIAL DISPLACEMENTS ROUGH PLATENS TEST 150

axial displacements for the lead shot in the various columns are plotted. The distribution is nearest uniform for the columns near the outside edge of the sample while the columns near the centre show zero or near zero strain at the platens and very high strains at mid height.

Strain patterns for plane strain tests using lubricated platens on the other hand were largely uniform and the advantages of lubricated platens over rough platens in providing homogeneous strain states was again illustrated. The lateral and axial displacement data, not shown, are similar to that provided by cube compression tests.

V.-CONCLUSIONS

The following remarks summarise the findings obtained from the tests which were performed under drained conditions on medium-dense Leighton Buzzard sand.

The tests show that the use of lubricated end platens has the desirable feature of producing homogeneous strain states in the cubical and cylindrical triaxial extension test samples, also in the cubical triaxial compression test sample and in the prismatic plane strain compression test sample.

The use of rough platens on the other hand results in the development of highly non-uniform strain states in these tests and should be avoided in situations where reliable stress-strain information is required.

These conclusions are identical to those reached previously in relation to the use of lubricated and rough platens in the cylindrical triaxial compression sample.

Extension samples behaved well as long as the end platens were of the same size or slightly smaller than the initial diameter of the sample. In these cases the sides of the sample deformed relatively uniformly so that a reasonably accurate estimate of the cross sectional area could be obtained for use in the calculation of axial stress. The use of enlarged end platens led to the development of localised necking failures which resulted in inaccuracies in the estimation of cross sectional areas and would give errors in the calculation of stress.

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