A Method for the Application of Soil Mechanics to Non-Homogeneous Soils

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SUMMARY. A statistically based model is developed to overcome the uncertainties in the selection of design parameters for soils arising from their non-homogenuity. Experimental verification of this model is presented. It is shown that conventional methods of selection of design parameters lead to an indefinite situation with respect to factor of safety. A method for the selection of design parameters and factor of safety is proposed, based on ensuring an acceptable probability of satisfactory performance.

1 INTRODUCTION

The need for a sceptical approach in the application of soil mechanics to practical problems has been widely accepted by engineers, as a consequence of the variability of real soil performance with respect to the idealised materials on which soil mechanics is based. This paper presents a statistically based method of applying such theories under these conditions.

2 NOTATION

a	experimental value $\bar{\bar{y}}_s$ / $\bar{\bar{y}}_e$
ehu	Equivalent homogeneous unit
$F_{g,s,r}$	Partial factors of safety
k	number of ehus in sui
m_e, m_s	means of log property values ehu and sui populations
n	number of ehus in sample set
p	number of suis in site
q	result of design or analysis
sui	Significant unit of influence
S_e, S_s	standard deviations of log property values of ehu and sui sample sets
t	Student's t
$\bar{x}_e, \bar{x}_s$	means of log property values of ehu and sui sample sets
$\bar{x}_{e_s}$	mean of log property values of ehus in sui
y_s	design property value for sui
$\bar{y}_e, \bar{\bar{y}}_e$	geometric and arithmetric means of ehu property values
$\bar{y}_s, \bar{\bar{y}}_s$	geometric and arithmetric means of sui property values
$\bar{y}_{e_s}, \bar{\bar{y}}_{e_s}$	geometric and arithmetic means of property values of ehus in sui
α	Co-efficient defining limitations of theory in defining real soil performance
β	Stress factor

3 THE PROBLEMS

Properties of engineering interest, such as shear strength or compressibility, are frequently found to vary by 50% or more of their mean values. The selection of design parameters under these conditions has customarily been based on engineering judgement, with a view to analysing the "worst conditions" likely to occur as a result of non-homogenuity.

A number of guide lines have been suggested to assist in this selection, such as the use of the lower quartile value of a set of test results. Tomlinson has suggested that the mean value of the test results can be used as the design parameter, if the range of test results is less than 50% of their mean, but that the minimum test result value should be used where their range is greater than 50% of the mean (Ref. 1).

It has frequently been found that the results of analysis based on theories for homogeneous materials do not agree with the observed performance of real, non-homogeneous soils (e.g. as a result of brittle behaviour, the presence of fissures or other defects, etc. (Ref. 2.) , and empirical co-efficients have been introduced to adjust these differences. For example, it has been found that estimates of the ultimate bearing capacity of pier foundations in London Clay, based on the results of laboratory tests on conventional size samples, have to be multiplied by a factor of 0.75 to 0.8 to reconcile with the results of larger scale field loading tests on prototype foundations or test plates (Ref. 3).

In some cases, particularly where major developments have been involved, the magnitude of this discrepancy has been determined experimentally. However, in very many instances, it has been assumed that any discrepancy in performance from that predicted by theory for a homogeneous material will be accommodated within the factor of safety used.

The use of engineering judgement in the selection of design parameters or factors of safety is unsatisfactory in that the degree of conservatism or risk in subsequent design cannot be quantified. In addition the application of such judgement must be doubtful for conditions other than those under which it was developed.

4 THE SOLUTION

(a) Analysable Models

Knowledge of sub-soil conditions is gained from observations, and synthesised into a mental concept of these conditions. This mental concept frequently involves such factors as irregular stratum boundaries and non-homogeneous soil properties and will not yield readily to analysis. It is therefore replaced by a simplified analysable model such as the homogeneous, isotropic, elastic solid, yielding to continuum mechanics analysis.

Soil mechanics theories are concerned with the analysis of such analogous models, as are other engineering theories. The successful application of soil mechanics theories to non-homogeneous soils therefore requires a method for the development of appropriate mental concepts and models.

(b) The Equivalent Homogeneous Unit

Quantitative information on soil properties usually comprises the results of field or laboratory tests on a number of relatively small, discrete elements of the soil, and inductive rather than deductive methods should be used in developing mental concepts. Quantitative measurement of non-homogenuity of properties is on the scale of these test specimens. Thus, a soil stratum may be conveniently represented inductively by a mental concept of a material comprising homogeneous elements of similar size, and properties to the test specimens. To these elements we will give the name Equivalent Homogeneous Units (ehus).

If a mathematical expression for the range of ehu properties in a given stratum can be found, then it will be possible to quantify the non-homogenuity of that stratum. The use of statistics suggests itself, and as most site investigations yield small numbers of quantitative results, it would be desirable if a statistical normal relationship could be established, to allow the use of statistical techniques for handling small sample sets.

The Author has analysed the distribution of test results for a number of soil types and has observed that properties such as shear strength and compressibility do not fit a normal distribution on a Chi-square test (Fig. 1). However, it has been found that the logarithms of these property values provide a good fit to the normal distribution (Fig. 2). It is not usually possible to establish spatial trends within the scope of routine site investigations for other than obvious cases such as shear strength-depth for normally consolidated soils.

(c) The Significant Unit of Influence

In most engineering problems the soil will be influenced by the proposed development in discrete units only, such as, for example, the soil zones beneath foundations, zones influenced by excavations, etc. These are the zones for which parameters are required for soil mechanics analysis, and they will be given the name of Significant Units of Influence (suis).

It would be expected that the appropriate design value for any sui property would be a function of the average of the property values of the ehus in it. i.e.

$$y_s = \alpha \bar{\bar{y}}_{e_s} \qquad (1)$$

Since x_e is normally distributed, the function $(\bar{x}_{e_s} - \bar{x}_e)$ will have a Student's t distribution where n or k is small (Fig. 3). i.e.

$$t\phi = \bar{x}_{e_s} - \bar{x}_e \quad \text{where } \phi^2 = S_e^2\left(\frac{1}{n} + \frac{1}{k}\right) \qquad (2)$$

By using the approximation

$$\frac{\bar{y}_{e_s}}{\bar{y}_e} = \frac{\bar{\bar{y}}_{e_s}}{\bar{\bar{y}}_e} \qquad (3)$$

(2) may be re-written as

$$y_s / \alpha \bar{\bar{y}}_e = \exp(\pm t\phi) \qquad (4)$$

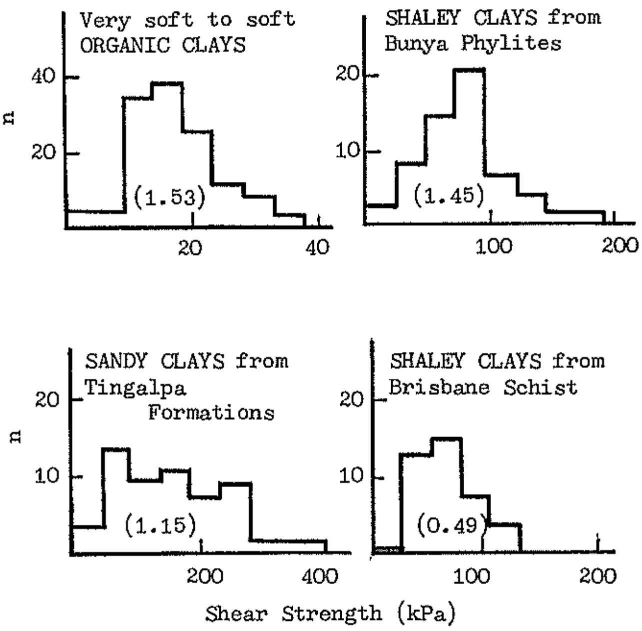

Fig. 1 Distribution of shear strength values for various soil types.

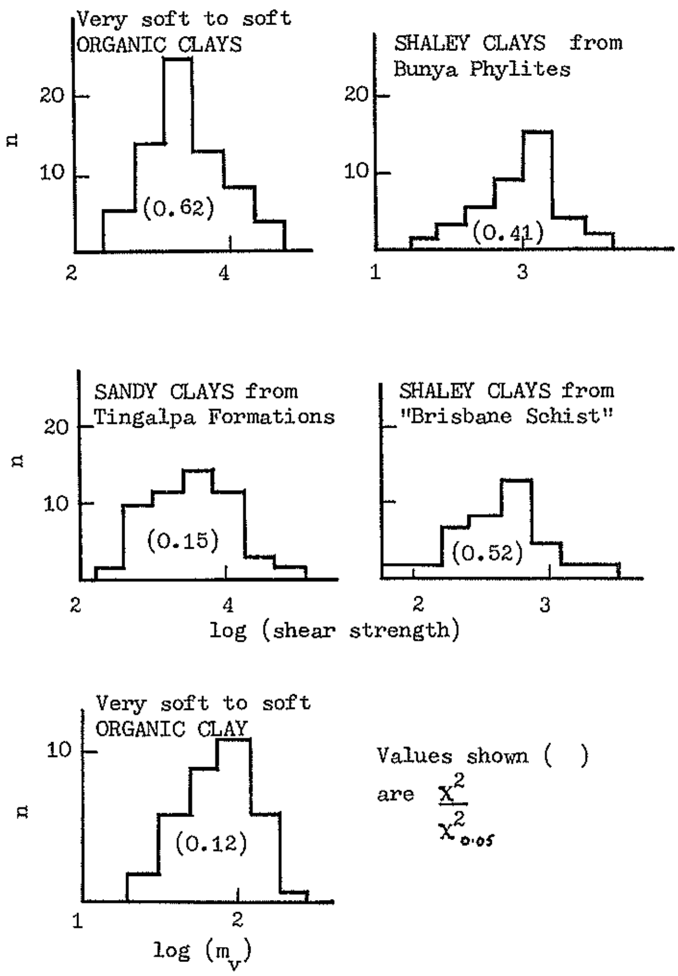

Fig. 2 Distribution of log-property values for various soil types.

TABLE I

DETAILS OF CASE STUDIES OF PREDICTED AND PROTOTYPE PERFORMANCES

CASE	SITE	EHU UNIT	EHU n	SUI UNIT	SUI p	k
	<u>Bearing Capacity</u>					
1	Plate load tests on Adelaide clays (Ref. 4)	Set 3/38 mm dia. triaxial specimens	73	Stress zone for 425 mm square plate	4	10
2	Plate load tests on residual clays, Gladstone (Ref. 5)	Single 50 mm dia. triax ial specimen, stage test	5	Stress zone for 1.07 m square plate	4	3000
3	Deep plate load tests on stiff London Clay (Ref. 6)	Single 38 mm dia. triax-ial specimen	18	Stress zone for 865 mm dia. plate	6	3000
4	Deep plate load tests on stiff London Clay (Ref. 6)	Single 98 mm dia. triax-ial specimen	19	Stress zone for 865 mm dia. plate	6	170
	<u>Uplift Resistence</u>					
5	Pull-out tests in stiff clay at Kerang (Ref. 7)	100 mm by 50 mm in-situ vane test or triaxial test specimen	11	Stress zone for base enlargement from 900 mm to 1200 mm on bored pier	3	320
6	Pull-out tests in stiff clay at Keilor (Ref. 8)	Single 38 mm dia. triax-ial specimen	Not Known	Adhesion zone around 600 mm to 750 mm dia. pile of average depth 4 m	7	2000
	<u>Settlement</u>					
7	Plate load tests on highly weathered tuff, Collins-ville (Ref. 9)	Single 50 mm dia. triax-ial consolidation spec.	7	Stress zone for 375 mm square plate	4	130
8	Plate load tests on residual clays, Gladstone (Ref.5)	Single 50 mm dia. triax-ial consolidation spec.	5	Stress zone for 1.07 m square plate	5	3000
	<u>Effect of sample size</u>					
9	Deep plate load tests on stiff London Clay (Ref.6)	Single 38 mm dia. triax-ial specimen	18	Single 98 mm dia. triax-ial specimen	19	17
10	Pile load tests in stiff clay (Ref. 10)	Single 38 mm dia. triax-ial specimen	8	Single 113 mm dia. by 113 mm high triaxial spec.	7	10

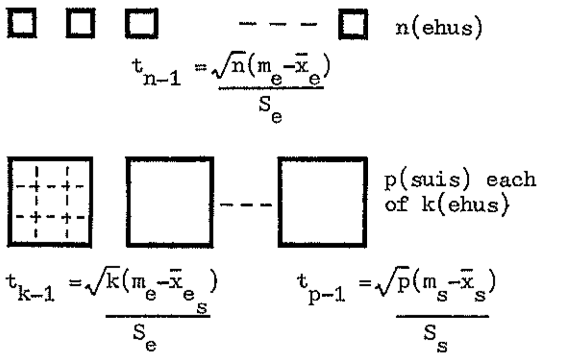

Fig. 3 Relationship of ehu and sui properties

The following expressions may be similarly derived

$$\alpha = a.\exp(\pm t\not{f}) \quad \text{where } \not{f}^2 = \frac{S_e^2}{n} + \frac{S_s^2}{p} \qquad (5)$$

$$y_s/\bar{\bar{y}}_e = a.\exp(\pm t\,\theta) \quad \text{where } \theta^2 = \phi^2 + \not{f}^2 \qquad (6)$$

Thus, the range of sui property values can be estimated from (4 or 6), to any required probability level, and the mental concept of a soil stratum composed of elements of varying properties may be replaced by an analysable model of a homogeneous material to represent the limits of the real soil performance.

(d) Experimental Verification

The observed performance of a number of non-homogeneous soils under prototype tests have been compared with performances predicted from laboratory tests, covering bearing capacity, uplift resistence, settlement and the effect of sample sizes. Details of these cases are given in Table I.

The value of S_s may be estimated at the 95% probability level from (2) as

$$S_{s_{est}} = \frac{S_e t}{1.96}\left(\frac{1}{n} + \frac{1}{k}\right)^{\frac{1}{2}} \qquad (7)$$

Observed values of S_s have been compared with values estimated on this basis, and with observed values of S_e, using the F test for homogeneity, and the results of these comparisons are given in Table II. These results indicate that the hypothesis that x_e and x_s values have been drawn from identical populations must be rejected at the 5% probability level in some cases, but that the hypothesis that observed and estimated values of x_s have been drawn from identical populations can be accepted in every case.

Values of 'a' have been estimated from the observed values of $\bar{\bar{y}}_s$ and $\bar{\bar{y}}_e$. These values are given in Table III. It is worthy of note that they differ significantly from unity in many cases and for the more extreme values in this range to be accommodated in design without evaluation would require the use of very conservative factors of safety. The evaluation of α, in areas of major development at least, would therefore appear to be warranted.

The estimated ranges of y_s values from (6) are given in Table III for the 95% probability level, and compared with the respective observed ranges. There is a good correspondence between these ranges in all cases.

TABLE II

RESULTS OF ANALYSIS OF VARIANCES FOR CASE STUDIES

CASE	COMPARISON S_e & S_s		COMPARISON $S_{s_{est}}$ & S_s	
	F *	F/F'	F *	F/F'
	Bearing Capacity			
1	2.94	0.21	2.60	0.79
2	4.10	0.27	1.39	0.09
3	20.3	3.2	1.23	0.19
4	5.42	0.85	2.90	0.86
	Uplift Resistence			
5	2.92	0.07	3.05	0.56
6	8.50	>1	2.12	<1
	Settlement			
7	1.62	0.11	3.05	0.46
8	1.41	0.15	4.20	0.44
	Effect of Sample Size			
9	3.75	1.44	2.18	0.79
10	3.05	0.54	1.30	0.25

* F = Ratio of variances
F/F' of greater than one indicates probability level of less than 5% that variances are from populations with identical variances.

TABLE III

COMPARISON OF ESTIMATED AND OBSERVED PROTOTYPE PERFORMANCE

CASE	a	ESTIMATED RANGE OF PROTOTYPE PERFORMANCE 95% Probability Level	OBSERVED RANGE OF PROTOTYPE PERFORMANCE
	Ultimate Bearing Pressure (kPa)		
1	1.36	385 - 1130	485 - 855
2	0.38	345 - 1430	460 - 780
3	0.62	715 - 1165	810 - 960
4	0.59	755 - 1225	810 - 960
	Uplift Resistence (kN)		
5	1.04	200 - 485	226 - 345
	Ultimate Shaft Adhesion (kPa)		
6	0.63*	36 - 117	59 - 84
	Settlement (mm)		
7	0.44	0.57 - 2.00	0.68 - 1.48
8	0.16	1.32 - 3.82	1.99 - 2.55
	Effect of Sample Size on Shear Strength (kPa)		
9	1.06	125 - 210	130 - 223
10	1.08	59 - 151	89 - 118

* Ratio of ultimate shaft adhesion to shear strength of soil.

5 APPLICATIONS

(a) Conventional Design Parameters

The foregoing conclusions can be used to examine some of the conventionally employed methods of selecting design parameters.

Minimum probable values of $y_s/\alpha\bar{\bar{y}}_e$ have been estimated for various values of $\bar{n}$ and $\bar{k}$, based on the ehu sample set lying within a range of 50% of its mean by taking this range as a measure of S_e, and these values are shown in Fig. 4. (Although these are not continuous functions they have been shown as such for convenience.) Similar minimumm probable values $y_s/\alpha\bar{\bar{y}}_e$ are given in Fig. 5 for the range of the sample set equal to its mean.

From these figures it follows that the probability of y_s being less than say 2/3 of the respective commonly adopted design values of the mean or lowest test result, when $\alpha = 1$, would only be less than 1:20 if the number of test results in the prescribed range was 4 or more. Similarly, at least 8 to 9 test results would be required in the prescribed range, for the probability of y_s less than 2/3 the corresponding commonly adopted design value to be less than 1:2000. Thus, where the number of test results is small and/or α is significantly less than unity, there would be a significant probability of safety factors occurring in practice which were much lower than used in design.

Some soil mechanics design, such as for temporary structures and slope stability, is often carried out to low factors of safety, of 1.5 or less. If the procedures examined above were used to select design parameters for such low factors of safety, then it must be concluded that a significant probability of failure could exist.

(b) Selection of Factor of Safety and Design Parameters

The probability that an sui will exceed the estimated limits of performance, or fail, suggests that the factor of safety should be selected to limit this probability to an acceptable level (Ref. 2,11,12). The factor selected will depend upon the value of the corresponding design parameter, and in the following, the conventional approach of selecting the design parameter on the basis of "worst conditions" will be followed.

It has been proposed (Ref. 11) that the conditions to be covered by the factor of safety can be divided into gross, systematic and random uncertainties for each of the three aspects of design of loading, material properties and analysis.

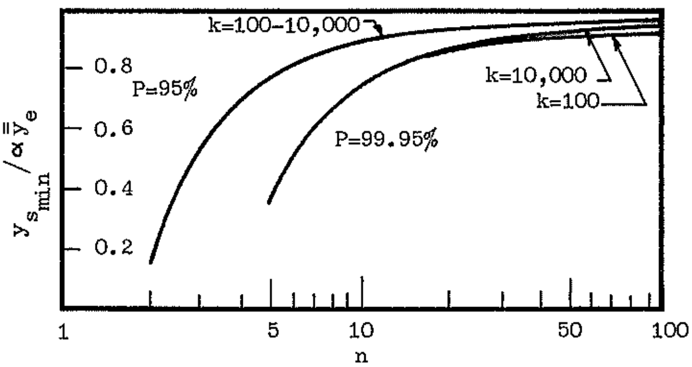

Fig. 4 Minimum probable sui property values for range of ehu values equal 50% of their mean

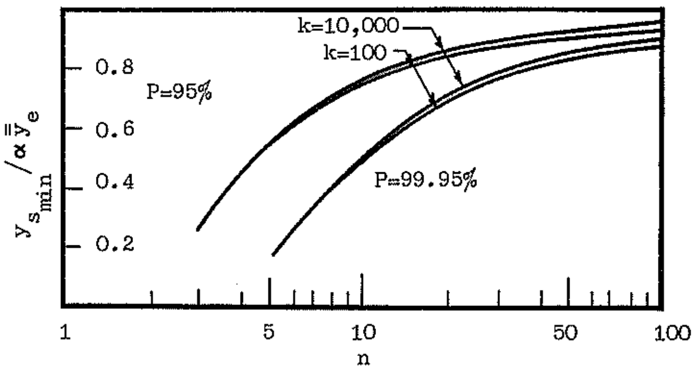

Fig. 5 Minimum probable sui property values for range of ehu values equal their mean

Thus, for design to ensure an acceptably low probability of failure in any sui the design result (q) should be a function of

$$q = f(F_g \cdot F_s \cdot F_r, y_{s_1}) \qquad (8)$$

where y_{s_1} is the "worst condition" soil parameter corresponding to the acceptable probability of failure.

Systematic uncertainties or errors would include effects of sampling disturbance under material properties and the limitations of design theories in modelling the performance of non-homogeneous soils under analysis. If the uncertainties are defined to the required probability level, by α, then the partial factor of safety (F_s) to cover systematic uncertainties would have only to cover such uncertainties with respect to loads.

If the design value of the relevant soil property is selected to correspond to the "worst conditions" of soil properties at the acceptable probability level of failure, then the partial factor of safety (F_r) with respect to random uncertainties would have to cover such uncertainties only with respect to loads and analysis.

It is also frequently a design requirement to ensure that stress levels do not exceed a given proportion (β) of the ultimate value (e.g. to avoid high local deflections, non-linear behaviour or creep). To meet this condition, q should be a function of

$$q = f(F_g \cdot F_s \cdot F_r, \beta\, y_{s_2}) \qquad (9)$$

where y_{s_2} is the "worst condition" soil parameter corresponding to the acceptable probability of exceeding the selected stress level.

Where both conditions (8) and (9) apply in design, the more conservative value of q should be the one adopted.

6 CONCLUSIONS

(a) It has been postulated and demonstrated that the variation in performance of non-homogeneous soils from that predicted by theories for homogeneous materials may be considered in two parts: a factor (α) defining the limitations of the theory in fitting the real conditions, and a range of performance produced by spatial variation of soil properties.

(b) The factor (α) can be estimated from prototype tests and the range of prototype performance can be estimated, from the range of results from conventional investigation tests, to any required probability level.

(c) It has been suggested that design parameters and safety factors are best selected in combination, to ensure an acceptable probability of satisfactory performance. Use of Figs. 4 and 5 should ensure that sufficient test results are obtained for the proposed statistical analysis to be effective.

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8 REFERENCES

1. Tomlinson M.J.: "Foundation Design and Construction". Pitman & Sons Ltd., 1963.
2. Lumb, P.:"Application of Statistics to Soil Mechanics". Soil Mechanics. New Horizons., Ed. I.K. Lee, Butterworth. 1974
3. Whitaker T. & Cooke R.W.: "An Investigation of the Shaft and Base Resistence of Large Bored Piles in London Clay". Symp. on Large Bored Piles, Inst. C.E., London, 1966.
4. Cox J.G.: "A Review of the Geotechnical Aspects of the Soils in the Adelaide City Area". Symp. on Soils and Earth Structures in Arid Climates. Inst. Eng. Aust., Adelaide, 1970.
5. Reports by Ground Test Pty. Ltd. to Messrs. MacDonald Wagner & Priddle Pty. Ltd., and the Southern Electric Authority of Queensland on Site Investigation Work for Gladstone Power Station. 1971.
6. Marsland A.: "The Use of In-situ Tests in a Study of the Effects of Fissures on the Properties of Stiff Clays". 1st A.N.Z. Geomech. Conf., Melbourne, 1971.
7. Clements B.J.: "Full Scale Pull-out Tests on Transmission Line Tower Foundations". 3rd A.N.Z. Soil Mech. & F.E. Conf., Sydney, 1960.
8. McKenzie R.J.: "Uplift Testing of Prototype Transmission Tower Footings". 1st A.N.Z. Geomech. Conf., Melbourne, 1971.
9. Report by Ground Test Pty. Ltd. to Northern Electric Authority of Queensland on Site Investigation for Stage III of Collinsville Power Station. 1971.
10. Bromham S.B. & Styles J.R.: "An Analysis of Pile Loading Tests in a Stiff Clay". 1st A.N.Z. Geomech. Conf., Melbourne, 1971.
11. Basler A.: "Analysis of Structural Safety". Probabilistic Design of Reinforced Concrete Buildings, Am. Conc. Inst. Pub. SP-31, 1972.
12. McMahon B.K.: "A Statistical Method for the Design of Rock Slopes". 1st A.N.Z. Geomech. Conf., Melbourne, 1971.