The Significance of Structure-Foundation Interaction

by
P. T. BROWN, B.Sc., B.E., M.Eng.Sc., Ph.D., M.I.E.Aust.
Senior Lecturer, School of Civil Engineering, University of Sydney

SUMMARY. An examination of the importance of structure-foundation-soil interaction with regard to the effect on differential displacement of the column bases, bending moments in the structure and column loads is presented. The structures considered are plane frames with pin-based columns, the foundations considered are strip footings of finite length, and the soil is regarded as being an isotropic homogeneous perfectly elastic continuum of infinite depth.

The purpose of the paper is to show which factors are most important in controlling the magnitude of interaction effects and to provide graphs of change in differential displacement, bending moment and column load for a range of the most important of these variables. From these results it should be possible to decide when a full interaction analysis needs to be carried out for the structure under consideration, or to decide that interaction effects are of little significance and a complete analysis can be avoided.

1 INTRODUCTION

Design of a framed building requires a decision regarding the importance of structure-foundation-soil interaction on the behaviour of the building. When the differential displacement of the column bases is sufficiently small, interaction effects can safely be ignored and the building can be analysed as an independent entity. When interaction effects are significant, differential displacements will be found to decrease, and a full interaction analysis will lead to safer or more economical design of the building frame or the foundation.

Previously published papers on interaction such as those by Meyerhof (Ref.1), Lee and Harrison (Ref. 2), Lee and Brown (Ref.3), Hain and Lee (Ref.4) and Poulos (Ref.5), have considered techniques for interaction analysis and some of the trends in behaviour as various parameters are changed. However no published paper appears to contain a survey of the effects of stiffness, number of storeys and fraction of foundation covered by the building, as is presented here.

The present paper does not aim to provide a method of interaction analysis, or graphs whose use will generally enable interaction analysis to be avoided. Instead it attempts to provide a broad survey of interaction effects, which it is hoped will provide a sound basis for estimation of the significance of the various types of interaction effect.

2 NOTATION

B = strip footing breadth
E = Young's modulus of structural material
E_r = Young's modulus of footing material
E_s = Young's modulus of soil
I = second moment of area of beams
I_r = second moment of area of footing
K_b = stiffness of building
K_r = stiffness of strip footing
K_s = stiffness of soil
K_{br} = relative stiffness of building to strip footing
K_{rs} = relative stiffness of strip footing to soil
K_{sb} = relative stiffness of soil to building
ℓ = span length of beams
L = length of strip footing
n = number of storeys
t = fraction of strip footing covered by building
x = column height per storey/ℓ (when $I_{col} = I$)
ν_s = Poisson's ratio of soil.

3 THE ANALYSIS

In order to reduce the range of cases to manageable proportions it has been assumed that

(i) the soil can be represented by an isotropic homogeneous perfectly elastic continuum of infinite depth,

(ii) the foundations are strip footings of finite length and are unaffected by horizontal loads or reactions,

(iii) the structures are symmetrical frames of rectangular form, with pin-based columns and uniform loading on every beam.

Analysis of the frames considered was carried out using the expressions provided by Kleinlogel (Ref.6) where possible, and by a standard computer program for the other cases. Foundation load-settlement results were taken from Brown (Ref.7) and the interaction analyses were carried out using the method described by Poulos (Ref.5).

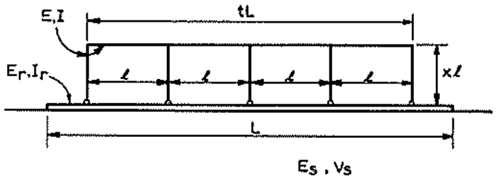

Fig. 1 The Type of Problem Considered

The cases (see Fig.1) studied include footings whose length/breadth ratio is 5 and 10, structures whose columns have the same moment of inertia as the beams and lengths equal to or half that of the beams (x = 1.0 or 0.5), and the fraction of the footing covered by the building (t) ranges from

0.90 to 0.98. In the case of two-bay frames one storey and four storeys were considered, for three-bay frames one storey and three storeys, and for four-bay frames one storey was considered. The full set of results consists of various combinations of the above factors.

Preliminary studies showed that interaction behaviour was dependent on relative stiffnesses rather than absolute stiffnesses. The relative stiffnesses are ratios of the stiffnesses of the building (K_b), the strip foundation (K_r) and the soil (K_s) and are defined as follows

$$K_b = \frac{nEI}{\ell^4}, \quad K_r = \frac{E_r I_r}{L^4}, \quad K_s = \frac{E_s}{1-\nu_s^2}$$

where n = number of storeys
E = Young's modulus of structural material
I = second moment of area of beams
ℓ = length of beams
E_r = Young's modulus of footing material
I_r = second moment of area of footing
L = length of footing
E_s = Young's modulus of soil
ν_s = Poisson's ratio of soil,

and the adopted ratios are $K_{rs} = K_r/K_s$, $K_{sb} = K_s/K_b$ and $K_{br} = K_b/K_r$.

4 DISCUSSION OF RESULTS

Although the three relative stiffnesses are not independent, it is convenient to plot the results in terms of all three stiffnesses in the manner shown in Fig.2. However any two relative stiffnesses are sufficient for reading the graphs.

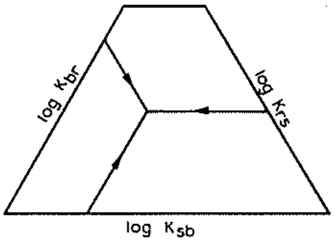

Fig. 2 The Coordinate Directions on Graphs

(a) Differential Displacement

The differential displacement considered is the largest differential displacement between two adjacent column bases. The variation due to interaction is specified as the percentage by which the actual differential displacement is a reduction of the differential displacement which would arise if the column loads were unaltered by interaction effects.

The reduction in differential displacement is principally dependent on the number of bays in the frame, and is virtually independent of the length/breadth ratio of the footing. Fig.3 shows the combinations of relative stiffness at which the reduction in differential displacement is 50% and 80% for 2-, 3- and 4-bay single-storey frames when $t = 0.90$, beam and column lengths are equal ($x = 1$) and $L/B = 5$ (B = footing breadth). Some minor reductions in addition to those shown in Fig.3 arise due to increases in column stiffness, n and t. With the most adverse combination of these factors and relative stiffnesses, Fig.3 could indicate a reduction of 50% in differential displacement when the actual reduction was 80%, but such extreme combinations and errors are likely to be rare. Thus the results shown will almost certainly be conservative for normal problems.

(b) Bending Moment

The means selected for describing the variation of bending moment in the structure due to interaction is the percentage increase in magnitude of eaves moment compared with the result of a normal structural analysis. For 3- and 4-bay frames, the increase in eaves moments is virtually identical and is almost independent of column stiffness, n and L/B. However the increase in eaves moment is significantly less for $t = 0.98$ than for $t = 0.90$. The results plotted in Fig.4 are for 3- and 4-bay single storey frames when $t = 0.90$ and 0.98, beam and column lengths are equal ($x = 1$) and $L/B = 5$, and indicate the conditions for 5% and 20% increase in eaves moment.

For 2-bay frames the results are quite similar except that the effect of variation of t is somewhat greater for the case plotted. However the 2-bay interaction effects are reduced more by an increase in column stiffness and number of storeys than the 3- and 4-bay results and for certain combinations of parameters the increase in eaves moment due to interaction may be extremely small. This situation arises because when $t \geq 0.90$ the distribution of column loads is similar to that required to produce zero differential displacement. Consequently a small change in the structure, particularly for the larger values of t, can cause differential displacement and eaves moment to change considerably. For this reason the results shown in Fig.5, which apply for $t = 0.90$ and 0.98, $x = 1$, $n = 1$ and $L/B = 5$ indicate considerably higher increases in eaves moment than occur in certain other cases. However the considerable number of parameters involved precludes the presentation of more than the conservative set of curves provided.

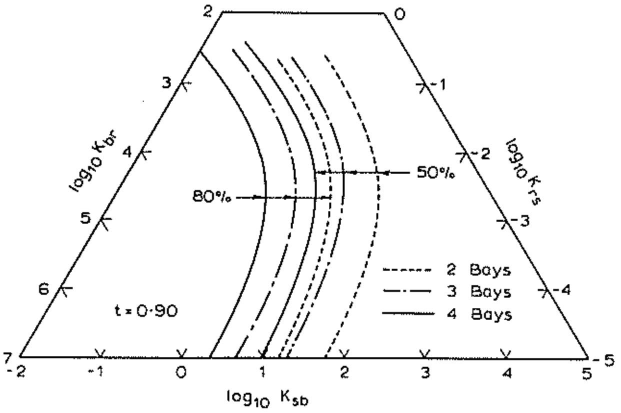

Fig. 3 Decrease in Differential Displacement

(c) Column Loads

The means selected for describing the variation of column loads due to interaction is the percentage increase in outer column load. The results shown in Fig.6 are for 3-bay single storey frames when $t = 0.90$ and 0.98, $x = 1$, $L/B = 5$. These curves may be applied to 3 bay and 4-bay frames without involving errors of more than a few percent, regardless of the values of other parameters. When applied to 2-bay frames, the curves in Fig.6 are quite accurate for some combinations of parameters, while in other cases they grossly overestimate the increase in outer column load for the reasons indicated in the preceding paragraph.

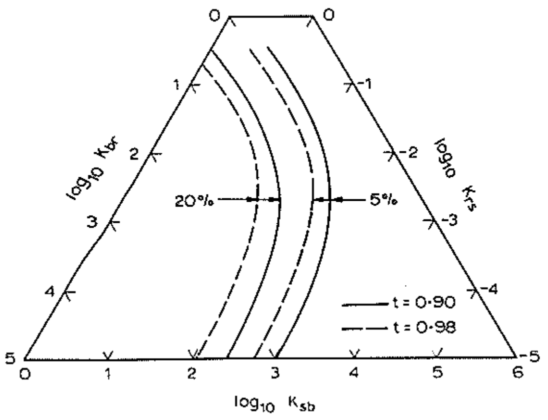

Fig. 4 Increase in Eaves Moment for 3 and 4 bay frames

Values of column load are required both for design of the columns and for determination of the behaviour of the foundation.

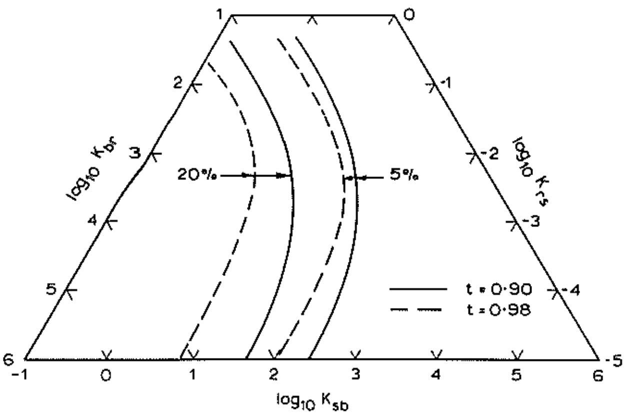

Fig. 6 Increase in Outer Column Load

(d) Extension of Application

Analysis of two types of multi-storey frame indicated that the results presented in this paper could well be applied to such frames when their columns are pin-based. Curves presented by Hain and Lee (Ref.4) suggest that fixity of the column bases will have little effect on the interaction behaviour and hence on the applicability of the graphs provided. Satisfactory results from application of the curves to pitched-roof portal frames would be expected, since modern trends are to use such low pitch angles that the effect of the non-rectangularity is very small in the cases examined.

5 EXAMPLE

The use of the results provided may be clarified by consideration of the example shown in Fig.7. The building consists of rectangular portal frames at intervals of 4m, each frame being 4m high, consisting of three 6m bays with column centres 1m from the edge of the raft, and is constructed of universal beams UB 460 of 74 kg/m whose second moment of area is 334 x 10^6mm^4. The reinforced concrete raft is 20m wide and for the present purpose is considered to consist of 4m wide strips 20m long each carrying one portal frame. The effective depth of the raft is 150mm and was designed on the basis of a modular ratio of 12 and allowable stresses in concrete and steel of 9 MPa and 138 MPa respectively, and balanced design with full cracking. This gives I_r = 1125 x10^6mm^{4}and K_r = 0.12 kPa. The underlying soil has a Poisson's ratio of zero and a Young's modulus of 0.65 MPa and hence K_s = 650 kPa. Since the beams have I = 334 x 10^6mm^4 and ℓ = 6m, K_b = 53 kPa, and the relative stiffnesses are K_{rs} = 1.84 x 10^{-4}, K_{sb} = 12.2 and K_{br} = 446.

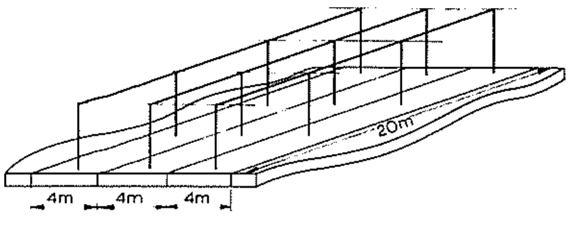

Fig. 7 The Example Considered

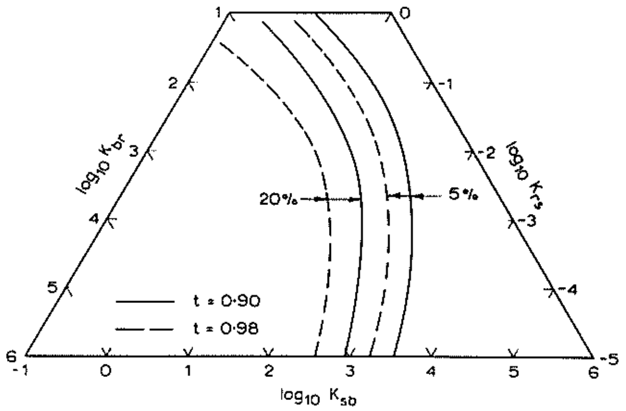

Fig. 5 Increase in Eaves Moment for 2 bay frames

The uniform loading to be supported by the building is 5 kPa and if there is no differential settlement of the columns Kleinlogel's formulae indicate that the outer and inner column loads are 54 and 126 kN respectively and the bending moments at the eaves and inner ends of the outer beam span are -33.3 and -69.1 kNm respectively. Application of these column loads to the strip footing considered produces a differential displacement in the outer bay of 20mm or 1/300 of the span of that bay.

Making use of the calculated relative stiffnesses and the fact that t = 0.90, inspection of Figs.3, 4 and 6 indicates that the following interaction effects are to be expected:

(i) a decrease in differential displacement of approximately 50%,

(ii) an increase in eaves moment considerably greater than 20%,

(iii) an increase in outer column load somewhat greater than 20%.

These results indicate that a complete interaction analysis is needed. Such an analysis gave the following results:-

(i) a decrease in differential displacement of 56% reducing it from 20mm to approximately 9mm which should cause no cracking.

(ii) an increase in eaves moment of 138% from -33.3 to -79.3 kNm and at the inner end of the beam a decrease from -69.1 to -18.3 kNm.

(iii) an increase in outer column load of 30% from 54 to 70 kN, and a decrease in the inner column load from 126 to 110 kN.

The bending moments in the raft can be determined using the latter values of column load.

6 CONCLUSIONS

A series of curves indicating the effect of structure-foundation-soil interaction on the major aspects of the behaviour of the structure have been presented for plane rigid-jointed structures on strip footings on a deep bed of soil. These can be used as an indication of which situations merit an interaction analysis, although in certain cases the actual interaction effects may be much smaller than those predicted from the curves

7 ACKNOWLEDGEMENTS

The work described in this paper forms part of a general programme of research into the settlement of all types of foundations being carried out at the University of Sydney under the general direction of Professor E.H. Davis, Professor of Civil Engineering (Soil Mechanics) and was supported by a grant from the Australian Research Grants Committee. The author is indebted to Mr. P. Ansourian and Dr. H.G. Poulos for their helpful comments during the progress of this work.

8 REFERENCES

1. MEYERHOF, G.G. Some Recent Foundation Research and its Application to Design. Struct.Engr., Vol.13, June 1953, pp.151-167.

2. LEE, I.K. and HARRISON, H.B. Structure and Foundation Interaction Theory. J.Struct.Divn, ASCE, Vol.96, No.ST2, Proc.Paper 7059, Feb.1970, pp.177-198.

3. LEE, I.K. and BROWN, P.T. Structure-Foundation Interaction Analyses. J.Struct.Divn,ASCE, Vol. 98, No.ST11, Proc.Paper 9352, Nov.,1972, pp. 2413-2431.

4. HAIN, S.J. and LEE, I.K. Rational Analysis of Raft Foundations. J.Geotech.Divn,ASCE, Vol. 100, No.GT7, Proc.Paper 10683, July 1974, pp. 843-860.

5. POULOS, H.G. Settlement Analysis of Structural Foundation Systems, University of Sydney, Australia, Civil Eng.Res.Rep.R248, 1974.

6. KLEINLOGEL, A. Mehrstielige Rahmen. New York, Ungar, 1948.

7. BROWN, P.T. Strip Footings with Concentrated Loads on Deep Elastic Foundations. University of Sydney, Australia, Civil Eng.Res.Rep.R225, 1973.