Interaction of Foundation and Base Upon Swelling

by

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SUMMARY. Using the approach of the thermodynamic potential for moisture state in swelling soils,the method for calculation the displacements the elastic and flexible foundations has been developed. The displacements due to swell of soil take place in conditions of artifical wetting,shielding of the surface, increase of water table. Applying the collocation method the contact pressures distribution upon swelling has been determined. To obtain necessary parameters for calculation, the procedure has been described.

SYMBOLS

z = vertical coordinate direction
x = horisontal coordinate direction
Z = water table depth
Φ=total potential
Ψ = moisture potential
Ω = overburden potential
z = gravitational potential
e = void ratio
w = water content
w_s = water content of swelling
w_{sh}= water content of shrinkage
γ_s = bulk density
γ = unit weight of soil
γ_w = unit weight of water equal to one
S_t = total base displacement
S_o = elastic base settlement
f = base upheave due to swelling
μ = Poisson's ratio
E_o = deformation modulus
p = external vertical pressure
p_ℓ = external lateral pressure
q = contact pressure
P = concentrated force
M = moment
h = upper boundary of swelling layer
H = lower boundary of swelling layer
m_v= factor of volume swell
m_1= factor of conditions

1 INTRODUCTION

The economic and safe construction on swelling soils is in large part dependent upon the prediction of foundation displacements. To estimate the foundation strength, it is necessary to determine of contact pressures redistribution under foundation. It can be achieved by solving the problem of interaction of foundation and base upon swelling.

Construction and operating experience has shown that both increase of soil moisture and swelling occur,when artificial wetting,shielding of the surface, increase of the water table take place (Ref.1-5).

In the case of artificial wetting the method of calculation flexible foundation upheave has been developed (Ref.6).

The paper present the general method of calculation both the displacements of elastic or flexible foundations and contact pressures redistribution which are caused by swelling.

2 THEORY

The calculation of displacements due to swelling may be carried out at two stages. At first stage we should determine the equilibrium (or more accurately quasiequilibrium) moisture distribution in the vertical given water table depth, loading on the surface and due to soil's own weight. At second stage the volume changes and the vertical displacements will be calculated according to moisture changes.

Equilibrium moisture distribution has been determined using the hydrostatics of swelling soils (Ref.7). The moisture state can be characterized by the quantity of the total potential

$$\Phi = \Psi - z + \Omega \qquad (1)$$

At the thermodynamic approach the swelling soil differs from nonswelling soil owing to presence of overburden potential in Equation 1. In onedimensional case Ω has the form as follows

$$\Omega = \frac{1}{\gamma_s}\left(p + \int_0^Z \gamma dz\right)\left(\frac{\partial e}{\partial w}\right)_\sigma \qquad (2)$$

In two-and three-dimensional case of the loading on surface the following expression for Ω has been suggested

$$\Omega = \frac{1}{\gamma_s}\left(\sigma_z' + \int_0^Z \gamma dz\right)\left(\frac{\partial e}{\partial w}\right)_\sigma \qquad (3)$$

where σ_z' is additional vertical stress due to a load on the surface which may be calculated by means of stresses distribution theory. The form of Equation 3 is based on the following hypothesis: when moisture content increases the swell in the horisontal is negligible in comparison with vertical one. Such assumption follows from our research of swell under general stress system.

In equilibrium moisture state the value of Φ equal at all points. It is suitable to accept as a constant the inverse of the water table depth. The equation describing invisibly equilibrium moisture distribution in the vertical will take the form

$$\Psi - z + \frac{1}{\gamma_s}\left(\sigma_z' + \int_0^z \gamma dz\right)\left(\frac{\partial e}{\partial w}\right)_\sigma = -Z \qquad (4)$$

Let us come now to the second stage of calculation. In the elementary cube with rib equal to one the moisture increase on infinitesimal quantity dw leads to volume and porosity increases on dv, that can be written in the form

$$dv = \left(\frac{\partial v}{\partial w}\right)_\sigma dw = \frac{1}{1+e_0}\left(\frac{\partial e}{\partial w}\right)_\sigma dw \qquad (5)$$

The index σ shows that the vertical changes occur under constant stress state. The rib increase has been obtained by multiplication of volume increase on the factor of volume swell

$$dh = m_v dv = \frac{m_v}{1+e_0}\left(\frac{\partial e}{\partial w}\right)_\sigma dw \qquad (6)$$

The factor m_v has been found in laboratory. It is ratio of volume change in the vertical to the total volume change. For the homogeneous soil m_v is equal to 1/3. The anisotropy of real soils has a considerable effect on m_v. If the lateral expansion is restrained, the value of m_v is approximated to one when the swell of monolithic soil occurs.

The moisture increase from initial w_i to equilibrium water content w increases rib height by Δ

$$\Delta = \int_{w_i}^{w} \frac{m_v}{1+e_0}\left(\frac{\partial e}{\partial w}\right)_\sigma dw \qquad (7)$$

Upheave of a layer at a depth z may be obtained by integrating over z from z to H

$$f(z) = \int_z^H \Delta dz = \int_z^H \left\{\int_{w_i}^{w} \frac{m_v}{1+e_0}\left(\frac{\partial e}{\partial w}\right)_\sigma dw\right\} dz \qquad (8)$$

Foundation upheave has been calculated by integrating Equation 8 with z=h.

To determine both the upheave of elastic foundation and the contact pressures distribution due to swelling, the condition of equalization of base displacements and foundation deflection y in the vertical is used

$$y = S_t \qquad (9)$$

In the case of swelling soils the total base displacement S_t has been presented in the form

$$S_t = S_o - f \qquad (10)$$

The settlement S_o is downwards and it depends on both the base deformation properties and the type of contact pressures distribution. The upheave f is caused due to swelling in the course of building operation. Directly after the completion of construction f is equal to zero. In this case the foundation settlements and contact pressures distribution are determined by known methods, for example, by means of Zhemochkin's method. The minus sign before f in Equation 10 indicates that the direction of f is opposite to S_o when artificial wetting, shielding of the surface, increase of the water table take place.

3 DETERMINATION OF NECESSARY PARAMETERS FOR SWELLING SOILS

(a) Moisture Potential $\Psi(w)$

Method of determination relation between Ψ and w is based on soil's property to swell under load. Relationship between Ψ and w is affected by soil structure and its density. Considering this fact, such relationship had been obtained on undisturbed samples. The oedometer expansion test corresponds on one-dimensional case of loading. After stabilisation the process of swelling the equilibrium moisture distribution is described by Equation 4, where σ_z' is the external pressure p. The analysis of term values shows that the gravitational potential and the load due to the soil's own weight on account of the small ring height could be negligible. The swell being stabilized, the filling degree is equal to one, i.e. $\gamma_w/\gamma_s(\partial e/\partial w)_\sigma = 1$
In test the free water level is at sample surface, i.e. Z=0. So, the Equation 4 takes the following form

$$\Psi(w) = -P/\gamma_w \qquad (11)$$

where w corresponds on water content after the end of swell stabilization under the external pressure p. Equation 11 has the physical sense as follows: the swell upon moisture increasing is ceased when the equilibrium between external pressure and pushing apart action of the water films is achieved.

The relationship $\Psi(w)$ is obtained upon moisture increasing and under the constant load. At similar conditions it may be used for the equilibrium moisture distribution to calculate, for example, when artificial wetting, shielding of the surface, increase of water table take place.

Figure 1 shows a relationship w of Ψ of Khvalynian clay obtained by data processing.

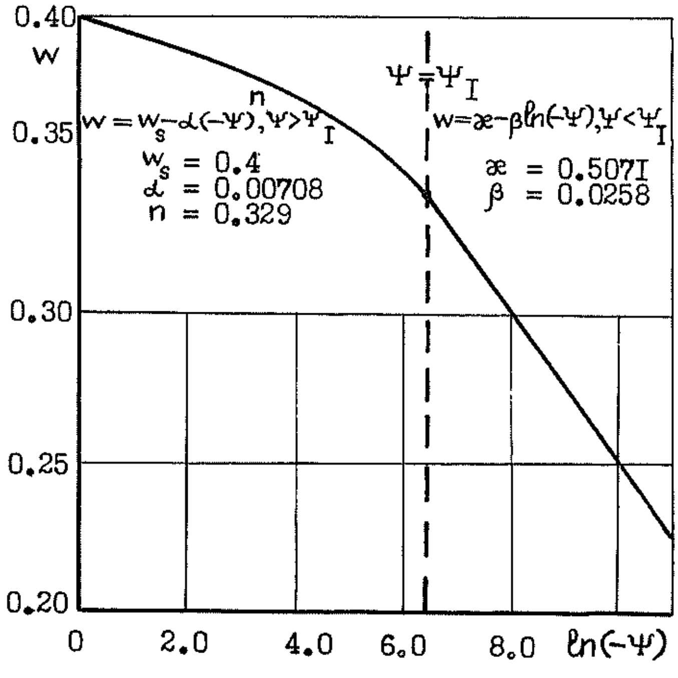

Fig.1 Relationship between water content and moisture potential of Khvalynian clay

(b) Relationship Between e and w.

The function soil porosity of water content is determined by statistical processing of data yielded by the engineering-geological investigations. Thereby, the effect of nonhomogeneous soil properties in site is considered. The following relationship e of w is found as

$$e = kw + a \qquad (12)$$

For swelling clays the correlative coefficient proved more than 0.87. The value of k in Equation 12 is dependent upon the properties of considered clays. For monolithic and homogeneous clay the value of k is approximated to γ_s/γ_w.

For example, figure 2 shows experimental data of Khvalynian clay

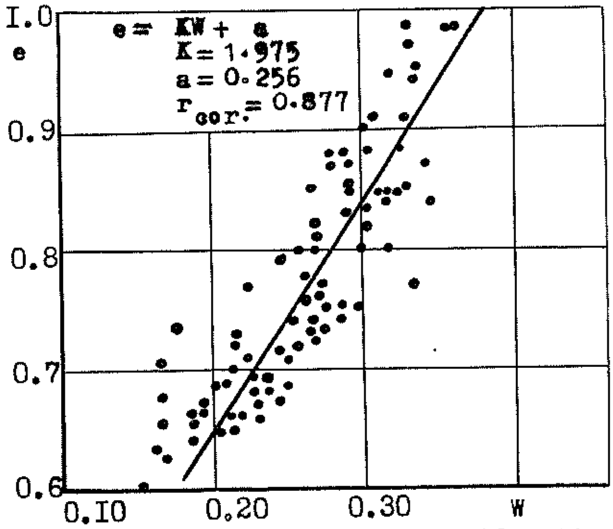

Fig.2 Relationship between void ratio and water content of Khvalynian clay.

(c) Factor m_v

The factor m_v is determined by processing of data obtained in oedometer test at shrinkage of undisturbed clay samples. The samples were overburden by plate weight that permit upon slow drying to prevent the formation of fissures. For example, figure 3 shows the relationship between the linear sizes of sample of Khvalynian clay and average water content. The expression for m_v is determined by formula

$$m_v = \frac{\frac{1}{h}\frac{dh}{dw}}{\frac{1}{V}\frac{dV}{dw}} = \frac{1}{1+2\frac{h}{r}\frac{dr}{dh}} = \frac{1}{1+2\frac{\bar{h}}{\bar{r}}\frac{\Delta r}{\Delta h}} \qquad (13)$$

where Δh and Δr are the height and radius changes upon decreasing of water content from w to $w-\Delta w$; $\bar{h}$ and $\bar{r}$ are average values h and r in such water content interval. The experimental data shows: 1) upon shrinkage from the water content of swelling to the water content of shrinkage the factor m_v is practically independent on water content; 2) the undisturbed swelling clay with disposition of layers in the horisontal shows the shrinkage anisotropy, which is more across layers direction than along them.

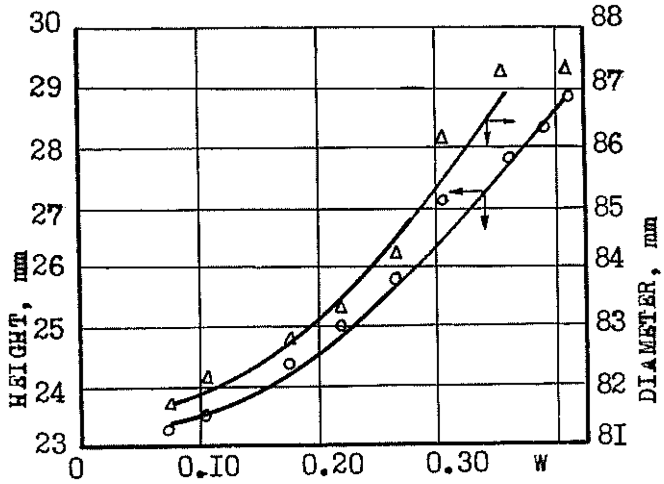

Fig.3 Changes of linear sample sizes upon shrinkage

(d) Expansion under General Stress System

Swelling investigation under the vertical and lateral pressures as a constant has been carried out on stabilometer. During the test the additional device (the swimming piston) with indicator were used to establish and keep a constant lateral pressure. The lateral expansion value $\Delta r/r_o$ was computated by the removing water volume from stabilometer. The results of experiment of Khavalynian clay are given in Table I.

The sample N 11 was cutted out in the horisontal direction. The line showing W_s has the numerator which shows water content of swelling obtained on stabilometer test and the denominator which shows the water content of oedometer test.

The data show that the clay having the horisontal structure finds upon swelling the anisotropic property; the lateral swelling

TABLE I

EXPANSION UNDER GENERAL STRESS SYSTEM

Remarks	Specimens										
	1	2	3	4	5	6	7	8	9	10	11
p, MPa	0	0.05	0.1	0.15	0.2	0.05	0.15	0.15	0.15	0.1	0.05
p_e, MPa	0	0	0	0	0	0.05	0.10	0.05	0.05	0.1	0
w_e, %	23.2	24.4	22.8	24.8	22.6	24.4	25.0	22.7	23.7	24.5	24.2
γ, g/cm^3	1.95	1.98	2.00	2.01	2.00	2.01	2.01	2.01	2.02	2.01	1.96
W_s, %	$\frac{41.2}{40.0}$	$\frac{36.3}{34.6}$	$\frac{33.2}{32.7}$	$\frac{33.3}{31.5}$	$\frac{31.0}{30.9}$	$\frac{35.0}{34.6}$	$\frac{34.1}{34.6}$	$\frac{32.4}{31.5}$	$\frac{33.8}{32.7}$	$\frac{33.8}{32.7}$	$\frac{35.7}{34.6}$
$\Delta h/h_o$, %	13.3	6.3	5.5	4.3	3.4	9.4	8.6	6.8	7.2	6.6	2.0
$\Delta r/r_o$, %	4.25	3.8	4.15	3.25	2.45	0.85	0.75	0.65	0.7	0.75	6.3
$\Delta V/V_o$, %	21.8	13.9	13.8	10.8	8.3	11.1	10.1	8.1	8.6	8,1	14.6

is negligible in comparison with the swell value in the vertical while a sample is overburden by lateral pressure. In the case of the halfspace this conclusion proves the using Equation 3 for overburden potential proposed by Philip.

4 CALCULATION OF FLEXIBLE FOUNDATION UPHEAVE

In the practical analytical scheme the following simplifying assumptions were made: the relationship between e and w is taken as linear; factor m_v and e_o in Equations 7-9 are constants; the overburden potential is determined by Equation 3, where the additional vertical stress due to a load on the surface is calculated by elasticity theory; γ and γ_s are constants and equal to average values; the displacements calculated from Equation 8 are multiplied by factor of conditions. m_1, which takes into accound not only bench and in site data differences, but also the effects of other factors.

All considered things, Equation 8 takes the following form

$$f(z)=\frac{m_1 m_v k}{1+e_o}\int_z^H (w-w_o)\,dz \qquad (14)$$

In Equation 14 water content $w_o = w_{sh}$ at $w_i < w_{sh}$ and $w_o = w_i$ at $w_i > w_{sh}$.

To determine the equilibrium moisture distribution $w(z)$ we use the Equation 4 and the experimental relationship w of Ψ represented by Figure 1.

In one-dimensional case we find from Equation 4 the value of z, where $\Psi = \Psi_1$

$$z_1=\frac{Z+\Psi_1+(k/\gamma_s)p}{1-(k\gamma/\gamma_s)} \qquad (15)$$

Performing the integration in Equation 14, we obtain the following formulas for determining f

$$f=\frac{m_v m_1 k}{1+e_o}\left\{w_s(H-z)-\int_z^H w_o dz+\frac{d\gamma_s}{\gamma_s-k\gamma}\cdot\frac{(-\Psi_H)^{n+1}-(-\Psi_z)^{n+1}}{n+1}\right\},\quad H<z_1 \qquad (16)$$

$$f=\frac{m_v m_1 k}{1+e_o}\left\{\varkappa(H-z)-\int_z^H w_o dz+\frac{\beta\gamma_s}{\gamma_s-k\gamma}\cdot\left[\Psi_z(\ln-\Psi_z-1)-\Psi_H(\ln-\Psi_H-1)\right]\right\},\quad z_1<z \qquad (17)$$

To determine the flexible foundation upheave at $z<z_1<H$ both Equation 16 and Equation 17 are used. In Equations 16 and 17 Ψ_z and Ψ_H are the moisture potential at the depth z and H respectively.

For example, figure 4 shows the relationship between upheave of the surface, the water table depth and the loading on the surface. The layer of Khvalynian clay is 300cm thick. The initial water content distribution is assumed being uniform (30%) with depth. The limits of integration in Equation 14 are as follows: the lower is zero and the upper is 300 cm. The data are: m_1 =0.8, m_v=0.58, k =1,975, e_o=0.85, γ =1,85 g/cm^3, γ_s =2.77 g/cm^3.

Represented by figure 4 the curves permit to find the upheave of surface upon shielding when the water table is at the constant depth or at other position.

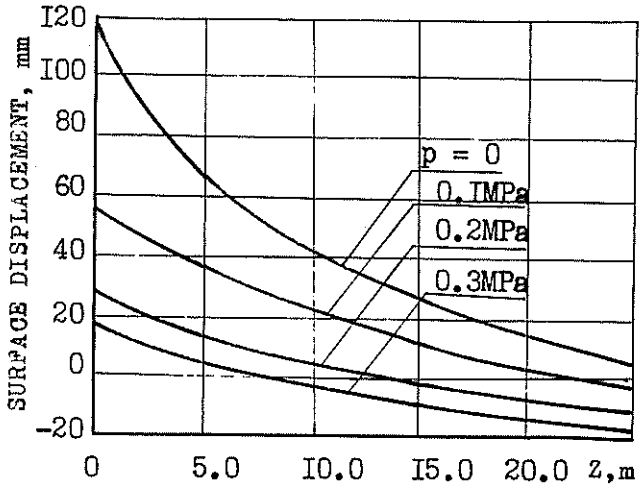

Fig.4 Relationship between surface displacements, water table depth and loading on the surface

In two-and three-dimensional cases of surface loading the additional vertical stress σ_z' depends unlinearly on z. In such cases the integration in Equation 15 is replaced by summation over the elementary layers thickness ℓ_i

$$f(z)=\frac{m_1 m_v k}{1+e_o}\sum_z^H (w-w_{oi})\ell_i \qquad (18)$$

In Equation 18, w is found from Equation 4, in which σ_z' is calculated for the middle of (i) elementary layer. The value of w_i is given by known initial moisture distribution.

5 COMPUTATION OF ELASTIC FOUNDATION DISPLACEMENTS AND CONTACT PRESSURES DISTRIBUTION UPON SWELLING

For determination the displacements of elastic foundation and the contact pressures distribution upon swelling we use the general condition given by Equation 10. The assumptions, accepted in calculating the upheave of flexible foundations, are complemented as follows: the foundation member is elastic, the vertical forces develop between foundation and base only.

Then we consider a problem of contact pressures distribution under beam upon swelling. For simplicity, the flexural rigidity of beam EJ is taken constant. The beam b in width and L length is divided into n equal zones of length $2a$ so that $2an=L$. Within (i) zone the uniformly distributed external p_i and contact q_i pressures are assumed constant. The concentrated forces and moments act between (i) and $(i-1)$ zones. The base deformation properties within (i) zone are characterised by deformation modulus E_{oi} and Poisson's ratio μ_i.

The initial moisture distribution $w_i(z)$ with depth, the upper h_i and lower H_i values of the boundaries of the swelling layer are known from the data yielded by engineering-geological investigations.

Within the (i) zone the differential equation of beam is

$$\frac{d^4 y_i}{dx^4}=\frac{1}{EJ}(p_i-q_i) \qquad (19)$$

Performing the integration in Equation 19 we obtain the Equation of beam elastic line as:

$$y_i = \frac{p_i-q_i}{24EJ}[x-a(2i-1)]^4+\frac{C_{1,i}}{6}[x-a(2i-1)]^3+\frac{C_{2,i}}{2}[x-a(2i-1)]^2+C_{3,i}[x-a(2i-1)]+C_{4,i} \quad (20)$$

For determination the constants of integration $C_{1,i}, C_{2,i}, C_{3,i}, C_{4,i}$ the conditions expressing the equality of deflections, angles of rotation, shear forces and moments between (i) and (i −1) zones are used

$$y_{i-1}=y_i\ ,\ y'_{i-1}=y'_i,\ y''_{i-1}=y''_i+\frac{M_i}{EJ}\ , \quad (21)$$

$$y'''_{i-1}=y_i-\frac{P_i}{EJ}\quad (i=1,2,\dots,n)$$

The constants of integration $C_{3,i}$ and $C_{4,i}$ are the angle of rotation and beam deflection at point $x=a(2i-1)$. Take as unknowns the deflection y_1 and the angle of rotation φ_1 at point $x=a$. Using the conditions (21) the expression of deflection at points $x=a(2i-1)$ will take the form

$$y_i = y_1+2a\varphi_1(i-1)+2a^2(i-1)^2C_{2,1}+\frac{4a^3(i-1)^3C_{1,1}}{3} \quad (22)$$
$$+\frac{a^3}{6EJ}\sum_{j=2}^{i}[2(i-j)+1]^3P_j-\frac{a^2}{2EJ}\sum_{j=2}^{i}[2(i-j)+1]^2M_j+$$
$$+\frac{a^4}{24EJ}\sum_{j=1}^{i}d_j(p_{i+1-j}-q_{i+1-j}),\ (i=1,2,\dots,n)$$

Here the value of coefficient d_j for $j=1,2\dots,15$ are as follows: 0,15,175,671,1695, 3439,6095,9855,14911,21455,29679,39775, 51935,66351,83215.

The constants of integration $C_{1,1}$ and $C_{2,1}$ are determined by conditions on the left and right edges of beam. If the beam edges are not restrained from deflections and rotation:

$$y'''_{|x=0}=\frac{P_1}{EJ}\quad ,\quad y''_{|x=0}=-\frac{M_1}{EJ} \quad (23)$$

we obtain the following expressions for $C_{1,1}$ and $C_{2,1}$

$$C_{1,1}=\frac{P_1}{EJ}+\frac{a(p_1-q_1)}{EJ},\ C_{2,1}=\frac{aP_1}{EJ}-\frac{M_1}{EJ}+\frac{a^2(p_1-q_1)}{EJ} \quad (24)$$

The expression of deflection at point $x=a(2i-1)$ will take a form

$$y_i=y_1+2a\varphi_1(i-1)-\frac{2a^2(i-1)^2M_1}{EJ}-\frac{a^2}{2EJ}\sum_{j=2}^{i}[2(i-j)+1]^2M_j+ \quad (25)$$
$$+\frac{a^3P_1}{EJ}[2(i-1)^2+\frac{4(i-1)^3}{3}]+\frac{a^3}{6EJ}\sum_{j=2}^{i}[2(i-j)+1]^3P_j+\frac{a^4(p_1-q_1)}{EJ}\cdot$$
$$\cdot[(i-1)^2+\frac{4(i-1)^3}{3}]+\frac{a^4}{24EJ}\sum_{j=1}^{i}d_j(p_{i+1-j}-q_{i+1-j}),$$
$$i=2,3,\dots,n$$

The base's settlements at point $x=a(2i-1)$ are.

$$S_{oi}=\frac{1-\mu_i^2}{E_{oi}}\sum_{j=1}^{n}F_jq_j \quad (26)$$

The functions F_j depend upon the distance between zones where the contact pressure q_j is acting and points where the settlement is determined.

At point $x=a(2i-1)$ the part of total base displacement due to swelling is determined from Equation 14, where the lower and upper integration limits are h_i and H_i respectively. The equilibrium moisture distribution $w(z)$ is determined by Equation 4, where the additional vertical stress due to contact pressures q_i is calculated by means of angle points method.

To find the unknowns $y_1, \varphi_1, q_1, q_2, \dots, q_n$ we have n-conditions showing the equality of beam and base displacements at points $x=a(2i-1)$ and the equilibrium equations

$$y_i=S_{oi}-f_i\ ,\ i=1,2,\dots,n \quad (27)$$
$$\sum_{i=1}^{n}q_i=\sum_{i=1}^{n}p_i+\frac{1}{2ab}\sum_{i=1}^{n+1}P_i$$
$$\sum_{i=1}^{n}[2(i-1)+1]q_i=\sum_{i=1}^{n}[2(i-1)+1]p_i+\frac{1}{ab}\sum_{i=1}^{n+1}(i-1)P_i+\frac{1}{2a^2b}\sum_{i=1}^{n+1}M_i$$

The nonlinear system of Equations 27 has been solved with the help of computer using numerical techniques.

For example, figure 5 shows the results of calculation of the contact pressures distribution during soil wetting under the middle of the beam. The soil is Khvalynian swelling clay. The initial moisture profile is uniform with depth (30%). The data are: L=24m, n =12, a =1m, b=0,6m, EJ =5GN.m^2 p_i =0,2MPa, P_i =M_i=0, E_{oi} =20MPa, μ_i=0,35 (i=1,2,...,12). The wetting is occured within 6-th and 7-th zones. The wetting zone is 4 meters wedth.

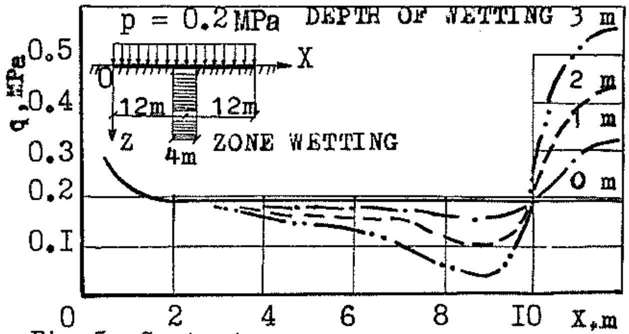

Fig.5 Contact pressures distribution upon swelling

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