An Analysis of Stability of Embankments on Soft Clays

by
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SUMMARY. The Paper presents a simple analysis for stability of finite and infinite embankments on soft clays which takes into account the strength anisotropy, dessication crust, variation of cohesion with depth and the surcharge load. Nomographs for rapid evaluation of the factor of safety for simplified cases are also presented. The results show that the strength anisotropy has a significant effect on the factor of safety. The width of the embankment becomes important only in the case of constant cohesion with depth. Finally, a simple method based on in-situ double vane test is suggested for the evaluation of the strength anisotropy.

1 INTRODUCTION

Because of the economic exigencies of land usage, there is, at present, an ever increasing necessity of planning transportation facilities, e.g. highways, pipelines etc. on marginal land. In the specific case of the design of embankments on soft clays, one of the critical aspects is their stability during and at the end of construction. Hence any analysis which attempts to optimise the design in relation to the soil properties is of immediate interest. Many authors, for example, Lo (Ref.1), Nakase (Ref.2), Reddy and Srinivasan (Ref.3), Medeiros (Ref.4) and others have discussed the effect of factors such as the variation of cohesion with depth, dessication crust, strength anisotropy and surcharge load on the stability of slopes on soft clays. This Paper presents a simple analysis of low embankments by considering all the above factors including the effect of the width of embankment. The effect of the shear strength of the fill has not been considered since it is likely to be small. Medeiros (Ref.4) found in an example that taking into consideration the dessication crust and strength anisotropy of a 6m high embankment of infinite width, the variation in factor of safety by neglecting the fill undrained strength of 2.5 t/m^2 was only 2%.

2 NOTATION

- Ch, Cv : strength corresponding to the horizontal and vertical directions, respectively, of the major principal stress.
- Ci : strength corresponding to the inclination i of the major principal stress with the horizontal direction- $Ci = Ch + (Cv - Ch)\,Sin^2 i$. (Ref.5).
- Chs : Ch at the surface.
- Sv, Sh : strength in vertical and horizontal planes respectively.
- d : depth of dessication crust.
- q : stress due to self weight of the fill.
- n : surcharge factor.
- Dc : depth of critical circle.
- K : strength anisotropy, Cv/Ch, (Ref.1).
- MR : resisting moment,
- MD : disturbing moment.
- F : factor of safety, MR/MD.
- ψ : angle between direction of failure plane and major principal stress.
- α : angle between failure plane to horizontal.

Other geometrical parameters are defined in Fig.1.

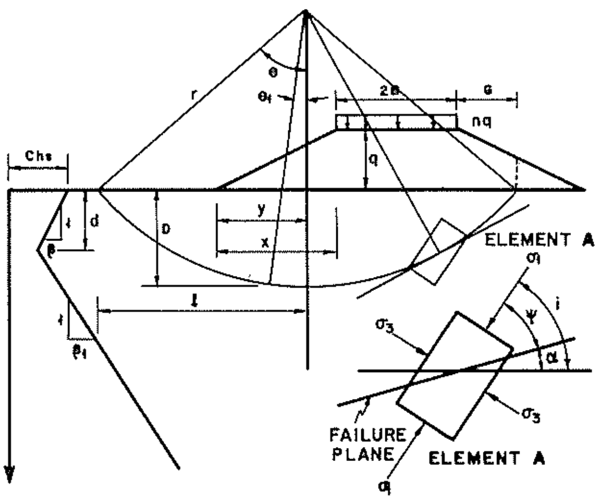

Fig.1 Definition of parameters

3 PRESENTATION OF THE ANALYSIS

Consider the case of a low embankment as shown in Fig.1, where the factor of safety for any slip circle is given by the ratio MR/MD. Neglecting the shear strength of the fill, the resisting moment MR, which depends solely on the foundation strength parameters, is given by the following (Ref.3):

$$MR = r^2 \int_{\theta_1}^{\theta} \{Chs + \beta r Cos\alpha - r Cos\theta\}\{1 + (K-1)(Sin^2\alpha\, Cos^2\psi + Sin^2\psi\, Cos^2\alpha + 2 Sin\alpha\, Sin\psi\, Cos\alpha\, Cos\psi)\}\,d\alpha \quad (1)$$

Hence for a given chord $2l$ parameters r and θ which yield a minimum value of MR may be evaluated. The disturbing moment, MD, depends on the geometry of the slip circle with respect to the embankment profile and may be evaluated for six possible cases schematically shown in Fig.2.

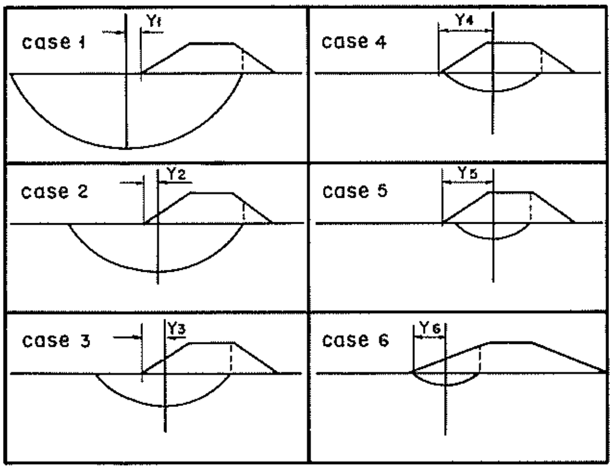

Fig.2 Slip circle locations

Case 1. Conditions: $l-y>x+2B$ and $y<l$.
The disturbing moment is given by:

$$MD_1=\{qx(y+2x/3)/2\}+\{q(1+n)(x+y+B)2B\}+ \left|\{(1-2G/3)qG/2\}+\{qG(x-G)(1-G/3)/2x\}\right| \quad (2)$$

Max. disturbing moment is given by $\partial MD_1/\partial y=0$

$$\left|\{(1-2G/3)qG/2\}+\{qG(x-G)(1-G/3)/2x\}\right|=A \quad (3)$$

$$\partial MD_1/\partial y=(qx/2)+2qB(1+n)+\partial A/\partial y=0 \quad (4)$$

From Eq.(4), $y=-2(B+x)\pm\sqrt{\{l^2+2x^2+4Bx(1+n)\}}$ (5)

Substituting $p=\sqrt{\{l^2+2x^2+4Bx(1+n)\}}$ (6)

$$y=-2(B+x)\pm p$$

It may be shown that the value of y which satisfies the conditions for this case is:

$$y_1=-2(B+x)+p \quad (7)$$

Substituting Eq.(7) in Eq.(2) gives maximum value of disturbing moment for a given value of l.

Case 2 Conditions: $l+y>x+2B$ and $y<l$.

$$MD_2=\{qx(2x-3y)/6\}+\{2Bq(1+n)(x-y+B)\}+A \quad (8)$$

Solution of $\partial MD_2/\partial y=0$ yields $y=2(B+x)\pm p$ (9)
Eq.(9) satisfies the conditions only when

$$y_2=2(B+x)-p \quad (10)$$

Case 3. Conditions: $l+y\leq x+2B$ and $y\leq l$.
Solution to this case has been given by Nakase (Ref.2) and is given by:

$$MD_3=q\left|\{l^2(1+n)/2\}-\{x^2(1+4n)/24(1+n)\}\right| \quad (11)$$

and $y_3=x(1+2n)/2(1+n)$ (12)

Note that Eq.(11) and (12) are also valid for the case of an embankment of infinite width.

Case 4. Conditions: $l+y>x+2B$ and $y>l$.

$$MD_4=\{qx(2x-3y)/6\}+\{2Bq(1+n)(x-y+B)\}+A+E \quad (13)$$

where $E=q(y^3-3yl^2+2l^3)/6x$ (14)

$\partial MD_4/\partial y=0$ yields $y=B+x\pm J$ (15)

where $J=\sqrt{\{l^2+B(2xn-B)\}}$ (16)

Three subcases may be analysed to determine y_4 which yields the maximum value of MD_4.

(i) $J\geq 0$ and $y_4=B+x+J$. This subcase is not valid since the conditions for Case 4 are not satisfied.
(ii) $J\geq 0$ and $y_4=B+x-J$. The conditions for case 4 are satisfied only when $2xn<B$.
(iii) $J<0$. For a circle to lie in Case 4, $2l>2B$, i.e. $l^2-B^2>0$. Since $2Bxn\geq 0$, this subcase is also not valid.

Case 5. Conditions: $l+y\leq x+2B$ and $y>l$.

$$MD_5=\{qx(2x-3y)/6\}+\{q(1+n)(l-x+y)(l+x-y)\}+E \quad (17)$$

and $y=x(1+n)\pm\sqrt{\{l^2+x^2n^2\}}$ (18)

The maximum disturbing moment is given by:

$$y_5=x(1+n)-\sqrt{\{l^2+x^2n^2\}} \quad (19)$$

Note that Case 5 is also valid for infinitely wide embankments since the maximum disturbing moment is independent of the width.

Case 6. Conditions: $2l<x$ and $y=l$.

$$MD_6=2ql^3/3x \quad (20)$$

and $y_6=l$ (21)

4 SUGGESTED PROCEDURE AND PRESENTATION OF RESULTS

The procedure is simple as outlined below and the analysis can readily be done with an electronic computer.

(i) Assume a trial value of l.
(ii) Calculate the minimum MR for this l by varying r.
(iii) Verify in which of the six cases the problem lies and calculate maximum MD.
(iv) Determine F.
(v) Adopt successive trial values of l and repeat steps (ii) to (iv) until a minimum value of F is obtained.

Figs.3 to 6 present simplified nomographs for embankment width, 2B, varying from 5m to ∞. For the sake of convenience of presentation of results in a limited space, the depth of dessication crust and surcharge load have not been considered. Values of the increase of cohesion with depth, β_1, in the range of 0 to 0.4t/m³ have been taken into consideration. Medeiros (Ref.4) in an analysis of published data on 23 fills on soft clays found β_1 varying from 0 to 0.35t/m³, with most of the values lying between 0 to 0.15 t/m³. Note that the graph of the variation of depth Dc of the critical circle with the width of slope, x, in each figure is common to all the three cases of anisotropy considered, since it was found that the variation in strength anisotropy practically does not alter the geometry of the critical circle.

5 DISCUSSION OF RESULTS

The nomographs clearly show the influence of strength anisotropy on the factor of safety. In general, a 50% variation in K (0.8 to 1.2) leads to a variation of approximately 25% in F. For example, it is of interest to consider the case of a highway embankment with the following characteristics: 2B=10m; x=30m; Chs=1t/m²; β_1=0.05t/m³ and q=7t/m². From Fig.4, factors of safety equal to 1.17, 1.31 and 1.50 for values of K of 0.8, 1.0 and 1.2 respectively are obtained. The importance of the determination of strength anisotropy is obvious.

The nomographs show the interesting result that the consideration of an embankment of finite width as infinite (commonly done in practice) may lead to an underestimation of the factor of safety by approximately 15% in the case of constant cohesion with depth. However this underestimation of F is shown to be insignificant when the clay shows even a slight increase of cohesion with depth, (β_1>0.05t/m³). Fig.7 shows the above trend for the typical case of an embankment with 2B=5m, x=60m and K11.0. It may further be observed that the above results donot alter with K.

It has already been mentioned that the depth of critical circle, Dc, is found to be virtually independent of K. Further it may be noted that the critical depth is independent of cohesion when the latter does not vary with depth (β_1=0).The width of the embankment can significantly alter Dc only in the case of constant cohesion.Since most natural clay deposits show an increase of cohesion with depth,the above effect will be small.

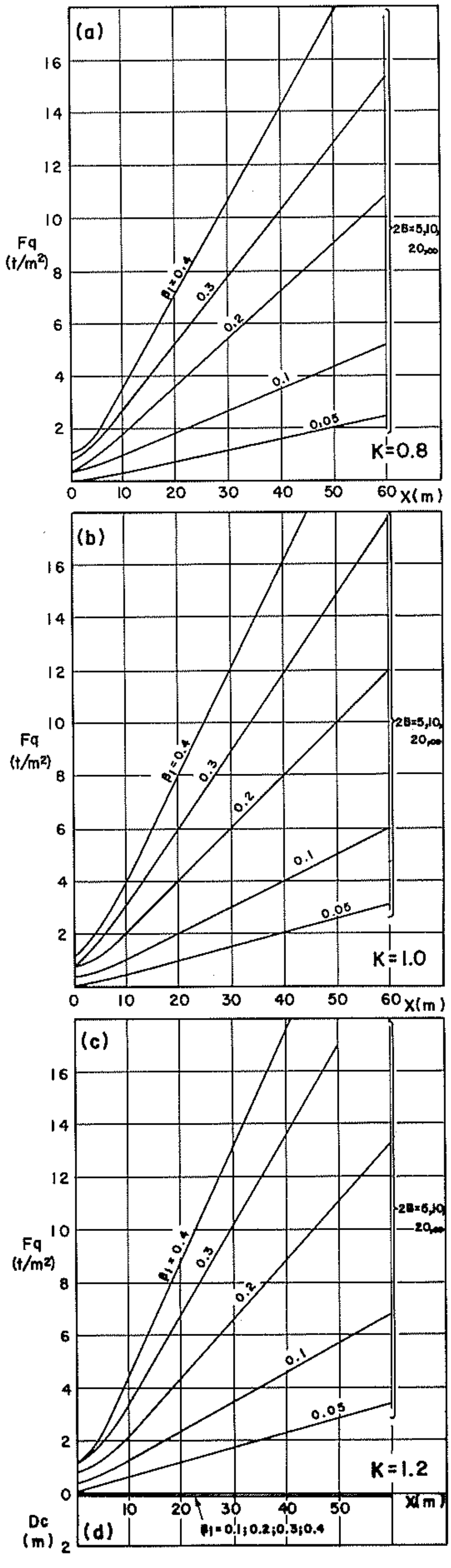

Fig.3 Stability graph for Chs=0

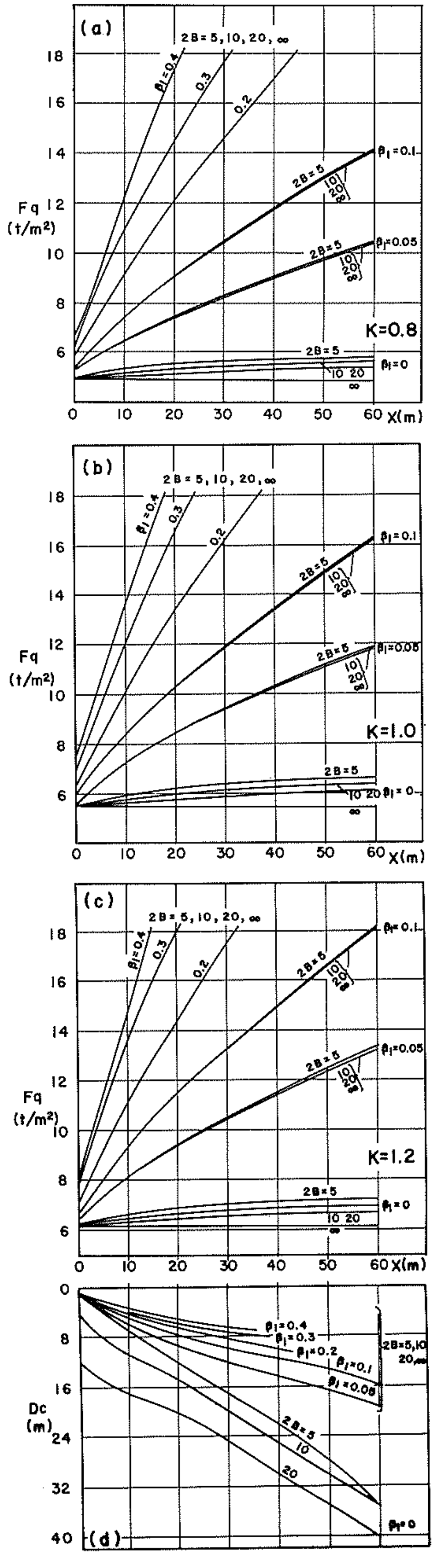

Fig.4 Stability graph for Chs=1.0t/m^2

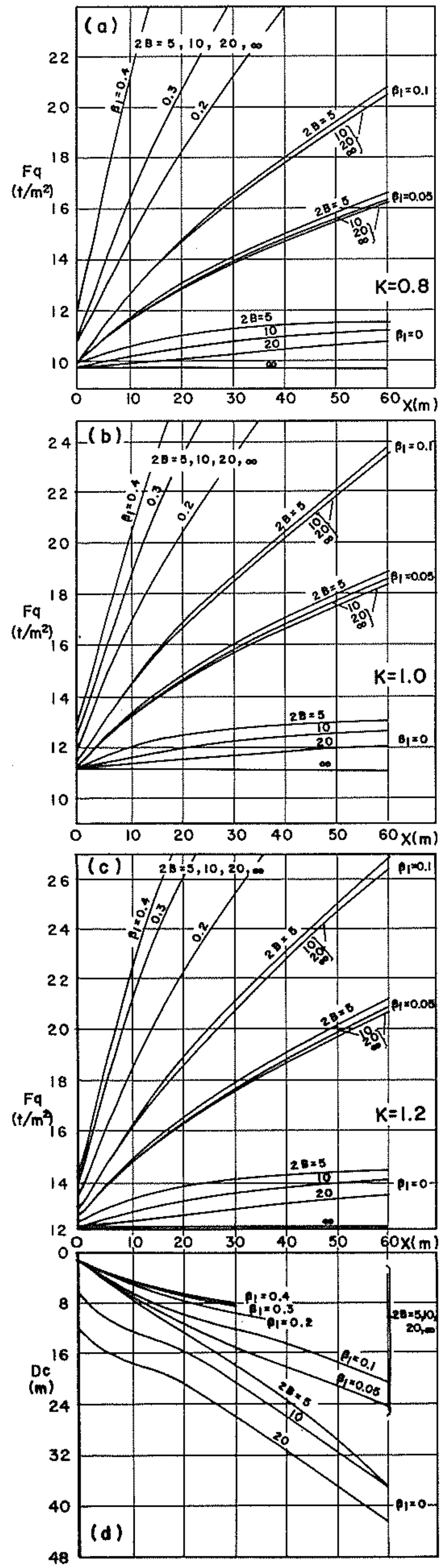

Fig.5 Stability graph for Chs=2.0t/m^2

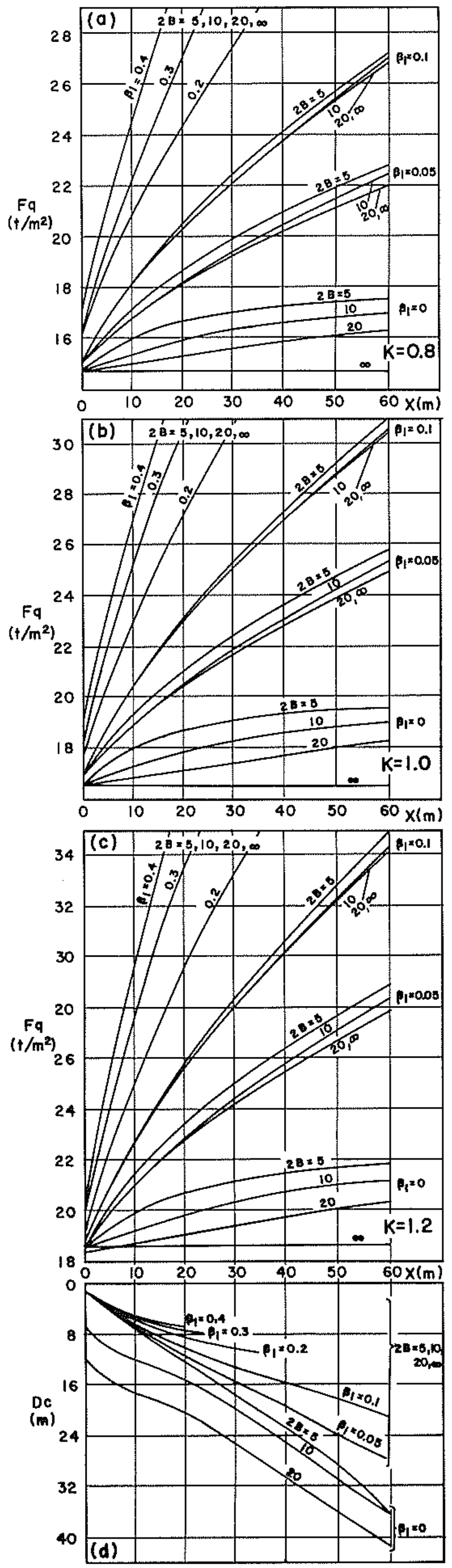

Fig.6 Stability graph for Chs=3.0t/m^2

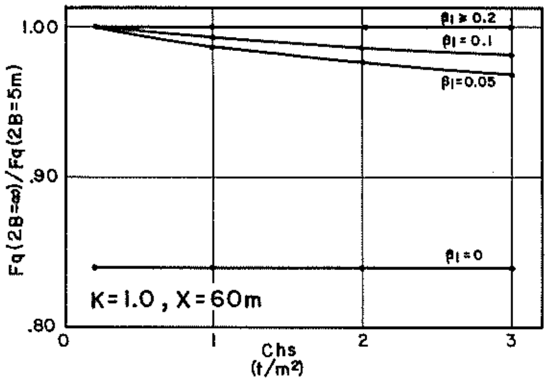

Fig. 7 Effect of embankment width and β_1 on F.

6 DETERMINATION OF STRENGTH ANISOTROPY, K.

Because of the difficulty in obtaining undisturbed samples of soft clays, it is desirable to determine the strength parameters (Cv and Ch) in-situ. These may conveniently be derived from determination of strengths in vertical and horizontal planes (Sv and Sh) by using two vanes of different sizes (Ref.6). Note that Cv and Ch are different from Sv and Sh. Sv shall be approximately equal to Ch since the major principal stress acts on the horizontal plane in both cases (Ref.1). From considerations of the element A in Fig.1 it may be observed that for failure along the horizontal plane, when the strength corresponds to Sh (α=0), the relationship $C_i = C_\psi$ = Sh holds. Hence the expression for C_i (see notation) transforms to:

$$C_i = C_\psi = Sh = Sv + Sv(K-1)\sin^2\psi \qquad (22)$$

ψ has been shown to be approximately equal to 34° for a number of clays, (Refs.1 and 7).

7 CONCLUSIONS

The proposed method of analysis shows the remarkable effect of strength anisotropy on the factor of safety. Hence it is of practical importance to evaluate this parameter, and a method based on the use of in-situ double vane test is suggested. The strength anisotropy, however, does not significantly alter the geometry of the critical circle. The effect of the width of the embankment becomes important only in the case of constant cohesion with depth.

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