Effects of the Pile Cap on the Load Displacement Behaviour of Pile Groups When Subjected to Eccentric Loading

by

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SUMMARY. The paper describes a method of analysis of problems associated with the design of cap-bearing pile groups when subjected to eccentric loading. The analysis is based on an algorithm developed by using the theory of multi-dimensional singular integral equations with Mindlin's solution for a point load within a semi-infinite solid as a kernel function. The materials of the piles and the soil medium are both idealised as linearly elastic solids. The results of the analysis for 2x1, 2x2 and 3x3 pile groups are presented in nondimensional form to show the effects of the various geometrical variables as well as the soil properties and the properties of the pile material on the behaviour of the system. The results of the analysis have been compared with available experimental data.

1 INTRODUCTION

Although the problem of the load displacement behaviour of pile groups with the cap in contact with the ground is one of very common occurrence in foundation engineering yet few systematic attempts have been made in predicting their behaviour either by experimental or by analytical and numerical methods.

Some experimental investigation of axially loaded cap-bearing pile groups have been carried out by Whitaker (Ref. 1), Koizumi and Ito (Ref. 2), Brand et al (Ref. 3) and Hooper (Ref. 4). Whitaker conducted a large number of small scale laboratory tests on axially loaded cap-bearing pile groups embedded in remoulded London clay. He studied the effects of the variables such as the length to diameter ratio, spacing to diameter ratio and the geometry of the group and found the settlement response of such groups is almost identical to that of free-standing groups. Koizumi and Ito (Ref. 2) carried out tests on a single pile and a cap-bearing 3x3 pile groups embedded in a thick uniform layer of silty clay. Hooper (Ref. 4) instrumented the foundations of a tall multi-storeyed building supported on 51 under-reamed piles.

Theoretical analyses of the problem of the load-displacement behaviour of axially loaded cap-bearing pile groups embedded in ideal elastic sub-soil have been carried out by Butterfield and Banerjee (Ref. 5) and Davis and Poulos (Ref. 6) These theoretical analyses showed that it is possible to predict the behaviour of such complex foundations under working load by use of relativelv sophisticated theoretical analyses.

Although relatively large number of pile group problems commonly encountered in practice are that of eccentric loadings, virtually no attempts have been made to analyse such problems. The most commonly used method for calculating the distribution of pressures under the cap and the load distribution in piles is the reinforced concrete beam-column theory, in which the piles are represented by using a modular ratio concept. It has been shown by Butterfield and Banerjee (Ref. 5) and also Davis and Poulos (Ref. 6) that the stiffness of the pile soil system depends not only on the properties of the pile but also on the spacings, length, diameter and properties of the soil. Therefore the simple modular ratio concept such as that used in beam-column theory cannot be used satisfactorily.

2 NOTATION

The following is a list of global notation used in the text, in addition, there are a few symbols which are defined locally :

D	diameter of piles
G	shear modulus of the soil medium
H	the thickness of the soil layer
$[K]$	the matrix calculated from $K(A_c,B_s)$, $K(A_s,B_c)$, and so on
$K(A,B_s)$	the kernel functions evaluated from Mindlin's solution for a point load designated by the co-ordinates in brackets
M_o	the moment applied to the system
M_c	the moment developed due to the stress intensity at the cap-soil interface
L	the length of the pile
m,n,p	number of elements representing the discrete cap-soil, pile-soil and soil-rigid layer interfaces respectively
N	number of piles in the group
P_o	the total vertical load applied to the system
P_c	the vertical load carried by the cap
S,C,R	the pile-soil, the cap-soil, the rigid layer-soil interfaces respectively
$U(A_c)$, $U(A_s)$ etc	the displacements at the point designated by the co-ordinates in brackets
U_o	the vertical displacement of the cap

{U} the matrix representation of $U(A_c)$, $U(A_s)$ etc

Φ the fictitious stress intensities

Φ_c, Φ_s, Φ_r the fictitious stress intensities on the cap-soil, the pile-soil and the soil-rigid layer interfaces respectively

{Φ} the matrix representation of Φ_c, Φ_s etc

λ the ratio of the Young's modulus of the pile material to the shear modulus of soil

μ Poisson's ratio of the soil

Θ the rotation of the cap

3 METHOD OF ANALYSIS

The displacement vector $U_i(A)$ at a point A due to the forces $P_j(B)$ acting at a point B in the interior of a semi-infinite solid can be written as

$$U_i(A) = K_{ij}(A,B)\ P_j(B) \tag{1}$$

where

$$U_i(A) = \begin{Bmatrix} U_x(A) \\ U_y(A) \\ U_z(A) \end{Bmatrix}, \quad P_j(B) = \begin{Bmatrix} P_x(B) \\ P_y(B) \\ P_z(B) \end{Bmatrix}$$

$$K_{ij}(A,B) = \begin{bmatrix} K_{xx}(A,B) & K_{xy}(A,B) & K_{xz}(A,B) \\ K_{yx}(A,B) & K_{yy}(A,B) & K_{yz}(A,B) \\ K_{zx}(A,B) & K_{zy}(A,B) & K_{zz}(A,B) \end{bmatrix} \tag{2}$$

The elements of the 3x3 $K_{ij}(A,B)$ matrix are symmetrical about the diagonal and are functions of the positions of A and B with respect to a cartesian co-ordinate system.

If we distribute the fictitious stress intensities Φ_c over the cap-soil interface C, the stress intensity Φ_s over the pile-soil interface S, and Φ_r over the soil-rigid layer interface R, we can write the displacements at any point A within the soil layer as (see Figure 1) :

$$U_i(A) = \int_C (\Phi_c)_j\ K_{ij}(A,B_c)\ dC + \int_S (\Phi_s)_j\ K_{ij}(A,B_s)\ dS + \int_R (\Phi_r)_j\ K_{ij}(A,B_r)\ dR \tag{3}$$

where dC, dS and dR are the elements of the surfaces C, S and R respectively, and $(\Phi_c)_j$, $(\Phi_s)_j$ and $(\Phi_r)_j$ each have three components in the direction of the three co-ordinate axes.

By bringing the point A onto the surfaces C, S and R, we can obtain the three integral equations for displacements of the three surfaces C, S and R as (see Figure 1) :

$$U_i(A_c) = \int_C (\Phi_c)_j\ K_{ij}(A_c,B_c)\ dC + \int_S (\Phi_s)_j\ K_{ij}(A_c,B_s)\ dS + \int_R (\Phi_r)_j\ K_{ij}(A_c,B_r)\ dR \tag{4}$$

$$U_i(A_s) = \int_C (\Phi_c)_j\ K_{ij}(A_s,B_c)\ dC + \int_S (\Phi_s)_j\ K_{ij}(A_s,B_s)\ dS + \int_R (\Phi_r)_j\ K_{ij}(A_s,B_r)\ dR \tag{5}$$

and

$$U_i(A_r) = \int_C (\Phi_c)_j\ K_{ij}(A_r,B_c)\ dC + \int_S (\Phi_s)_j\ K_{ij}(A_r,B_s)\ dS + \int_R (\Phi_r)_j\ K_{ij}(A_r,B_r)\ dR \tag{6}$$

Equations (4), (5) and (6) have singularities in the kernel functions K_{ij} when either A_c and B_c or A_s and B_s or A_r and B_r coincide but the integral themselves exist in the normal sense because the singularities involved are of the order 1/r, where r is the distance between A and B. These equations also satisfy equilibrium and compatibility everywhere and the boundary conditions on the unloaded ground surface. Therefore solutions of (4), (5) and (6) will represent exact solution of the problem for specified displacements $U_i(A_c)$, $U_i(A_s)$ and $U_i(A_r)$.

It has been shown by Butterfield and Banerjee (Ref. 7) and Banerjee (Ref. 9) that the following assumptions :

(i) the cap-soil surface is smooth

(ii) the rigid layer-soil interface is smooth

(iii) the lateral compatibility may be ignored for vertical piles in a floating pile group

leads to less than 10% error in the load displacement response for the range of problems commonly encountered in practice. This results in reduction of computational effort by a factor of about 9.

When these approximations are introduced to the analysis, equations (4), (5) and (6) which represents 9 simultaneous integral equations, reduces to 3 simultaneous integral equations. These equations then can be written in matrix notation for m elements of C, n elements of S and p elements of R (see Refs. 5, 7 and 9) as :

$$\{U\} = [K]\{\Phi\} \tag{7}$$

Formal solution of which can be written as :

$$\{\Phi\} = [K]^{-1}\{U\} \tag{8}$$

Equation (8) provides the stress intensities at the cap-soil interface, the stresses at the pile-soil interface and the fictitious stress intensity at the rigid layer-soil layer interface (which has no physical significance) provided {U} is evaluated from the specified boundary conditions.

For a rigid cap-rigid pile ($\lambda = \infty$) system, we can write the boundary conditions as :

(a) for specified unit vertical displacement of the cap

$U(A_c) = 1$
$U(A_s) = 1$
$U(A_r) = 0$

(b) for specified unit rotation of the cap in the XZ plane (say)

$U(A_c) = x_i$, where x_i is the x co-ordinate of the cap element i from the centroid of the cap

$U(A_s) = x_i$, where x_i is the x co-ordinate of the ith pile centre from the centroid of the cap

$U(A_r) = 0$

TABLE I

RESULTS FOR 2x1 GROUPS

(cap size = 5Dx2.5D H/L = 2, μ = 0.5, spacing/diameter = 2.5)

L/D	λ	STIFFNESSES		LOAD DISTRIBUTION		COMBINED EFFECTS of P_o & M_o	
		P_o/GU_oD	$M_o/G\theta D^3$	P_c/P_o %	M_c/M_o %	e_a	e_b
20	100	24	98	54	62	0.195	0.62
	1000	42	190	22	28	0.180	0.55
	10000	50	274	15	18	0.200	0.52
	∞	52	291	14	17	0.200	0.51
40	100	25	104	49	61	0.195	0.65
	1000	50	215	16	27	0.160	0.57
	10000	75	441	14	18	0.175	0.53
	∞	80	538	14	17	0.185	0.52

TABLE II

RESULTS FOR 2x2 GROUPS

(Cap size = 5Dx5D, H/L=2, μ=0.5 Spacing/diameter=2.5)

L/D	λ	STIFFNESSES		LOAD DISTRIBUTION		COMBINED EFFECTS of P_o & M_o	
		P_o/GU_oD	$M_o/G\theta D^3$	(P_c/P_o)%	(M_c/M_o)%	e_a	e_b
20	100	35	166	54	61	0.22	0.58
	1000	55	325	25	27	0.22	0.52
	10000	62	455	19	18	0.26	0.49
	∞	63	478	18	17	0.28	0.49
40	100	35	175	48	59	0.22	0.64
	1000	68	372	16	24	0.22	0.56
	10000	90	742	18	18	0.23	0.51
	∞	94	883	17	16	0.30	0.51

TABLE III

RESULTS FOR 3x3 GROUPS

(Cap size = 7.5Dx7.5D H/L=2, μ=0.5 spacing/diameter = 2.5)

L/D	λ	STIFFNESSES		LOAD DISTRIBUTION		COMBINED EFFECTS OF P_o & M_o		
		P_o/GU_oD	$M_o/G\theta D^3$	(P_c/P_o)%	(M_c/M_o)%	e_a	e_b	e_c
20	100	54	612	55	53	0.23	0.37	0.31
	1000	77	1198	28	21	0.26	0.39	0.29
40	100	54	642	47	51	0.24	0.40	0.36
	1000	94	1476	17	18	0.24	0.40	0.34

Having obtained the stress intensities, one can easily evaluate the loads at various pile sections, and that taken by the cap by applying simple principles of statical equilibrium.

For compressible piles, solutions may be obtained by using the solution for a rigid pile as a first approximation in an iterative procedure described by Butterfield and Banerjee (Ref. 7).

4 RESULTS OF THE ANALYSIS

Numerical accuracy and convergence of the solution would depend on how accurately the integrals in equations (4), (5) and (6) are replaced by the numerical integration formulae. In the present analysis, these were evaluated by using the trapezoidal rule. From a series of trial computations it was found that m = 36, n = 7xN (6 for the shaft and one for the base area for each pile in the group), and p = 100 (taken over an area of 4 times the pile length square) was satisfactory.

The effects of the length to diameter ratio, the group size, the ratio of the Young's modulus of the pile material to the shear modulus of the soil, the influence of the cap on the vertical and rotational stiffnesses of the system have been studied and typical results for 2x1, 2x2 and 3x3 groups are presented in Tables I, II and III respectively (see also Figure 2 for pile geometries).

The Poisson's ratio of soil has been assumed to be 0.5, and the ratio of the depth of the soil layer to the length of the piles has been adopted as 2.0 throughout.

In order to study the combined effects of vertical load and moments, the following values of eccentricities were calculated :

(a) e_a = the ratio of the minimum eccentricity to cause tension at the cap-soil interface to the width of the cap in the direction of the applied eccentricity

(b) e_b = the ratio of the minimum eccentricity to cause tension at the pile head of the corner piles to the distance between the two outer rows of piles measured in the direction of the eccentricity

(c) e_c = same as e_b, but calculated for the middle of the outer row of piles for 3x3 groups.

5 COMPARISON WITH EXPERIMENTAL RESULTS

There are no reliable and well documented experimental results available for the load displacement behaviour of eccentrically loaded cap-bearing pile groups. However, some experimental results for axially loaded cap-bearing pile groups have been reported by Whitaker (Ref. 1), Koizumi and Ito (Ref. 2), Brand et al (Ref. 3) and Hooper (Ref. 4). The groups tested by Whitaker and Hooper are too large to be analysed by using the present computer program economically. Therefore the present analysis has been compared with the test results reported by Koizumi and Ito (Ref. 2) and Brand et al (Ref. 3). Koizumi and Ito carried out full scale tests in a single pile and a 3x3 pile group embedded in a uniform layer of silty clay with shells. The geometry of the pile group tested are shown below :

3x3 floating pile group with rigid cap

The length to diameter ratio (L/D) = 18.5

The spacing to diameter ratio (S/D) = 3.0

Outside diameter of piles = 30 cm

Inside diameter of piles = 29.36 cm

The piles were made of closed-end steel tubes. The experimental value for the settlement ratio (defined as the ratio of the settlement of the group at a load of NxP_o to that of a single pile in the same soil under a load of P_o) calculated at a factor of safety of 3 was 3.81, which compared well with the theoretical settlement ratio of 4.01 (for H/L = 2.0, μ = 0.5).

Brand et al (Ref. 3) conducted a series of full scale tests on 2x2 cap-bearing pile groups embedded in normally consolidated soft clay. The piles were 15 cm diameter and 6 metre long timber piles. They also reported test results on an isolated single pile for comparison. Table IV shows the comparison of the theoretical settlement ratios (H/L = 2.0, μ = 0.5) and experimental values calculated for a factor of safety of 3 on the ultimate group load.

TABLE IV

COMPARISON WITH BRAND ET AL (Ref. 3)

S/D	EXPERIMENTAL SETTLEMENT RATIO	THEORETICAL SETTLEMENT RATIO
2	2.6	2.70
2.5	2.2	2.61
3	2.2	2.35
4	1.8	2.00
5	1.0	1.60

6 CONCLUSIONS

(i) New method of analysis based on the theory of integral equations, which in principle can be applied to obtain solutions of any three dimensional elastic problem is presented.

(ii) Numerical results for nondimensional vertical stiffness and rotational stiffnesses for 2x1, 2x2 and 3x3 pile groups have been tabulated. These can be used as boundary conditions for the analysis of the super-structure.

(iii) It has been shown that the pile cap can support as much as 60% of the applied loading.

(iv) The minimum value of the eccentricity required to produce tension at the cap-soil interface or that the pile head is always higher than $\frac{1}{6}$th of the width of the foundation.

(v) The theoretical results agreed reasonably well with the experimental data for some axially loaded cap-bearing pile groups to suggest that such an analysis can be used to design pile groups.

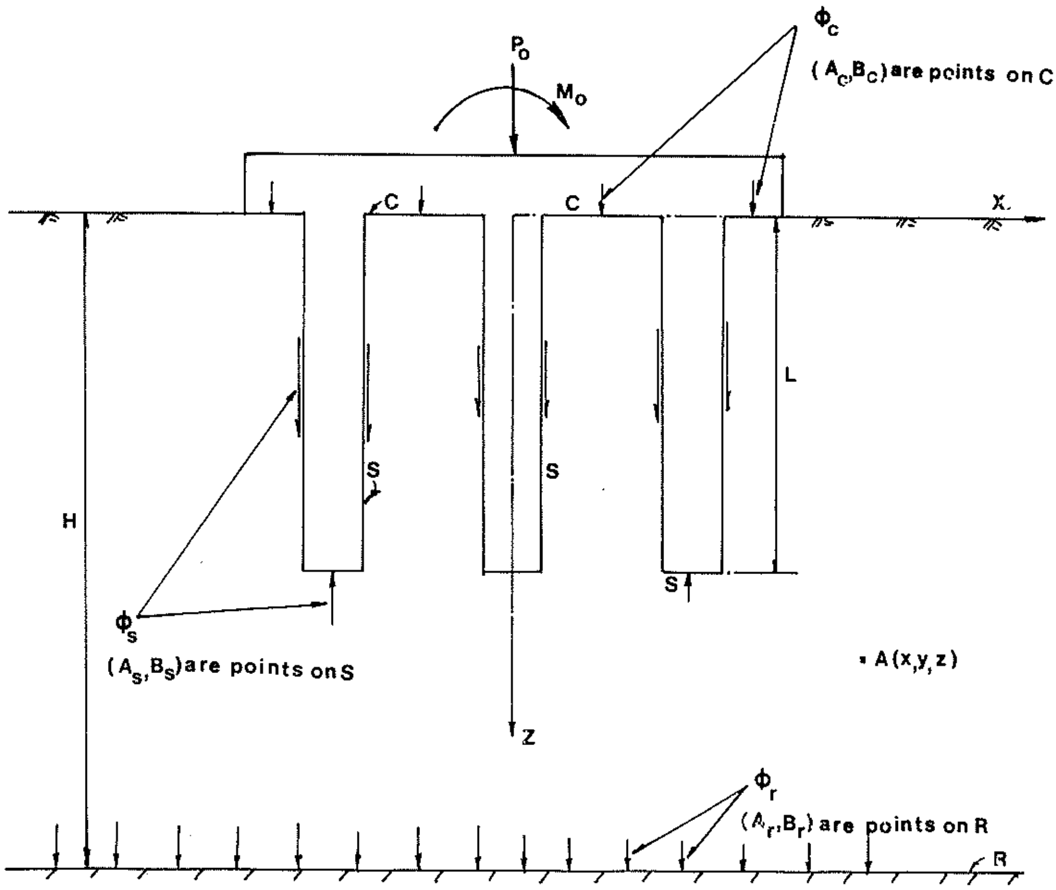

Fig. 1 General arrangement of the pile cap-pile group system

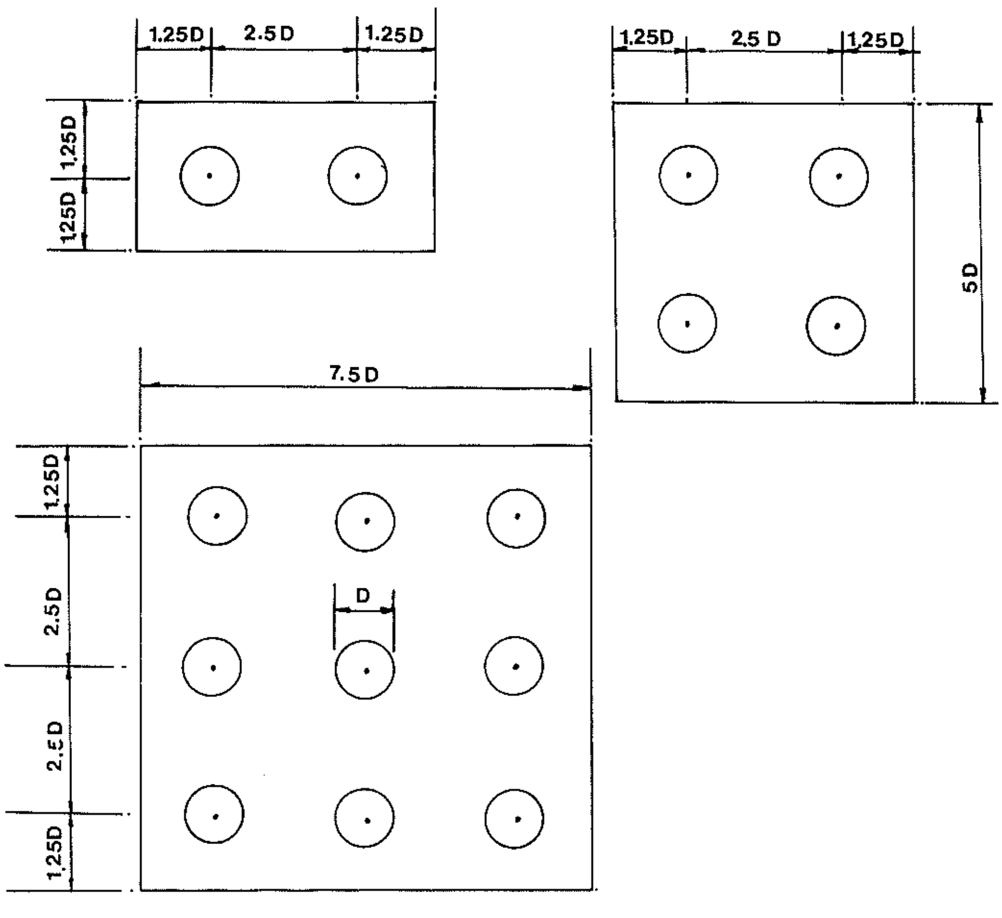

Fig. 2 Typical pile groups which were analysed

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