INTERNATIONAL SOCIETY FOR
SOIL MECHANICS AND
GEOTECHNICAL ENGINEERING

SIMSG ISSMGE

This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

https://www.issmge.org/publications/online-library

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

Separable Yield Surfaces to Correlate Axi-Symmetric, Plane Strain, Simple Shear and Multiple Stage Tests

by

T. K. CHAPLIN, M.A., Ph.D.

Senior Lecturer, University of Birmingham, England

SUMMARY . Separable yield surfaces were first considered in 1971; no particular expression for plastic work is needed. Plastic straining of 'wet' clay is taken as (a) anisotropic consolidation steps, using the normality law, plus (b) straining in one mode of deformation while there is no strain in one or more other modes. The stress-strain curve for each mode is empirically related to the corresponding flow rate function. The yield surface is assumed unchanged by the deformations.

In plane strain and shearing, when a single mode of deformation is applied, two or more cohesive resistances are reduced; the new *steepest-ascent principle* shows how the reduction might be shared. Examples of different volume change conditions are given. It is hoped that improved forecasts of three-dimensional deformations under working stresses will be made possible by this fully-computable 'Bir-Clay' model.

1 INTRODUCTION

The concept of a separable yield surface was first presented at the Roscoe Memorial Symposium at Cambridge in 1971 (Ref. 1). Few clays on the 'wet' side of the critical state seem likely to have a yield surface (locus) differing much from a separable approximation. In this paper, simpler modes of deformation for soil testing are studied with separable yield surfaces, a new *steepest-ascent principle* and an empirical correlation to get a stress-strain curve from the flow rate function.

There may not be any reversal of shear strain or volumetric strain. Strains giving contraction in length or volume are taken as positive. Changes in the shape of the yield surface are ignored. It is assumed that the normality law applied to changes at constant stress ratio, and that they are added to other plastic strains. Elastic strains are taken as zero. Close to the critical state, the yield surface need *not* be parallel to the space diagonal in principal effective stress space.

2 NOTATION

The p superscript denotes plastic strain. The dot notation need *not* mean constant-rate change.

c' Cohesion intercept in effective stresses.
e Void ratio, or 2.71828...
$f'(s)$ A flow rate function.
$f(s)$ A yield function.
p Mean effective stress $(\sigma'_x+\sigma'_y+\sigma'_z)/3$.
p_e Equivalent isotropic consolidation pressure.
p_u Critical pressure, i.e. mean effective stress for given void ratio.
q Stress difference $\sigma_x-\sigma_z$.
s A mobilization ratio (of the cohesive resistance opposing some mode of deformation), e.g. q/q_{max}.
t Time.
u Pore water pressure.
x, y, z Axes, e.g. x for the sample axis.
A Cross-sectional area.
C_c Compression index in consolidation test.
K Coefficient of earth pressure, σ'_3/σ'_1.
K_o K for zero lateral strain
L Length.
V Volume.
α Factor of proportionality.
γ Engineering shear strain.
δ Change in following quantity.
ε Strain, also $\varepsilon_a - \varepsilon_v/3$.
ε_a Axial strain.
ε_c A strain measure.
ε_v Volumetric strain = $\varepsilon_x+\varepsilon_y+\varepsilon_z$.
ζ Constant relating actual strains in one mode to theoretical strains.
η q/p.
κ Gradient of swelling line.
λ Scaling factor for a set of strain-increments; gradient of compression line in consolidation.
σ Total direct stress.
σ' Effective direct stress.
σ_a Axial stress in axi-symmetric test.
σ_c A stress measure.
σ_{crmax} Maximum value of σ_{cr} at current void ratio.
σ_e Equivalent pressure (isotropic consolidation).
σ_m Mean total stress $(\sigma_x+\sigma_y+\sigma_z)/3$.
σ_r Radial stress in axi-symmetric test.
$(\sigma_x, \sigma_y, \sigma_z$ Total direct stresses
$(\sigma'_x, \sigma'_y, \sigma'_z$ Effective direct stresses.
τ Shear stress.
ϕ' Angle of shearing resistance for effective stresses.
Δ Change in following quantity.
M (capital μ) Critical state constant q_{max}/p_u.

3 MAIN DEFINITIONS

Separable yield surface: can be put for n axes as

$$F \equiv f_1(s_1) + f_2(s_2) + \dots f_n(s_n) \qquad (1)$$

where each term is a positive *yield function.*

Mode of deformation: has fixed ratios of all direct and shear strain increments.

Measure of strain: A linear combination of direct and shear strains (or strain-increments), expressed by a single quantity, e.g. volumetric strain.

Measure of stress: A linear combination of stresses, direct and shear. Examples are the stress difference $q = \sigma_x - \sigma_z$ and the mean effective stress.

Cohesive resistance: One which could be made negligibly small by large strains in a mode orthogonal to the one it opposes. The critical pressure p_u is *not* a cohesive resistance.

Mobilization ratio: Ratio of a stress measure to its maximum for the *current* void ratio.

Orthogonality: In principle, change in a measure of stress (or strain) would not change any measure orthogonal to it.

TABLE I
BASIC ORTHOGONAL SETS FOR STRESS AND STRAIN

σ_x	σ_y	σ_z	τ_{xy}	τ_{yz}	τ_{zx}
ε_x	ε_y	ε_z	γ_{xy}	γ_{yz}	γ_{zx}

Admissible measures: The sum of each stress measure times the corresponding strain measure accounts for all the work done; the normality law applies to admissible measures.

4 SEPARABLE YIELD SURFACES

For Mode No. 1 of deformation the mobilization ratio is s_1, and so on.

In Equation 1, each term varies from 0 to 1 as s varies from 0 to 1. Because F is separable and $\partial f_r(s_r)/\partial s_k = 0$ for $r \neq k$, the rth term in the partial (or total) differential of F is

$$f_r'(s_r) = \frac{df_r(s_r)}{ds_r} \tag{2}$$

called a *flow rate function* with unit area above the s axis from s = 0 to s = 1, with non-negative slope to ensure that the yield surface is convex.

Example

For a 'triaxial' compression test we take Mode 1 as axi-symmetric shearing at constant volume in Mode M1, defined as

$$\dot{\varepsilon}_x : \dot{\varepsilon}_y : \dot{\varepsilon}_z = \tfrac{1}{3} : -\tfrac{1}{6} : -\tfrac{1}{6} . \tag{3}$$

Uniform inwards strain, Mode M2 (as in isotropic consolidation), could be defined as

$$\dot{\varepsilon}_x : \dot{\varepsilon}_y : \dot{\varepsilon}_z = 1 : 1 : 1 . \tag{4}$$

and used for Mode 2 in this test. The flow rate functions are

$$f_1'(s_1) = 2s_1 \tag{5a}$$

$$f_2'(s_2) = 1.0, \tag{5b}$$

as shown in Fig. 1. These correspond to a Tresca type behaviour in Mode 2 and the Lévy-Mises flow-rule in Mode 1. The yield functions are

$$f_1(s_1) = s_1^2 \tag{6a}$$

$$f_2(s_2) = s_2 \tag{6b}$$

as in Fig. 2. The separable yield surface in s space (Fig. 3) is then

$$s_1^2 + s_2 = 1 \tag{7}$$

At the end of consolidation the conditions are: $s_1 = 0$, $s_2 = 1$, $q = 0$, $p = p_e$. At the end of the test, we have $s_1 = 1$, $s_2 = 0$, $q = q_{max}$, $p = p_u$. In between the values of q and p vary linearly with s_1 and s_2.

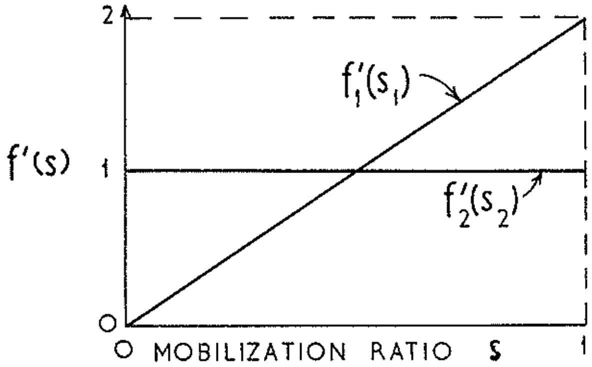

Fig. 1 Flow rate functions

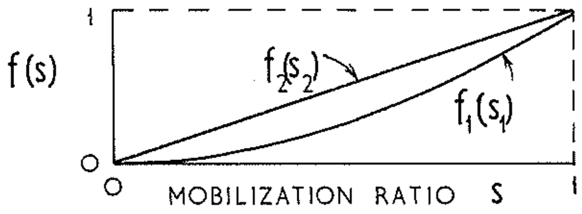

Fig. 2 Yield functions

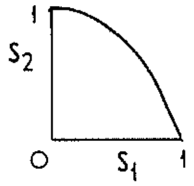

Fig. 3 Separable yield surface

5 EMPIRICAL DERIVATION OF STRESS-STRAIN CURVES

An empirical transformation to derive the Voce stress-strain curve (the simplest) from the Tresca type of flow rate curve, was given in Ref. 2 as

$$\varepsilon_r^p = \zeta_r \int_0^{s_r} \frac{f_r'(t)}{1 - t}\, dt \tag{8}$$

where t is merely a variable replacing s_r, and the constant ζ_r scales down the strains to those for a real soil, being typically around 0.01 in shear.

When Equation 8 was applied to the Levy-Mises flow rule, Equation 5a, it was shown in Fig. 2-74 of Ref. 3 to give good agreement with actual stress-strain curves for two clays.

6 ORTHOGONAL MODES OF DEFORMATION

If two quantities are orthogonal to each other, *in principle* changes in one do not affect the other. Their vectors will be normal to each other. What third measure of strain would be orthogonal to those of Equations 3 and 4? Assume $\dot{\varepsilon}_x : \dot{\varepsilon}_y : \dot{\varepsilon}_z = A : B : -(A+B)$ to match Equation 4.
Equation 3 gives

$$-\tfrac{1}{3}A - \tfrac{1}{6}B + \tfrac{1}{6}(A+B) = 0$$

whence A = 0. One choice for Mode M3 could be

$$\dot{\varepsilon}_x : \dot{\varepsilon}_y : \dot{\varepsilon}_z = 0 : +\tfrac{1}{2} : -\tfrac{1}{2} . \tag{9}$$

7 MEASURES OF STRAIN-INCREMENT

To express with one quantity the strain in one mode of deformation, we define a quantity varying linearly with the component strains, with each coefficient proportional to the relative size of that component. Examples are

$$\dot{\varepsilon}_{c1} = \tfrac{1}{3}\dot{\varepsilon}_x - \tfrac{1}{6}\dot{\varepsilon}_y - \tfrac{1}{6}\dot{\varepsilon}_z \quad \text{(Mode M1)} \quad (10)$$

$$\dot{\varepsilon}_{c2} = \dot{\varepsilon}_v = \dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z \quad \text{(Mode M2)} \quad (11)$$

$$\dot{\varepsilon}_c = \tfrac{1}{2}\dot{\varepsilon}_y - \tfrac{1}{2}\dot{\varepsilon}_z \quad \text{(Mode M3)} \quad (12)$$

We could also have chosen stress measures first, then found admissible strain-increment measures.

8 ADMISSIBLE MEASURES OF STRESS

Assuming these exist, they will be correctly coupled with measures of plastic strain-increment by the following procedure:

(i) Set up the plastic work equation with known stresses and plastic strain-increments on one side, and the unknown ones on the other.

(ii) Compare terms in $\dot{\varepsilon}_x^p$ (if present) on both sides, to derive the first equation, and do the same for $\dot{\varepsilon}_y^p$, $\dot{\varepsilon}_z^p$, $\dot{\gamma}_{xz}^p$, etc.

(iii) Solve for the unknowns (called A, B, C).

For Modes M1 to M3, with stress measures given by Equations 10 to 12, the work equation is

$$\sigma_x'\dot{\varepsilon}_x^p + \sigma_y'\dot{\varepsilon}_y^p + \sigma_z'\dot{\varepsilon}_z^p = A\dot{\varepsilon}_{c1}^p + B\dot{\varepsilon}_{c2}^p + C\dot{\varepsilon}_{c3}^p \quad (13)$$

The three equations derived from Equation 13 give (with B as *excess* over p_u):

$$A = \sigma_{c1} = 2\sigma_x' - \sigma_y' - \sigma_z' \quad \text{(Mode M1)} \quad (14)$$

$$B = \sigma_{c2} = (\sigma_x' + \sigma_y' + \sigma_z')/3 \quad \text{(Mode M2)} \quad (15)$$

$$C = \sigma_{c3} + \sigma_y' - \sigma_z' \quad \text{(Mode M3)} \quad (16)$$

where σ_{c1} etc. are stress measures (orthogonal) which go with the orthogonal set of plastic strain-increment measures $\dot{\varepsilon}_{c1}^p$, etc.

9 PATHS OF STEEPEST ASCENT

During controlled strain-increments in one mode of deformation with stresses on the yield surface, if at least two other modes exist, how will their resistances decrease? The following postulate does not seem to have been tried in soil plasticity (or perhaps in any field).

We use *paths of steepest ascent* when finding stress paths over the yield surface, its equation being separable as Equation 1. Moving from $(s_1, s_2, \ldots s_n)$ to $(s_1+\Delta s_1, s_2+\Delta s_2, \ldots s_n+\Delta s_n)$, very close indeed, the Δs values obey Equation 17

$$\frac{\Delta s_1}{f_1'} = \frac{\Delta s_2}{f_2'} = \ldots \frac{\Delta s_r}{f_r'} . \quad (17)$$

where f_r' is the rth flow rate function. The mobilization ratios which can fall as they wish will permit the path of steepest ascent by changing *in direct proportion to components of the normal* to the yield surface. This makes no assumption about the orientation of principal stresses in relation to principal strain-increments.

10 FOLLOWING STRAIN PATHS AND STRESS PATHS

(a) Strain constants for shear strain measures

It is assumed that one mode of deformation is always inwards uniform straining (Mode M2), and all other modes are in shear alone. If ζ in Equation 8 has been found for one shear mode, what are the values for other shear modes in *isotropic* clay? Newmark (Ref. 2) suggested use of the octahedral shear strains given by Equation 18:

$$\tfrac{1}{2}\gamma_{oct} = \tfrac{1}{3}((\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2)^{\frac{1}{2}}. \quad (18)$$

where suffices refer to principal axes. Table II shows how octahedral strain-increments are rather similar for modes of shearing mentioned, with simple shear as Mode M4:

TABLE II

OCTAHEDRAL SHEAR STRAIN-INCREMENTS

Mode No.	Relative Sizes of Strain-Increments				$\dot{\gamma}_{oct}$
	$\dot{\varepsilon}_x$	$\dot{\varepsilon}_y$	$\dot{\varepsilon}_z$	$\dot{\gamma}_{xz}$	
M1	$+\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	0	0.4714
M3	0	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0.4914
M4	0	0	0	1	0.4914

(b) Yield surfaces in s space

It is sometimes convenient to work in a space of mobilization ratios, where $s_r = \sigma_{cr}/\sigma_{crmax}$, so that Equation 1 can be used directly. What should be chosen for plastic strain-increment measures? Normality applies with $f_r'(s_r) = \lambda.\dot{\varepsilon}_{cr}^p\sigma_{crmax}/p_e$, or a similar ratio unaffected by void ratio. (This point was not made in Ref. 1). We can then write for constant stress ratio

$$\dot{\varepsilon}_{cr} = \lambda.f'(s_r)p_e/\sigma_{crmax}. \quad (19)$$

(c) Prescribed strain-increments

From the given strain-increments, the plastic strain-increments are calculated in the measures of strain. Applying the normality rule from Equation 19, the largest possible set of *positive* plastic strain-increments is taken for a small step of K-consolidation at constant stress ratio. The excess (if any) is used to move round the (dimensionless) yield surface, which will reduce at least one cohesive resistance. A simple example shows a 'triaxial' test, where ε_2 is axial strain.

Example

	Shearing	Isotropic Straining
Mode No.	1	2
Stress measure	q	p
Strain measure	$\dot{\varepsilon} = \dot{\varepsilon}_a - \frac{1}{3}\dot{\varepsilon}_v$	$\dot{\varepsilon}_v$
Mobilization ratio	$s_1 = q/q_{max}$	$s_2 = \dfrac{p-p_u}{p_e-p_u}$
Flow rate function	$f_1'(s_1) = 2s_1$	$f_2'(s_2) = 1$
Yield function	$f_1(s_1) = s_1^2$	$f_2(s_2) = s$
Strain multiplier	$\zeta_1 = 0.006$	$\zeta_2 = 0.050$
Current state	$s_1 = 0.6$	$s_2 = 0.64$
Maxima, kN/m^2	$q_{max} = 162$	$p_e - p_u = 120$
Current values, kN/m^2		($p_e = 300$ ($p_u = 180$
Consolidation constant		$\frac{\lambda}{1+e} = 0.085$

The current stress measures are $q = q_{max} \times s_1$

$= 162 \times 0.6 = \underline{97.2 \text{ kN/m}^2}$; $p = p_u + (p_e - p_u) \times z_2 = 180 + (300-180) \times 0.64 = \underline{256.8 \text{ kN/m}^2}$.

We impose strain-increments $\Delta\varepsilon^p = 0.0010$, $\Delta\varepsilon^p_v = 0.0003$. What are the changes in q and p?

In K-consolidation, from Equation 19 we have $\dot{\varepsilon}^p/\dot{\varepsilon}^p_v = (f_1'(s_1)/f_2'(s_2) \times (p_e - p_u)/q_{max} = 0.889$. The largest multiples are 0.000266 (shear) and 0.00030 (volume change), leaving $\delta\varepsilon^p = 0.001000 - 0.000266 = 0.000734$ as excess shear strain to be used in moving round the yield surface. For a volumetric strain of 0.00030,

$$(\log_e(p+\delta p) - \log_e p) \frac{\lambda}{1+e} = 0.00030.$$

For small δp, using $\delta p/p = \delta q/q$ we have

$$\frac{\delta p}{p} \frac{\lambda}{1+e} = \frac{\delta q}{q} \frac{\lambda}{1+e} = 0.00030 \qquad (20)$$

whence $\delta p = 0.0003 \times 256.8 / 0.085 \ \underline{+ 0.91 \text{ kN/m}^2}$, and $\delta q = 0.0003 \times 97.2 / 0.085 = \underline{+0.34 \text{ kN/m}^2}$.

From Equation 8, for a small change in one mode,

$$\delta\varepsilon^p_r = \frac{\zeta_r f_r'(s_r)}{1 - s} \delta s_r \qquad (21)$$

As $f_1'(s_1) = 1.2$, $\zeta_1 = 0.006$ and $\delta\varepsilon^p = 0.000734$. In the 2nd stage $\delta s_1 = \frac{1 - 0.6}{0.006 \times 1.2} 0.000734 = 0.0408$, so $\delta q = 162 \times 0.0408 = \underline{+6.61 \text{ kN/m}^2}$. On the surface,

$$f_1'(s_1)\delta s_1 + \ldots f_n'(s_n)\delta s_n = 0 \qquad (22)$$

So $\delta s_2 = -\delta s_1 f_1'(s_1)/f_2'(s_2) = -0.0408 \times 1.2/1 = \underline{-0.049}$ and $\delta p = (p_e - p_u)\delta s_2 = 120(-0.049) = \underline{-5.88 \text{ kN/m}^2}$.

The totals are $\begin{cases} \delta q = +0.91 + 6.61 = +7.5 \text{ kN/m}^2, \\ \delta p = +0.34 - 5.88 = -5.5 \text{ kN/m}^2. \end{cases}$

(d) Prescribed stress increments

First come the changes proportional to present stresses, i.e. K-consolidation, then changes which increase some cohesive resistance. Equation 8 or 21 is used first for the strain-increments, then Equation 22 (and 17 if a steepest ascent path is followed) to find other changes.

11 AXI-SYMMETRIC TESTS RELATED TO PLANE STRAIN

(a) General

The shear mode of straining has to be closely reproduced by the assumed modes, and changed when necessary. K_o-consolidation uses axi-symmetric shearing, which can continue during 'triaxial' shearing to failure, with or without volume change. If plane strain shearing is used for compression, one shearing mode must be plane strain; the other orthogonal shearing mode has to be deliberately applied if there is any volume change.

(b) K_o-consolidation

Taking x as the sample axis, $\dot{\varepsilon}_y = \dot{\varepsilon}_z = 0$, and Mode 1 is axi-symmetric shear (defined earlier as Mode M1), Mode 2 is isotropic straining (Mode M2) and Mode 3 is an orthogonal shear mode (M3) which is not applied. For zero lateral strain, from Equations 10 and 11,

$$\dot{\varepsilon}_{c1} : \dot{\varepsilon}_{c2} : \dot{\varepsilon}_{c3} = 1 : 3 : 0 . \qquad (23)$$

Soil constants of the previous example in Equations 14-15 gives $\sigma_{c1max} = 2 \times 162 = 324$, $\sigma_{c2max} = 120 \text{ kN/m}^2$.

From Equation 19, the ratio of flow rate functions $f_1'(s_1)/f_2'(s_2) = (324 \times 1) / (120 \times 3) = 0.9$. With flow rate functions $2s_1$, 1, $2s_3$ and yield functions s_1^2, s_2, s_3^2, we find $s_1 = 0.45$, $s_3 = 0$. As the sum of the yield functions is always 1, $f_2(s_2) = 0.7975$, whence $s_2 = 0.7975$.

The stress measures are $\sigma_{c1} = 324 \times 0.45 = 145.8$, $\sigma_{c2} = 120 \times 0.798 = 95.7$ (above p_u), $\sigma_{c3} = 0$. At the void ratio giving $q_{max} = 162$ for *axi-symmetric* compression, $p_e = 300$, $p_u = 180 \text{ kN/m}^2$, during K_o consolidation $\sigma_x = 324.3$, $\sigma_y' = \sigma_z' = 251.4$, $q = 72.9$, $p = 275.7$, $K_o = 0.78$ (very high).

(c) Axi-symmetric shearing

After consolidation $e = 0.91$, with data mainly as the example in 10(c) but $\zeta_1 = 0.01$ and $\zeta_2 = 0.05$ based loosely on Fig. 7a of Ref. 3 and Fig. 2 of Ref. 4; latter shows London Clay follows Voce type of stress-strain curve very closely during swelling.

Undrained and drained 'triaxial' compression test simulations are shown in Fig. 4.

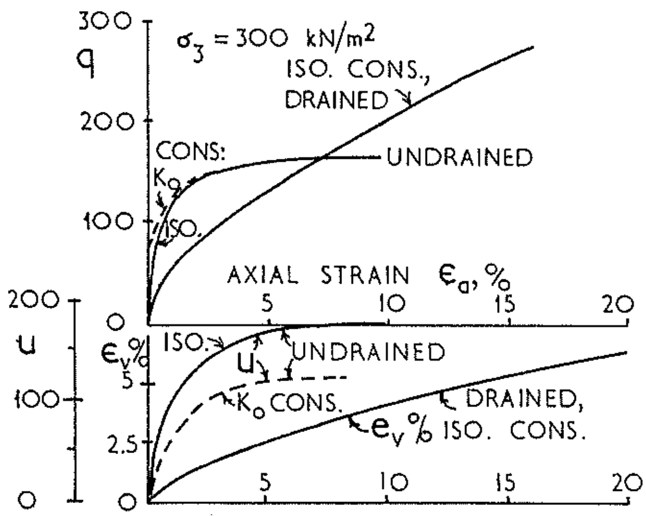

Fig. 4 Simulated 'triaxial' tests

(d) Plane strain tests

In shear, Modes 1 and 2 are Modes M5 & M6 below:

$$\dot{\varepsilon}_x : \dot{\varepsilon}_y : \dot{\varepsilon}_z = +\tfrac{1}{2} : 0 : -\tfrac{1}{2} \text{ (Mode M5)} \qquad (24)$$

$$\dot{\varepsilon}_x : \dot{\varepsilon}_y : \dot{\varepsilon}_z = +\tfrac{1}{6} : -\tfrac{1}{3} : +\tfrac{1}{6} \text{ (Mode M6)} \qquad (25)$$

Strain-increment measures, like Equation 10, are

$$\dot{\varepsilon}_{c5} = \tfrac{1}{2}\dot{\varepsilon}_x - \tfrac{1}{2}\dot{\varepsilon}_z \qquad \text{(Mode M5)} \qquad (26)$$

$$\dot{\varepsilon}_{c6} = -\tfrac{1}{6}\dot{\varepsilon}_x + \tfrac{1}{3}\dot{\varepsilon}_y - \tfrac{1}{6}\dot{\varepsilon}_z \qquad \text{(Mode M6)} \qquad (27)$$

Admissible stress measures, like Equation 14, are

$$\sigma_{c5} = \sigma_x' - \sigma_z' \qquad \text{(Mode M5)} \qquad (28)$$

$$\sigma_{c6} = -\sigma_x' + 2\sigma_y' - \sigma_z' \qquad \text{(Mode M6)} \qquad (29)$$

Their maxima are assumed to be $1.1547 \times 162 \approx 187.06$; $2 \times 162 = 324 \text{ kN/m}^2$.

Fig. 5 shows an undrained plane strain test after K_o consolidation as given in (b) above. In this case one might find the effective stresses at the end of K_o consolidation, and then to substitute the values directly into Equation 28 and 29. But they could equally well have been found by writing expressions for σ_{c5} and σ_{c6} in terms of the stress measures σ_{c1} and σ_{c2}, using symmetry.

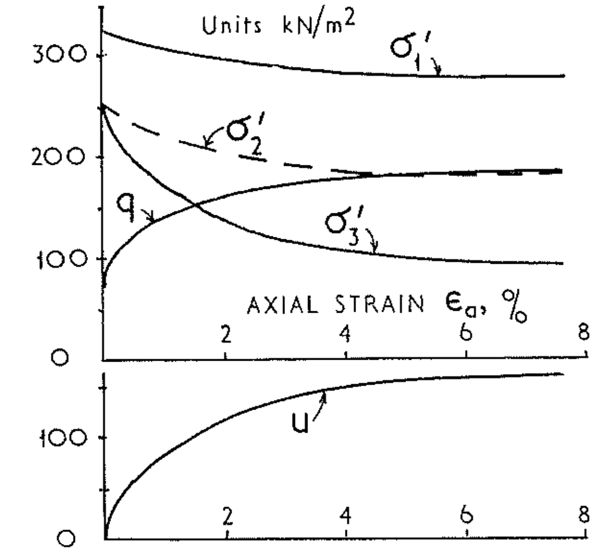

Fig. 5 Undrained plane strain test.

12 SIMPLE SHEAR TEST

As in K_o consolidation, $\dot{\varepsilon}_y = \dot{\varepsilon}_z = 0$. Mode 1 is axi-symmetric shear (M1), Mode 2 simple shear (M4), Mode 3 isotropic strain (M2). If $\dot{\varepsilon}_x/\dot{\gamma}_{xz} = f$, strain-increments are 2f : 1 : f/3, with measures of strain $\dot{\varepsilon}_{c1} : \dot{\varepsilon}_{c4} : \dot{\varepsilon}_{c2} = f/3 : 1 : f/3$. With constants as before, maxima of stress measures 162, 0.5x1.1547x162 = 93.53; 120 kN/m^2 (the latter is excess over p_u) and K_o consolidation, Fig. 6 shows an undrained test for constant σ_x.

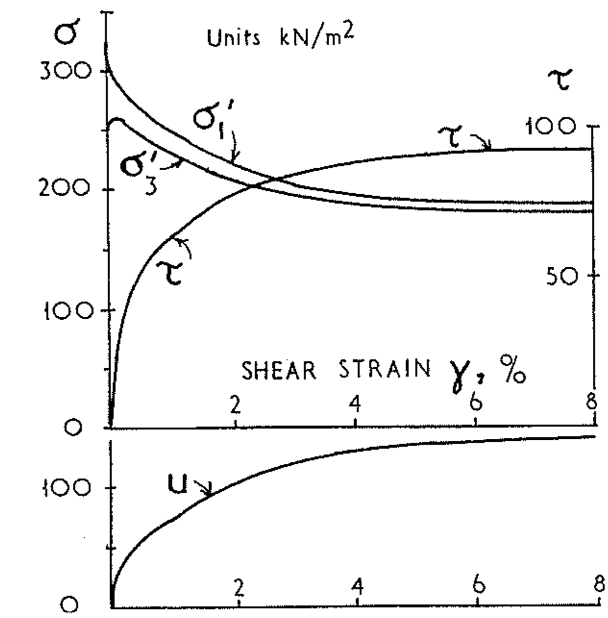

Fig. 6 Undrained simple shear test.

13 MULTIPLE STAGE TESTS

A simple technique is to raise cell pressure in a drained 'triaxial' compression test when the sample seems to be virtually at failure, and then again bring it almost to failure, perhaps repeating to get a third result. If one assumes that it is valid to draw Mohr circles from these 'failure' stress states, some c and ϕ values can be found.

The 'curve-hopping' technique of Schmertmann & Osterberg (Ref. 5) seems useful in clays with small strains to failure, but preliminary experiments do suggest curves would be unreliable in other clays. Obtaining virtually full stress-strain curves makes this technique particularly attractive.

14 CONCLUSIONS

An attempt has been made to develop a logical way of handling shear strains to compare various types of test. Using, say, octahedral shears to find maxima of different stress measures, all the 21 independent terms of a 6 x 6 incremental stiffness matrix could be found and used. Shear strain affects direct stress and vice versa, needing 9 terms which 'elastic' theory cannot give. One hopes new testing methods will give values for the 21 terms, at least crudely, throughout a test.

As an alternative to the finite element method, there seems to be a distinct possibility that an entirely different method could be developed for attaining compatible displacements and stresses in equilibrium. Since the soil moves according to the equilibrium equations, which only involve stress gradients, these can be related to strain gradients by the same 21 incremental stiffness terms mentioned above. At boundaries the fixed position is simple to handle, and so is a known displacement. Where the stress is controlled on the boundary, calculations of stress would begin; a least-squares analysis would give them.

As the elastic theory usually applied for finite elements cannot include dilatancy (which in this paper is implied to be an actual or potential contraction), this proposed 'point equilibrium' method would seem to permit any mass of soil or excavation in soil to be studied. It is hoped to follow up this possibility in the near future.

15 ACKNOWLEDGEMENTS

The Author wishes to thank Professor W.H. Wittrick, Head of the Department of Civil Engineering, for the research facilities provided.

16 REFERENCES

1. CHAPLIN, T.K. Separable Yield Surfaces in Clay. Stress-Strain Behaviour of Soils, ed. R.H.G. Parry, G.T. Foulis, Henley-on-Thames, 1972, pp. 234-240 & 264-269 (Disc.)

2. NEWMARK, N.M. Failure Hypotheses for Soils. Research Conference on Shear Strength of Cohesive Soils, Am. Soc. Civil Engrs., 1960 pp. 17-32.

3. BISHOP, A.W. and BJERRUM, L. The Relevance of the Triaxial Test to the Solution of Stability Problems. Research Conference on Shear Strength of Cohesive Soils. Amer. Soc. Civil Engrs., 1960, pp. 437-501. Also in Publication 43, Norw. Geotech. Inst.

4. HENKEL, D.J. The Effect of Overconsolidation on Clays During Shear. Géotechnique, Vol. 6, 1956, 139-150.

5. SCHMERTMANN, J.H. and OSTERBERG, J.O. An Experimental Study of the Development of Cohesive and Friction with Axial Strain in Saturated Cohesive Soils. Research Conference on Shear Strength of Cohesive Soils. Amer. Soc. Civil Engrs., 1960, pp. 643-694.