Some Case Histories of Computer Applications to Foundations

by

E. T. HAWS, M.A.
Director, Geocomp UK Ltd.

A. E. FURLEY, B.Sc.(Eng)
Geocomp UK Ltd.

R. DUNGAR, B.Sc., Ph.D.
Lecturer in Civil Engineering, University of Bristol, England

and

R. TABB, M.Sc.
Manager, Geocomp (Australia)

SUMMARY. This paper describes the application of computing techniques to the analysis of a deep anchored retaining wall, foundations for a very high multi-storey tower founded on piles, a series of large underground excavations and foundations for a mass concrete gravity dam. Three of the four examples involve the use of finite element techniques and the basis of this method is outlined with recent advances in modelling some complex geometries and loading conditions.

NOTATION.

B	Ratio of In-situ Vertical to Horizontal Stress
C	Plate Test Coefficient
D	Depth of Section
E	Young's Modulus
E_d	Young's Modulus in Drained Condition
E_u	Young's Modulus in Undrained Condition
E_H	Young's Modulus in Horizontal Direction
E_V	Young's Modulus in Vertical Direction
E_+, E_p	Moduli from Pressuremeter Test
F	Vector of Applied Forces
G	Shear Modulus
K	Stiffness Matrix for Complete System
K_o	At Rest Earth Pressure Coefficient
K_a	Active Earth Pressure Coefficient
K_p	Passive Earth Pressure Coefficient
k_h	Coefficient of Horizontal Subgrade Reaction
k_v	Coefficient of Vertical Subgrade Reaction
l_h	Constant of Horizontal Subgrade Reaction for Anchored Bulkhead with Free Earth Support
M	Mass Matrix for Complete System
M_s	Mass Matrix for Structure and Foundation
M_f	Mass Matrix for the Fluid
N	Ratio of E Values
n	Constant
r	Radius of Test Plate
t, u, v	Displacement Components for ith node
X	Displacement Vector
Y	Mode Shape Vector
Z	Depth
α	Constant
ϕ	Angle of Shearing Resistance
θ	Angular Co-ordinate about the Structural Axis
ν	Poisson's Ratio
ν_d	Poisson's Ratio (Drained)
ν_u	Poisson's Ratio (Undrained)
ν_H	Poisson's Ratio (Horizontal)
ν_V	Poisson's Ratio (Vertical)
ω	Natural Frequency of Vibration

1 INTRODUCTION

The use of computers in general and the finite element method in particular has recently gained much popularity in many branches of engineering including foundation analysis. Four examples of the application of computing techniques are described briefly in this paper. Three of the examples involve finite element techniques and the other used the structural stiffness approach. The basic theory of the finite element method is well documented and many specialised techniques are now available. Generally the matrix equation

$$K X = F \qquad (1)$$

is solved for simple static problems, where the stiffness matrix K is assembled from the stiffness properties of the subdivided system. This subdivision into a number of 'finite elements' enables various material properties, constitutive behaviour, geometric details, boundary conditions and changing stress contours to be represented conveniently. The displacements of the system X are calculated from equation (1), knowing the applied forces F, and a suitable solution routine is usually employed which takes advantage of the symmetric banded properties of the matrix K. In fact a Choleski decomposition method is used by the authors which takes advantage of these properties. A knowledge of the displacements, X, enables the strains and hence the stresses to be computed for each element.

2 CHOICE OF FINITE ELEMENT IDEALISATION

The correct choice of element idealisation is of utmost importance. To obtain a realistic solution to the posed problem the equivalent mathematical problem must be realistic. Perhaps the most simple example is the plane stress/plane strain triangle element with two degrees of freedom at each node. This element is capable of exhibiting only constant strain behaviour over the region which it is to represent. This is not a serious restriction if many elements are used for those areas where the strain field is changing rapidly.

A problem involving axisymmetric geometry should be represented by axisymmetric elements. The analysis of a tower foundation on this basis is described in one of the following sections. Special elements for dealing with axisymmetric loading such as self weight loading are used. The use of a full three dimensional element consumes a large amount of computer time and may not necessarily provide such an accurate solution, compared with other elements with appropriate assumptions. The application of non-axisymmetric loading (to model wind forces for example) on an axisymmetric tower foundation is also presented in the following text. Here, the displacement components are assumed to be dependent on the angular co-ordinate θ (see Fig. 1) such that

$$\begin{aligned} u_i &= \bar{u}_i \cos n\,\theta , \\ v_i &= \bar{v}_i \cos n\,\theta , \\ t_i &= \bar{t}_i \cos n\,\theta , \end{aligned} \qquad (2)$$

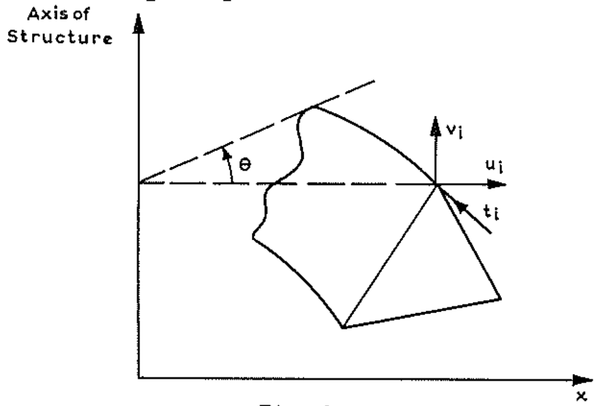

Fig. 1

In addition the forces applied to the tower are assumed to be a function of cos θ and sin θ, which enables a displacement solution for n = 1 to suffice. Again a special axisymmetric finite element is used to fully represent this "harmonic displacement and stress" situation. For the particular case of a foundation "reinforced" with piles, or a highly orthotropic material the relevant orthotropic properties may also be incorporated in this element.

3 NON-LINEAR CONSTITUTIVE BEHAVIOUR

The representation of non-linear material behaviour is of great importance to the foundation engineer and again the finite element method offers several possibilities. The two main methods involve either a modification to the matrix, K, of equation (1), or a modification to the applied force F. The modification of K, as a function of accrued strain is an expensive method because the equations must be resolved with a fresh X matrix for each stage of the solution. The modified force method is used by the authors because it enables the original matrix K and its Choleski decomposed form to be stored and any necessary changes to K are simulated by "modifying forces" which are added to the applied forces. An example of this technique giving a "no-tension" analysis of an underground cavern problem is presented in the following text.

The inclusion of elastic-plastic constitutive behaviour into the modified force method has also enabled a wide range of elasto-plastic problems to be covered. Various yield criteria are available such as the Von Mises and Extended Von Mises criterion, the latter representing fairly closely the Mohr-Coulomb yield criterion.

4 STAGE CONSTRUCTION ANALYSIS

The analysis of intermediate stages in the constructional sequence of structures, such as gravity and rockfilled dams, is often of importance to civil engineers. This analysis may be done by solving the matrix equation (1) for several stages of construction, the matrix K being formed for the structure and foundation as it exists at that stage. The loading, F, for each stage is taken as the load applied due to additional constructional material or due to the addition of other forces, such as hydrostatic or anchor forces. Again, non-linear material constitutive behaviour can be incorporated and if the material is assumed to be linear elastic during the course of any particular stage analysis, the results of strain and stress for the previous stage may be used to calculate the elastic properties for the current stage based on an input stress-strain relationship. The authors currently use a computer program which efficiently deals with this problem of stage raising.

5 DYNAMIC ANALYSIS

Dynamic displacements and stresses may be calculated for structures and foundations using finite elements. The response analysis of a large dam or an off shore oil platform both require the inclusion of the properties of the structure, the foundation and also the hydrodynamic forces of the reservoir water or sea. Here an efficient solution routine has been developed (Ref. 1). The equations of free vibration,

$$K\,Y = \omega^2 M\,Y, \qquad (3)$$

of the system must include the hydrodynamic "added mass matrix", M_f. If the structure foundation mass is M_s then $M = M_s + M_f$ and equation (3) may be rearranged as

$$K^{-1}(M_s + M_f)\,Y = \frac{1}{\omega^2}\,Y, \qquad (4)$$

where ω is the natural frequency and Y the corresponding mode shape. Reference 1 describes a method of solution of the above equations which incorporates an efficient use of the Choleski-decomposed banded symmetrical K matrix. The fluid "add-mass" M_f is not formed explicitly but is incorporated in the solution of equation (4), the fluid being represented also by a finite element idealisation. The resonant frequencies, mode shapes and stress vectors having been calculated from the lower modes of the system, the response to any given forcing system is calculated and the envelope of maximum response values stored and printed (Ref. 2 and 3).

6 BOUNDARY CONDITIONS

The mathematical model must be a realistic representation of the physical system. The analysis of an underground excavation is presented and a proportion of the surrounding rock mass is included in the analysis. The method chosen to represent this situation involved applying a known boundary stress condition in the form of equivalent boundary loading. To enable these stresses to develop, the boundary was fixed completely at the lower left hand corner and supported on rollers at the lower right hand corner. The enclosed volume of material was considered large enough when, with the excavations represented, the applied loading produced the required boundary stress and a uniform boundary displacement. That is, the effect of the excavation had negligible influence on the boundary deformation.

The importance of obtaining a correct representation for the boundary of both static and dynamic problems cannot be over estimated. The relevance of a normal mode analysis in dynamic problems, as outlined above, depends upon the relative stiffness of the structure and foundation. The extent of the required foundation representation must depend upon physical geotechnical characteristics, such as underlying strata and the internal damping of the soil or rock. In fact, a relatively small foundation mesh with a large stiff structure should be approached not from a normal mode concept but preferably by using a complex receptance method. The analysis of off-shore structures should be placed in this category.

7 CASE HISTORY NO. 1 - ANCHORED CONTIGUOUS BORED PILE RETAINING WALL

The construction of the foundations for the National Westminster Bank's new Head Office in London, U.K., necessitated that an excavation, through Thames Gravel into the underlying 'Blue' London Clay, be taken some 12 m below the foundations of adjacent 9 storey buildings. The heavy surcharge, together with restrictions on the allowable lateral deflection of the wall, required that a rigorous design procedure be followed. The adopted design method was to model the wall mathematically using a beam on elastic foundation approach. The program developed was able to

(i) Model a stage by stage excavation sequence and

(ii) Include springs above dredge level which both allowed pressure build up and pressure release together with springs which only allowed pressure build up. All springs below dredge level allowed for both pressure relief and build up.

Two methods of load application were initially investigated. The first was to start with K_0 pressures on the retaining wall and commence the excavation sequence. Lateral stress release would follow wall displacement towards the excavation. The second method was to load the wall with the pre-determined probable final active pressure forces and prevent load release from spring extensions. The first method had certain advantages since it offered a representation of the load release subsequent to excavation. However, even with a cut off for stress release at K_a conditions the method could lead to serious under estimation of intermediate lateral stresses. It was therefore decided to assume a constant state of lateral stress throughout wall deflection. A lateral earth pressure coefficient of 0.6 was selected for the London Clay, this value lying between K_0 and K_a conditions. For the overlying Thames Gravel the range between K_0 and K_a conditions is smaller and for conservatism the K_0 condition was chosen. These coefficients were used to calculate the three pressure distributions used in the analyses-triangular, trapezoidal and parabolic with account being taken of net lateral loading with deductions made for restraint on the passive face.

The values for the spring coefficients were calculated using the method proposed by Terzaghi (Ref. 4). The effects of a range of l_h values on the analyses were investigated for the dense Thames Gravel, from 7×10^3 kN/m^3 (Terzaghi Ref. 4) to 7×10^4 kN/m^3. k_h, the horizontal coefficient of subgrade reaction, is related to l_h by the equation

$$k_h = l_h \frac{Z}{D} \text{ where } \frac{Z}{D} \text{ is a depth ratio.} \qquad (5)$$

For the London Clay the selection of a suitable E/depth profile was required for the calculation of k_h. During the site investigation determination of insitu E values was undertaken using the Menard pressuremeter test where $E = \alpha E+$ and α for an overconsolidated clay is 1. There was some agreement between the pressuremeter E+ values and the back analysed E values published by Cole and Burland (Ref. 5). These values were used to prepare an upper bound E/depth profile. A lower bound E/depth profile was prepared using the pressuremeter E_p results. For completeness an intermediate profile was also selected for inclusion in the series of analyses. The relationship between E and k_v is

$$k_v = \frac{E}{C\ r} \qquad (6)$$

where C is a coefficient determined by experiment and r is the radius of the test plate (0.172 m) for equivalence to Terzaghi (4). The range of quoted C values lie between 1.18 to 1.425. A value of 1.2 was adopted for the wall analyses. The relationship between the vertical and horizontal coefficients of subgrade reaction was after Terzaghi (Ref. 4). The spring coefficients for the ground anchors were determined from consideration of the anchor free length and cable properties.

Using the trapezoidal pressure diagram a preliminary assessment of total anchor requirement was determined. These preliminary anchor loadings showed the probable requirement of 3 rows of anchors and this arrangement was used as input loading to the analysis.

During the preliminary analyses significant reductions in the lateral deflections was obtained by inserting a substantial wall penetration below the final dredge level. A final penetration of 5m was adopted for the deepest wall section. The analyses showed that the maximum bending moment was little affected by changes in the soil parameters (k_h) and the parabolic pressure distribution produced the maximum bending moment ordinate. In general the maximum anchor forces resulted from the trapezoidal distribution.

Ground anchor load cells, wall displacement tubes and extensometer lines normal to the wall were installed at the deep section of excavation (Ref.6). The recorded lateral movements have been somewhat lower than the predicted values, with analyses using the triangular pressure distribution best predicting the final deflected shape. The insertion of the ground anchors have generally held the wall at the installation position during subsequent excavation sequences. All measurements and relevant analyses will be presented elsewhere in due course.

8 CASE HISTORY NO. 2 - PILED FOUNDATIONS FOR A 190 METRE HIGH TOWER

Finite elements were used to model the short and long term behaviour of the piled foundations to a 190 metre high tower in London at present under construction. The building is founded on a twelve sided reinforced concrete raft supported by piles bored into the London Clay. The raft and pile foundations were modelled axisymmetrically. The modelling of the pile group posed an interesting problem. It was not possible to model adequately each individual pile beneath the circular raft and thus the concept of a "reinforced cylinder" of London Clay beneath the raft was used. The properties of the various materials used in the analysis are shown on Fig. 2. The properties of London Clay were determined by close examination of the results of pressuremeter tests done on the site itself and from a study of various plate-bearing tests done in the vicinity. The cylinder of London Clay effectively reinforced by bored piles was modelled as having anisotropic elastic properties, the vertical value of Young's Modulus being some 50 times greater than the average horizontal value depending to some extent upon the desnity of the piles. "Poisson's Ratios" in the two directions were related to each other in the manner required for compatibility with the ratio of horizontal to vertical Young's Moduli. e.g.

$$\left.\begin{aligned} \nu_V &= \tfrac{1}{2} \\ \nu_H &= 1 - \frac{E_H}{2\ E_V} \end{aligned}\right\} \text{for the axisymmetric undrained case} \qquad (7)$$

The Young's Moduli of London Clay and Woolwich and Reading Beds were varied between the short and long term conditions by using the relationship

$$E_d = \frac{(1 + \nu_d)}{(1 + \nu_u)} E_u \qquad (8)$$

Material	UNDRAINED				DRAINED			
	Young's Modulus MN/m^2		Poisson's ratio	Shear Modulus MN/m^2	Young's Modulus MN/m^2		Poisson's ratio	Shear Modulus MN/m^2
	Hor.	Vert.			Hor.	Vert.		
Core Wall	13000	13000	0·15		13000	13000	0·15	
Concrete Raft	19500	19500	0·15		19500	19500	0·15	
Piled Soil	See Diagram A	See Note	0·45	Drained Young's Modulus ÷ 2	See Diagram A	See Note	0·0	Drained Young's Modulus ÷ 2
London Clay	See Diagram A	See Diagram A	0·45		See Diagram A	See Diagram A	0·0	
Woolwich and Reading Beds	128	128	0·45		85·5	85·5	0·0	

Note: Vertical E value for piled soil = 2400-2700 MN/m^2

Fig. 2 Parameters used in Tower Analysis

It was assumed that Poisson's Ratio for these materials would vary from 0.5 in the short term to 0.0 in the long term.

The structual loads on the building were modelled as line loads acting at the top of the load bearing plinth walls and in addition the load due to the raft was modelled by the insertion of self weight into the analysis for the raft materials. Wind loadings and structural eccentric loadings on the building were modelled using variable vertical forces on the foundation at +10.9 m level. The vertical forces varied cosinusiodally around the perimeter of the ring wall with maximum positive and negative values in the direction of the applied moment. The associated shear force was also applied to the foundation at the appropriate level.

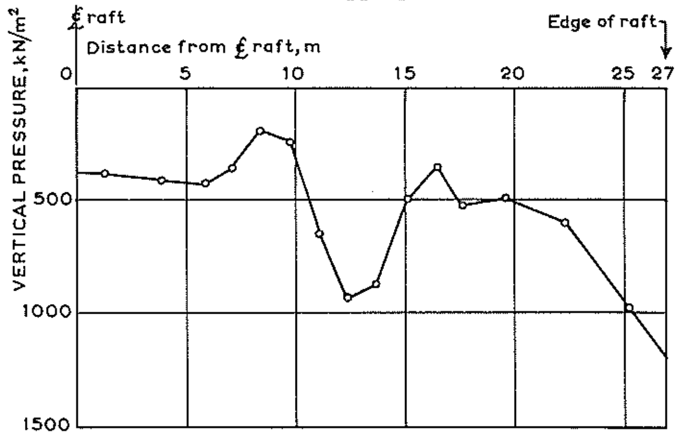

Fig. 3 Vertical Pressure on Raft/Piled Soil Interface

By means of the finite element analyses predictions have been made of horizontal movements, tilt, raft deformation, settlement and stress increases around adjacent underground railway tunnels as well as stresses within the raft under the various loading conditions. Analyses were done with various Young's Modulus values for London Clay and the reinforced concrete in the raft and these showed that the stresses were not sensitive to the E values for London Clay but were sensitive to changes in E values for the raft concrete. Settlements predicted by the finite element analysis were factored to allow for the effect of excavation for the raft. The pattern of vertical pressure between the raft and the "piled soil" is shown on Fig. 3. The pile group has been been designed, taking due account of load bearing reduction due to group factors, to take the whole of this load - in other words no allowance was made for any load transfer between the raft and the clay. The pile group under the raft comprises some 376 No. piles arranged in concentric rings at spacings from $1\frac{1}{2}$ pile diameters. A comprehensive set of instruments has been installed in the foundation the results from which will be published in due course.

9 CASE HISTORY NO. 3 - UNDERGROUND CAVERNS FOR A PUMPED STORAGE SCHEME

The pumped storage scheme at Dinorwic in North Wales will be one of the largest pumped storage schemes in Europe. The caverns are located some 100 metres under the ground surface in slate and at the time the preliminary analyses described herein were undertaken no measurements at depth had been taken of the in-situ ground pressures.

The preliminary analyses were done in order to assess the effect on the caverns and the surrounding rock mass of various ratios of in-situ vertical to horizontal stress. It was decided that since the preliminary analyses were exploratory in type a two dimensional plane strain type analysis was of sufficient accuracy although it was realised that a two dimensional model could only roughly approximate to the very complex system of underground excavations.

A literature search revealed that in similar analyses (Ref. 8, 9 and 10) done for other projects a diversity of boundary conditions and methods of obtaining in-situ stress fields had been chosen for finite element analyses. It was decided to test a set of boundary conditions and method of applying forces to the Dinorwic model in order to check that the in-situ stress field was actually obtained with out any caverns. The method used for obtaining the required in-situ stress field was to apply point loads on the "free" boundaries of a model but without any caverns. This trial was successful in that within a small distance from the corners of the mesh the stress field was accurately modelled and it was therefore decided to use the tested boundary conditions in the full analyses. The number of triangular elements in the mesh was approx. 1200. The nodes were numbered, and then optimised and checked using two separate computer programs. The parameters used for the rock (assumed isotropic) $E = 7 \times 10^4$ N/mm^2, $\nu = 0.10$. In addition the rock was not permitted to take any tensile stresses. The ratio's, B, of in-situ vertical to horizontal stress used varied from 2 to 0.25.

Clearly the preliminary two dimensional analysis does not truly model the underground situation since no allowance has been made for the effect of other chambers at right angles to those modelled and parallel to the model. If the situation between the extra chambers was modelled then at this position a plane stress condition with specified rock thickness surrounded by a plane strain condition would be one way to model the situation. On the line of the extra chambers a simple plane strain analysis with the extra chambers modelled as voids would give an idèa of stresses at this position. Various attempts (Ref. 10) have been made to model this particular set of circumstances by using a "reduced E" in the area of the extra chamber but in the authors view the results from such an analysis are not representative of any situation and as such are of little value. It is felt that the only way to obtain a true picture of stresses using finite elements would be to do a three dimensional analysis but at present this is very expensive in both computer time and data preparation.

10 CASE HISTORY NO. 4 - BUSALLETTA DAM, ITALY

Busalletta Dam is a mass concrete gravity dam some 60 metres in height at its deepest section. The dam is founded on schist which dips steeply downstream and has highly anisotropic properties parallel and at right angles to the bedding planes. A fault filled with softer weathered material lies under the dam. A number of sections of the dam were analysed using finite element techniques employing two-dimensional plane strain elements.

Predictions were made of the movements of the dam under various loading conditions such as impounding of reservoir, drawdown etc. using different assumed foundation parameters for each loading condition because of cyclic loading effects. The type of analysis used was elastic and thus the prediction of dam movements mentioned above during construction, through impounding, drawdown and reimpounding was build up by superposition.

The ratio of E values parallel to and perpendicular to the bedding planes of the foundation rock varied from five to one. This ratio influences both the shear modulus G and the Poisson's Ratios in the two directions. If we assume that E_1 is Young's Modulus parallel to the bedding plane, E_2 is Young's Modulus normal to the bedding plane, ν_1 is Poisson's Ratio in the plane of isotropy, ν_2 is Poisson's Ratio in the direction normal to the plane of isotropy due to a stress in the isotropic plane, ν_3 is Poisson's Ratio in the plane of isotropy due to a stress normal to this plane, then it can be shown that ν_2 varies between 0.7 and 0 for $\nu_1 = 0.3$. Using the relationship

$$\frac{\nu_2}{E_1} = \frac{\nu_3}{E_2} \tag{9}$$

and assuming realistic values of ν_2 of 0.45 and 0.1 we obtain

$$\nu_3 = \frac{0.45}{N} \quad \text{or} \quad \frac{0.1}{N} \quad \text{where } N = E_1/E_2.$$

Sensitivity analyses were carried out using both these values of ν_2 and it was found that there was little difference in the results. It was decided therefore to use $\nu_3 = \frac{0.45}{N}$. The value of shear modulus G is also related to E_1 and N and based on a series of N values the following relationship was used

$$G = \frac{E_1}{(0.0106N^2 + 0.956N + 1.0756)} \tag{10}$$

Pore pressures in the foundation were predicted using a finite difference computer program which has the facility for incorporating different permeabilities in two directions at right angles.

Instrumentation has been installed in the dam which is at present under construction and comparisons will be made between measured and predicted movements and it is hoped to publish results in due course.

11 CONCLUSIONS

A brief outline of the application of computing methods to the analysis of four civil engineering applications is presented. These analyses show that a very wide variety of geometries, loading conditions and material properties can be modelled successfully on the computer. By doing sensitivity analyses using a range of values for chosen parameters, the engineer can obtain a greater understanding of the influence of certain factors on prototype performance.

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