Probability of Failure and Expected Volume of Failure in High Rock Slopes

by

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SUMMARY. Long high rock slopes such as those obtained in large open pit mines are excavated in a series of lifts. Slope fialures can occur by sliding along fractures, or combinations of fractures, at any stage in the slopes' history. Procedures for calculating the probability of "operational failures", defined as failures above a certain size, and the expected volumes of such failures are presented. Procedures are also given for calculating the percentage of the area of a major slope affected by operational failures of berms reduced to a width less than that required by operational practice or government regulation. In projects where significant risk of failure can be accepted, the optimum slope is the one where the present value of the costs of initial excavation plus the expected costs resulting from failures are a minimum.

1 INTRODUCTION

High rock slopes such as those in open pit mines are normally excavated over a long period of time in a series of lifts, usually of the order of 10 to 15m high as shown in Figure 1. Berms are usually left after each lift but, where rock conditions permit, the slope can be steepened by leaving berms only after each second or third lift.

Slope failures may take place at any stage of the excavation. Failures of a single berm are serious only if they reduce the width of the berm to an unacceptable extent over a major part of the slope. A slide which involves several berms is obviously more serious than one which involves only a part of a single berm.

Rock slopes differ from slopes in soils and some very weak rocks in that failures are largely caused by sliding along inclined fractures or combinations of fractures. The exact locations and orientations of such weak fractures are rarely known prior to excavation of the slopes; although the statistical variation in their orientations and mechanical properties can usually be deduced. (References 1 and 2).

It follows that when uncertainty exists regarding the orientation, shear strength and length of the controlling fractures, uncertainty must also exist regarding the stability of any designed slope. A useful measure of this uncertainty is the *Probability of Failure*.

Economic design of rock slopes also requires that the timing and volume of material involved in the potential landslides be estimated. In projects where significant risk of failure can be accepted, the optimum slope is the one where the *present value* of the costs of initial excavation plus the *expected* costs resulting from slope failures are a minimum.

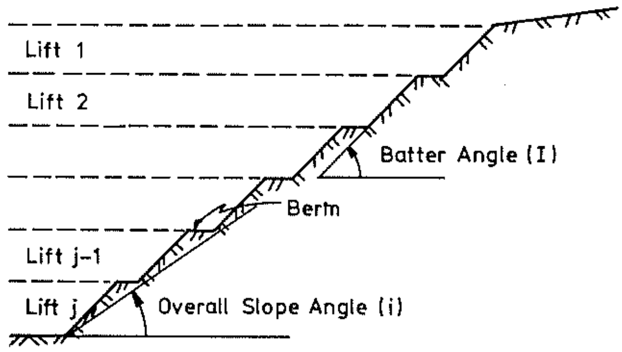

Fig. 1 A high rock slope excavated in a series of lifts.

2 NOTATION

α	Correction angle for loading conditions
β	Fracture dip (measured from horizontal)
β_c	Critical dip
c	Apparent cohesion
C	An expenditure in the Nth year.
C_{ij}	Cost of initial excavation of the jth lift at an overall slope angle i
C_{oj}	Component of the costs of repair of failures during the jth lift that is independent of volume of failure
C_F	Unit cost of re-excavation, anchorage etc. of failed section of the slope where these costs are proportional to the volume of failed material
EV_F	Expected total volume of failure over the whole life of the mine
EV_n^*	Expected volume of failure during excavation of the nth lift allowing for previous failures.

EV_j Expected volume of failure during jth lift assuming no previous failures

e A counter

g Acceleration of gravity

I Batter slope angle (measured from horizontal)

i Overall slope angle (measured from horizontal)

K Horizontal earthquake acceleration

K_s Correction factor for lateral restraint

k A counter

L_j Maximum length of fractures exposed by the jth lift.

L_r Fracture length required to allow failure by the specified mode during excavation of the jth lift.

m A counter

n A counter

N! Factorial of N

P Probability

P_o = 1, by definition

PC Present Cost

PC_{ij} Total present cost of expenditures during the jth lift on a slope of overall angle i

PC_i Total present cost of expenditures over all of n lifts on a slope of overall angle i

P_F Probability of one or more failures occurring during the life of the slope

P_j Probability that a failure occurs by the specified mode during excavation of the jth lift.

P_k Probability of failure through the kth slice

P_n^1 Probability that at least one failure of any size occurs during excavation of a total of n lifts

P_n^m Probability that at least one failure of size sufficient to involve at least m lifts occurs during the excavation of a total of n lifts.

$\prod_{j=1}^{n}$ The product of the following expression as j varies from 1 to n

$\sum_{j=1}^{n}$ The sum of the following expression as j varies from 1 to n

q A counter

r A counter or rate of interest depending on context.

σ_a Average Normal Stress

σ Normal stress

SF Safety Factor

ϕ Apparent Friction Angle

ϕ_T Tangential Angle of Friction

τ_a Average Shear stress at failure under normal stress σ_a

θ_A, θ_B Apparent dips of planes forming a wedge measured in the plane normal to the line of intersection of the wedge.

θ_k Dip (measured from the horizontal) of the base of the kth slice.

U Resultant groundwater force

W Weight of block or wedge.

V_k Volume of the kth slice.

V_j^* Volume which fails during excavation of the jth lift with probability P_j.

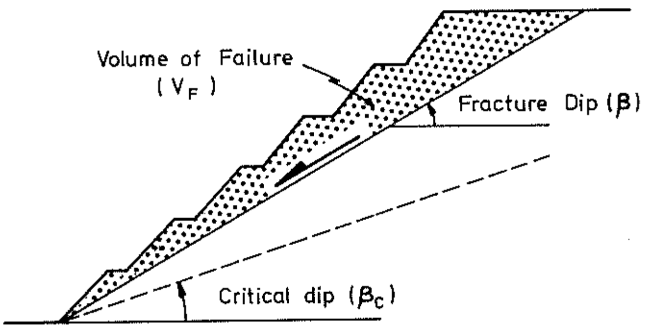

Fig. 2 Slope Failure by sliding on a single fracture.

3 CONDITIONS FOR SLOPE FAILURE

In a previous paper (Reference 2) several modes of rock slope failure were suggested. For simplicity, only one of these modes is considered here (Figure 2) although the design method can be extended to include the other modes. Also, for simplicity, the problem is considered in two-dimensions although extension to three-dimensions is possible by using the Schmidt method for analysis of fracture orientations (Reference 1).

As proposed in reference 2, a failure of the type shown in figure 2 requires that three conditions be satisfied:

(i) The average dip (β) of the fracture is steeper than the critical dip (β_c) as described in the Appendix
The probability that this condition is satisfied is represented by:

$$P_{(\beta_c < \beta)}$$

(ii) The average dip (β) of the fracture is less than the overall slope angle (i). The probability that this condition is satisfied is represented by:

$$P_{(\beta<i)}$$

(iii) The fracture length (L_j) is greater than the length required (L_R^j) to extend from the toe of the slope to the ground-surface.
The probability that this condition is satisfied is represented by:

$$P_{(L_j > L_R)}$$

4 PROBABILITY OF FAILURE

Slope failure occurs when the three above conditions are satisfied. Providing these conditions are independent of each other, the probability (P_j) that a failure occurs during excavation of the jth lift is given by:

$$P_j = P_{(\beta_c<\beta)} \cdot P_{(\beta<i)} \cdot P_{(L_j>L_r)} \qquad (1)$$

It follows that the probability that no failure occurs during the jth lift is:

$$(1 - P_j)$$

and that the probability that no failure occurs during the excavation of n lifts is:

$$\prod_{j=1}^{n}(1 - P_j)$$

The probability that at least one failure occurs during the excavation of n lifts is therefore:

$$P_F = 1 - \prod_{j=1}^{n}(1 - P_j) \qquad (2)$$

and furthermore, the probability that at least one failure involving more than m berms occurs during the excavation of n lifts is given by:

$$P_n^m = 1 - \prod_{j=m}^{n}(1 - P_j) \qquad (3)$$

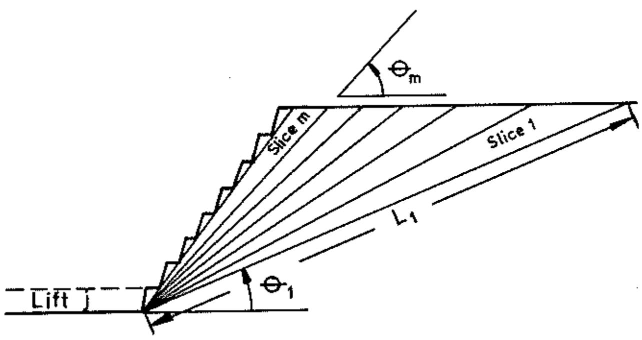

Fig. 3 Slope behind the jth lift considered as a series of m slices.

Since the fracture length required to allow failure is dependant on the dip of the fracture as shown in Figure 3, estimation of the probability of failure during the jth lift (P_j) can be refined by considering the slope behind the jth lift to be divided into a series of M slices, such that the base of the kth slice has a dip θ_k.

The probability (P_k) that a failure occurs within the kth slice is then:

$$P_k = P_{(\beta_c<\theta_k)} \cdot P_{(\theta_k<\beta<\theta_{k+1})} \cdot P_{(L_k>L_r)} \qquad (4)$$

By analogy with equation 2, the probability of at least one failure occurring during exposure of the jth lift is:

$$P_j = 1 - \prod_{k=1}^{M}(1 - P_k) \qquad (5)$$

5 RELATIONSHIP BETWEEN PROBABILITY OF FAILURE AND PERCENTAGE OF SLOPE FAILING

The probability of failure (P_F), given by equation 3, represents the best estimate (i.e. the mean probability) that any single section of a long rock slope will fail. If a long statistically homogeneous slope is considered to consist of N equal portions, then from the properties of the wellknown Binomial Distribution the probability that r portions (i.e. $\frac{100r}{N}\%$ of the slope length) will fail is given by the rth term of the binomial expansion of $(P_F + (1 - P_F))^N$, thus:

$$P(r) = C_r^N (P_F)^r \ (1-P_F)^{N-r} \qquad (6)$$

where $P(r)$ = the probability that $\frac{100r}{N}$ percent of the slope will fail

C_r^N = the number of combinations of N things taken r at a time

$$= \frac{N!}{r!(N-r)!}$$

It also follows from the properties of the Binomial Distribution that the mean number of portions failing is $N.P_F$, so that the most likely percentage of the slope to fail is $100P_F$ percent. Thus the probability of failure expressed as a percentage is equivalent to the best estimate of the percentage of the slope likely to fail.

6 SINGLE BATTER FAILURES

The usual practice of leaving berms results in sections of the slope being considerably steeper than the average overall slope as shown in Figure 4. Thus the probability of single batter failures is often much higher than the probability of failure of the overall slope.

Consider that the rock below the berm can be divided into m slices, as shown in Figure 4, such that the base of the bottom slice is parallel to the overall slope angle i and the top of the uppermost slice is parallel to the batter slope angle I.

Then, using relationships developed in equation 2, the probability of failure of the berm is given by:

$$P_F = 1 - \prod_{k=1}^{m}(1 - P_k) \qquad (7)$$

where $$P_k = P_{(\theta_k<\beta<\theta_{k+1})} \cdot P_{(\theta_k>\beta_c)} \qquad (8)$$

and, as explained in the previous section, this represents the best estimate of the proportion of the total batter length likely to fail.

Similarly, by considering only the uppermost slices, the percentage of the total batter length over which any proportion of the berm width will be lost can be calculated.

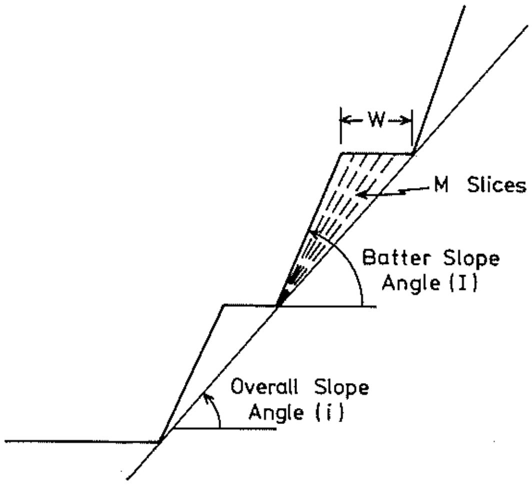

Fig. 4 Slope failures restricted to a single berm.

7 EXPECTED VOLUME OF FAILURE

Following the normal practice of optimal design methods the expected volume of failure (EV_F) is defined as:

$$EV_F = P_F V_F \qquad (9)$$

where P_F is the probability of failure and

V_F is the volume of material which would be involved in this failure should it occur.

The problem of estimating the expected volume of failure in high rock slopes is complicated by the fact that they do not suddenly appear, but are incrementally excavated by a series of lifts and the volumes involved in the failures of one lift cannot be considered independently of the failures resulting from the previous lifts.

However, omitting for the moment consideration of any previous failures, it is clear from Figure 3 that during excavation of the jth lift, a failure could take place through one or several of the slices shown. Because, however, the volume of material involved in any failure through an upper slice will also be included in the volume of failure on a lower slice, it is only necessary to consider the volume above the lowermost failure plane in the system.

During the jth lift the lowermost failure plane occurs:

(a) within the 1st Slice with probability P_1 and associated volume

$$\sum_{k=1}^{m} V_k$$

(b) within the 2nd Slice with probability $P_2\,(1-P_1)$ and associated volume

$$\sum_{k=2}^{m} V_k$$

(c) within the mth slice with probability

$$P_m \prod_{q=1}^{m-1}(1-P_q)$$

and associated volume V_m.

The expected volume of failure during the jth lift is therefore:

$$EV_j = \sum_{k=1}^{m}\left(P_k \prod_{q=1}^{k-1}(1-P_q) \sum_{e=k}^{m} V_e\right) \qquad (10)$$

At this stage, it is also possible to define a volume (V_j^*) which is the volume which fails during excavation of the jth lift with probability (P_j) as:

$$V_j^* = (EV_j - EV_{j-1})/P_j \qquad (11)$$

where P_j is the value computed from equation 5.

From Appendix 2, the expected volume of failure (EV_n^*) during excavation of the nth lift (taking into account all previous failures) is:

$$EV_n^* = P_n P_{n-1} V_n^* + \sum_{q=1}^{n-1}\left(\sum_{j=q}^{n} V_j^*\right) P_n P_{q-1} \prod_{k=q}^{n-1}(1-P_k)) \qquad (12)$$

and the total expected volume of failure throughout the life of the mine (n lifts)

$$EV_F = \sum_{j=1}^{n} EV_j^* \qquad (13)$$

This expression assumes that after each failure the slope is cleaned up or repaired and further excavation is on the original design angle (i). In practice, after a major failure, the design will be re-evaluated in the light of the additional data then available and the slope angle would possibly be changed. However, for design purposes, where it is necessary to place an economic value on a certain slope angle this assumption appears to be valid.

8 ECONOMIC ANALYSIS

From compound interest formulae the present value (PC) of an expenditure (C) in the Nth year at rate of interest (r) is given by:

$$PC = C/(1 + r)^N \qquad (14)$$

If a high slope with slope angle (i) is excavated at the rate of (m) lifts per year the jth lift will be excavated after (k = j/m) years. The present value of the costs PC_{ij} incurred during excavation of the jth lift will be:

$$PC_{ij} = (C_{ij} + P_j C_{oj} + EV_{ij}^* . C_F)/(1 + r)^k \qquad (15)$$

where EV_{ij}^* is the expected volume of failure occurring during the jth lift with a design overall slope angle (i).

The total present cost of a slope of angle (i) extended over (n) lifts is therefore:

$$PC_i = \sum_{j=1}^{n} PC_{ij} \qquad (16)$$

The economically optimum slope is the angle i for which the total present costs PC_i are a minimum.

The total present value can then be plotted against slope angle and the optimum design slope is the minimum value, providing the associated probability of failure is acceptable.

9 ACCEPTABILITY OF RISK OF FAILURE

Use of Probability of Failure as a design parameter has been criticised by several authors who favour Safety Factor, on the grounds that any design risk of failure is unacceptable in an engineered structure. However, experience shows that quoting a safety factor does not prevent failure, particularly if the analysis does not take into account uncertainty in fracture orientations and other important variables.

If a probabilistic analysis shows that there is a finite but unacceptable risk of failure for a required slope the solution is not to change the method of analysis to a more comforting one but either to remove the sources of uncertainty by investing in additional investigation or to abandon the project.

In many projects, particularly open pit mines, economic limitations prevent the type of investigation that would be appropriate to, say, an arch dam abutment. By requiring a less expensive and therefore less thorough investigation the owners implicitly accept a greater risk of failure.

Under these conditions it would seem prudent to clearly recognise, and at least try to estimate, the risk of failure, rather than claim an apparent margin of safety which may not eventuate.

10 CONCLUSIONS

This paper provides a procedure for estimating probability of failure, expected volume of failure and optimum slope angle for high rock slopes excavated over a long period of time.

The method is directly applicable only to slopes which fail on a single plane and must be extended if other modes of failure are possible.

Although the equations appear complex, they can be easily handled on a computer or programmable calculator and, for most projects, the time spent on analysis will be quite appropriate to the time spent on field investigations and the value of the project.

Any analysis made by these methods should be subjected to regular updating as more refined data becomes available during the progress of the excavation.

11 REFERENCES

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12 APPENDIX - NOTES ON THE CRITICAL DIP

The Critical Dip (β_c) is the dip at which the overlying block is at critical equilibrium under the loads applied. It is given by (Reference 2):

$$\tan \beta_c = K_s \tan (\phi-\alpha) \qquad (15)$$

The apparent friction angle (ϕ) is defined as the angle whose tangent is the ratio of the average shearing stress at failure (τa) to the average normal stress (σa), for sliding under dry conditions, that is:

$$\tan \phi = \frac{\tau a}{\sigma a} \qquad (16)$$

Sample values for this parameter may be obtained by direct shear tests on representative rock fractures carried out at the same average normal stress that is expected in the slope. Where this is not possible, conservative estimates can be obtained by testing at normal stresses higher than those expected in the slope. When the true failure envelope is curved as shown in Figure 5, the value of apparent friction angle so obtained is always conservative and is equivalent to the combination of the three widely used parameters *apparent cohesion, tangent angle of friction* and *safety factor* based on the tangent to the failure envelope, that is:

$$\tan \phi = \frac{c + \sigma \tan \phi_T}{SF.\sigma} \qquad (17)$$

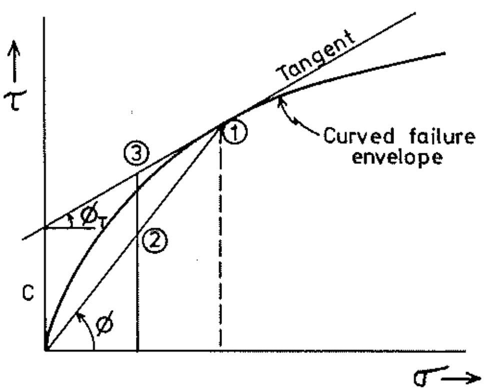

Fig. 5 Relationship between shear strengths calculated using the apparent angle of friction and strengths calculated using tangent angle of friction and apparent cohesion, where failure envelope is curved. At the average normal stress level, indicated by point (1) both methods indicate the same shear strength. At lower normal stresses the shear strength indicated by the apparent angle of frict- (point (2)) is lower than the shear strength indicated by the tangent (point 3).

The angle (α) is a correction angle for loading conditions such as groundwater forces, horizontal earthquake loading, anchor forces etc. (Reference 2). In particular, for slopes affected by groundwater forces:

$$\sin \alpha = \frac{U \sin \phi}{W} \tag{18}$$

and, for dry slopes affected by earthquakes:

$$\tan \alpha = {}^{K}/_{g} \tag{19}$$

The shape factor (Ks) corrects for the effects of lateral confinement, or lack thereof, due to the curvature of the slope or the shape of the block (or wedge). In the case of wedge shaped failures the critical dip is the angle of inclination of the line of intersection of the fractures forming the wedge. Where both fractures forming the wedge have the same angle of friction, Ks is given (Reference 3) by:

$$Ks = \frac{\sin\theta_A + \sin\theta_B}{\sin(\theta_A + \theta_B)} \tag{20}$$

In the case of block failures in straight slopes Ks equals one by definition.

An indication of the possible magnitude of Ks for concave slopes can be inferred from data provided by Piteau and Jennings (Reference 4) who refer to minimum slopes developed over periods of more than 75 years in the "Big Holes" of the Kimberley Area, South Africa. If it is assumed that these minimum slopes now approach the critical dip, the values for Ks can be calculated as shown in Table 1. It can be seen that a value of Ks equals 1.3 would be reasonable and probably conservative for a convex slope with radius of curvature (R) equal to 200 feet.

TABLE I

Comparison of Slope angles and radii of curvature
Data from Piteau and Jennings

Mine	Minimum Slope angles: Angle A R=200 ft degrees	Angle B R=1000 ft degrees	Estimated Ks
Kimberley	28.5	22.5	1.31
Kutoitspan	23.0	18.0	1.31
Bultfontein	34.5	21.0	1.79
Wesselton	36.0	29.0	1.31

Note: Estimated Ks = tan angle A/tan angle B.

Estimation of Ks for convex slopes can also only be made empirically at the present time. Experience in many open pit mines, indicates that slopes which are marginally stable at slopes of the order of 45^0 when straight must be excavated at angles 5^0 to 10^0 flatter when convex, suggesting that values of Ks between 0.7 and 0.85 would be reasonable, although not necessarily conservative, on present knowledge.

13 APPENDIX 2 - DERIVATION OF FORMULAE FOR EXPECTED VOLUME OF FAILURE.

Consider a slope excavated in n lifts divided into a series of (j) slices as shown in Figure 6 such that the volume (V_j^*) of each slice has a probability of failure (P_j) during excavation of the jth lift, and for mathematical convenience let $P_o = 1$, then:

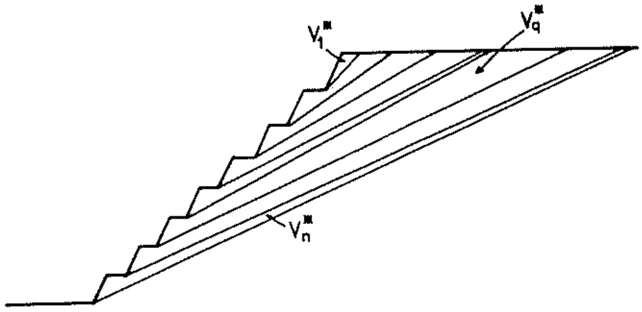

Fig. 6 Slope excavated in n lifts considered as a series of slices.

(a) *After excavation of the first lift:*

Volume V_1^* fails with probability P_1

No failure occurs with probability $1 - P_1$

(b) *After excavation of the second lift:*

Volume $V_2^* + V_1^*$ fails with probability $P_2(1-P_1)$

" V_2^* " " " $P_2 P_1$

No failure occurs with probability $(1 - P_2)$

(c) *After excavation of the third lift:*

Volume $V_3^*+V_2^*+V_1^*$ fails with probability $P_3(1-P_2)(1-P_1)$

" $V_3^*+V_2^*$ " " " $P_3(1-P_2)\,P_1$

" V_3^* " " " $P_3 P_2$

No failure occurs with probability $1 - P_3$

(d) *After excavation of the nth lift:*

Volume $V_n^*+V_{n-1}^*\ldots V_1^*$ fails with probability $P_n(1-P_{n-1})\ldots(1-P_1)$

Volume $V_n^*+V_{n-1}^*\ldots V_2^*$ fails with probability $P_n(1-P_{n-1})\ldots(1-P_2)P_1$

Volume $\sum_{j=q}^{n} V_j$ fails with probability $P_n P_{q-1} \prod_{j=q}^{n-1}(1-P_j)$

Volume $V_n^* + V_{n-1}^*$ fails with probability $P_n(1-P_{n-1})P_{n-2}$

Volume V_n^* fails with probability $P_n P_{n-1}$

No failure occurs with probability $1-P_n$.

The *expected volume of failure* EV_n^* during excavation of the nth lift is therefore:

$$EV_n^* = P_n P_{n-1} V_n^* + \sum_{q=1}^{n-1} \left(\sum_{j=q}^{n} V_j^* \right) P_n P_{q-1} \prod_{k=q}^{n-1} (1-P_k))$$