Fifteen years of geotechnical limit state design in Australia
Part II – Foundations

Patrick K. Wong
Coffey Geotechnics Pty Ltd, Sydney, Australia
Robert A. Day
Coffey Geotechnics Pty Ltd, Melbourne, Australia
Harry G. Poulos
Coffey Geotechnics Pty Ltd, Sydney, Australia

Keywords: Limit State Design, Footings, Pile Groups

ABSTRACT

Part I of this paper provided the background to limit state design and some of the problems and confusions associated with its use in geotechnical engineering, particularly for retaining structures. Despite these difficulties, there is definitely a place for limit state design in geotechnical engineering.

In this paper (Part II), the differences between Working Stress Design and Limit State Design (LSD) for foundation engineering are briefly described, together with a description of the factored resistance and factored strength approaches in limit state design. The benefits of limit state design in foundation engineering are illustrated through a number of examples involving a footing on stiff clay, a footing on medium strength rock, and a pile group.

The authors conclude that the use of limit state design in Geotechnical Engineering has benefits as long as the underlying design principles and soil-structure interaction effects are properly understood and communicated.

1 BACKGROUND

In engineering design, it is essential that a structure has a low probability of collapse under the worst load combination. It is equally important that deflections are within tolerable limits under normal operating conditions. Engineers use factors of safety in design to account for uncertainties and approximations that have to be made on material properties, loads and methods of design analysis.

Broadly speaking, factors of safety are applied in the following two methods of design:

Working Stress Design (lumped factor of safety approach):
The ultimate capacity, R_{ug}, is calculated without load and material factors, and an overall factor of safety, FOS, is applied to assess the allowable load R_a, and then compared to the Working Load (usually defined as the sum of the dead and live load without factoring up), such that:

$$P_w \leq R_a \tag{1}$$
$$R_a = R_{ug}/FOS \tag{2}$$

The Working Stress method is still widely used today; for example, a FOS of 3 is commonly used for design of foundations and a factor of between 1.25 and 1.5 is commonly used for design of slopes and dams.

Why then is there such a wide range of FOS, with a significantly higher FOS being used in building foundations compared to slopes? It turns out that the FOS of 3 used in foundations is not really a factor of safety on the strength of the foundation, but is a factor to limit the settlement based on experience of most soils for which relatively stiff and linear behaviour will persist if the stress levels are kept below about 1/3 their ultimate capacities (Atkinson, 2007). Understanding this principle is important as it will later explain how more cost-effective designs can be achieved when the Limit State Design approach is used for stiffer geotechnical elements.

Limit State Design (partial factors of safety approach):

The development and principles associated with limit state design have been discussed in Part I of this paper. In brief, the Limit State Design approach in Geotechnical Engineering comprises two parts:

Part 1 – Strength Limit State

The Strength Limit State requires that the following equation be satisfied under ultimate loading conditions:

$$R_{ug}^* \geq S^* \quad (3)$$

where: R_{ug}^* = design geotechnical strength
S^* = design action effect under all ultimate load combinations

However, there are several ways in which the values R_{ug}^* and S^* may be evaluated as described below:

(a) Factored Resistance Approach (North American approach, but also used in Australia)
The ultimate capacity, R_{ug}, is calculated without reduction of geotechnical parameters, and the calculated geotechnical resistance is factored down to the design ultimate, R_{ug}^*, so that:

$$R_{ug(1)}^* = R_{ug(1)} \times \Phi_g \quad (4)$$

$R_{ug(1)}$ = Ultimate capacity based on unfactored geotechnical parameters (5)

(b) Factored Strength Approach (European approach, but also used in Australia)
The design ultimate capacity, R_{ug}^*, is calculated with partial reduction factors applied to the geotechnical parameters so that:

$$R_{ug(2)}^* = R_{ug(2)} \times 1.0 \quad (6)$$

$R_{ug(2)}$ = Ultimate capacity based on factored geotechnical strength parameters (7)

In this paper, subscripts have been used for R_{ug} and R_{ug}^* to distinguish the difference between the different evaluation methods because they do not always produce the same results (e.g. where different partial factors are used in pile shaft friction and end bearing pressure in Method (b)). Further confusion is caused by the fact that the design action effect, S^* may also differ dependent on the way in which R_{ug}^* is calculated. For example, S^* is equal to the factored up ultimate load on a footing or an isolated pile foundation, and remains unchanged so long as S^* is less than R_{ug}^* regardless of which way R_{ug}^* is evaluated. However, this is not the case when S^* is, for example, the design action effect on a corner pile of a pile group under combined axial, lateral and moment loading (see Section 3) or the resulting bending moment on a retaining wall (see Section 4).

Part 2 – Serviceability Limit State

The Serviceability Limit State requires that under the serviceability loading conditions, the resulting deflection does not exceed the tolerable limit. The tolerable limit may be for the purpose of meeting operational, durability, or aesthetic requirements as specified by the owner or designer.

Serviceability loads are generally a combination of unfactored dead load plus a reduced component of the live load as specified in the Australian Loading Code AS1170.1 (1989). Because of the possible reduction in the live load component, the design serviceability load is <u>not</u> necessarily the same as the working load used in traditional working stress design method.

There is general agreement with designers that material parameters should not be reduced when the serviceability limit state is being considered. However, Pells et al (1998) recommended that the design elastic modulus for Sydney Sandstone and Shale be reduced by a factor of 0.75. The authors of this paper are of the opinion that this reduction factor on elastic modulus is not necessary; as long as a cautious approach has been adopted when selecting the elastic modulus value based on test results or experience, and that any stress level dependency has been properly considered (see Section 2.2 for example).

2 SHALLOW FOUNDATION

2.1 Footing on Stiff Clay Foundation

For this example, we will consider the following parameters

(a) Dead load, DL = 600kN
(b) Live load, LL = 200kN
(c) Partial load factor on DL, $\psi_D = 1.2$
(d) Partial load factor on LL, $\psi_L = 1.5$
(e) Live load reduction factor for serviceability assessment = 0.4
(f) Undrained shear strength of clay foundation, S_u = 100kPa, with an estimated bulk unit weight, γ, of 20kN/m^3
(g) Young's modulus of clay foundation, E_s = 15MPa
(h) Geotechnical strength reduction factor, $\Phi_g = 0.65$
(i) Square footing buried 1m below finished surface
(j) Groundwater level is more than 1m below finished surface
(k) Find required footing size so that long-term settlement ≤ 20mm

2.1.1 Traditional Working Stress Method

For this approach, the working load, P_w would be considered to be 600 + 200 = 800kN
The allowable bearing pressure, p_a , and the required footing width, B, would be calculated using a traditional FOS of 3, so that:

$p_a = (6 \times S_u + 1.0 \times \gamma)/3.0 = 207\text{kPa}$ (8)
$B = \surd(800/207) = 1.97\text{m}$ (9)

2.1.2 Limit State Design Approach

For this approach, the Strength Limit and Serviceability Limit States would be calculated as follows:

(a) Strength Limit State
$S^* = 1.2 \times 600 + 1.5 \times 200 = 1020\text{kN}$ (10)
$R_{ug} = (6 \times S_u + 1.0 \times \gamma) \times B^2 = 620B^2$ kN (11)
$R_{ug}^* = 0.65 \times R_{ug} = 403B^2$ kN (12)
$B = \surd(1020/403) = 1.59\text{m}$ (13)

(b) Serviceability Limit State
$S_{sls} = 1.0 \times 600 + 0.4 \times 200 = 680\text{kN}$ (14)
Settlement $= 0.9 \times S_{sls}/(B \times E_s) \leq 0.02\text{m}$ (15)
$B = 0.9 \times 680 \; / \; (15{,}000 \times 0.02) = 2.04\text{m}$ (governing case) (16)

2.1.3 Discussions

It can be seen that for the example of a shallow footing on a stiff clay foundation, both working stress and limit state design approaches result in a similar footing size of 2m. More importantly, it can bee seen from Section 2.1.2 that the footing size is governed by the deflection criterion, and not the strength of the foundation.

2.2 Footing on Medium Strength Sandstone

For this example, we will consider a pad footing founded on Class III Sandstone (Pells et al, 1998) with the following parameters

(a) Dead load, DL = 9000kN
(b) Live load, LL = 3000kN
(c) Partial load factor on DL, $\psi_D = 1.2$
(d) Partial load factor on LL, $\psi_L = 1.5$
(e) Live load reduction factor for serviceability assessment = 0.4
(f) Allowable bearing pressure = 6000kPa
(g) Ultimate bearing pressure = 35MPa
(h) Young's modulus of rock mass, E_R = 1000MPa

(i) Geotechnical strength reduction factor, Φ_g = 0.75
(j) Find required footing size so that settlement ≤ 10mm

2.2.1 Traditional Working Stress Method

For this approach, the working load, P_w would be considered to be 9,000 + 3,000 = 12,000kN
The required footing size, with the expectation that settlement would be less than 1% of the footing width (Pells et al, 1998) is:

$$B = \sqrt{(12{,}000/6{,}000)} = 1.41m \quad (17)$$

2.2.2 Limit State Design Approach

For this approach, the Strength Limit and Serviceability Limit States would be calculated as follows:

(a) Strength Limit State

S^* = 1.2 x 9,000 + 1.5 x 3,000 = 15,300kN (18)

R_{ug} = 35,000 x B^2 kN (19)

R_{ug}^* = 0.75 x R_{ug} = 26,250 x B^2 kN (20)

B = √(15,300/26,250) = 0.763m (21)

(b) Serviceability Limit State

S_{sls} = 1.0 x 9,000 + 0.4 x 3,000 = 10,200kN (22)

Settlement = 0.9 x S_{sls} /(B x E_R) ≤ 0.02m (note: no reduction in E_R adopted) (23)

B = 0.9 x 10,200 / (1,000,000 x 0.01) = 0.92m (governing case) (24)

Serviceability bearing pressure = 10,200 / B^2 = 12,104kPa (25)

2.2.3 Discussions

It can be seen that for the example of a pad footing on a medium strength sandstone foundation, deflection is also the governing case but this time the footing size assessed using the lmit state design method is significantly smaller than that assessed using the working stress method (i.e. 0.92m compared with 1.41m). The serviceability bearing pressure of about 12MPa assessed for the smaller footing is only slightly more than 1/3 of the ultimate bearing pressure of the rock, and unlike soils, yielding of competent rock is not expected to occur until much closer to the ultimate bearing pressure. Therefore, the solution found using the limit state method is considered to be acceptable and results in a more cost-effective design.

3 PILE FOUNDATION

For an isolated single pile foundation, the use of a limit state design approach in recent years has enabled designers and contractors in Australia to achieve more cost-effective solutions in a similar way as that discussed in Section 2.2, particularly for continuous auger piles or bored piles socketed in rock. Ultimate end bearing pressures of greater than 30MPa are nowadays frequently used in design of piles socketed into low to medium strength shale and sandstone, and with settlements of less than 15mm.

On the other hand, there is still some confusion, and opinions are divided, amongst geotechnical, structural engineers, and government authorities regarding the appropriate use of limit state design approach for pile groups under combined axial, lateral, and moment loading. This is a common problem for bridge pier foundations and is illustrated using the following example:

(a) Group of 3 x 3 driven precast 400mm square piles at 1.5m grid spacing driven through 15m of stiff clay and socketed 1m into weathered rock and terminating in low strength rock
(b) The soil and rock properties as summarised in Table 1.
(c) Ultimate axial load = 22.5MN
(d) Ultimate lateral load = 1.0MN
(e) Ultimate moment = 9MNm
(f) Geotechnical strength reduction factor, Φ_g = 0.7 (assuming sufficient dynamic load tests with signal matching would be carried out to assess R_{ug})

Table 1 – Soil and Rock Properties Used in Pile Group Example (Driven Piles)

Depth (m)	Material	Elastic Modulus (MPa) Axial	Elastic Modulus (MPa) Lateral	Ultimate shaft friction f_s (kPa)	Ultimate end bearing pressure f_b (kPa)	Ultimate lateral yield pressure f_y (kPa)
0 to 15	Stiff clay	20	14	50	-	200 to 900$^{(1)}$
15 to 16	Very low strength rock	100	70	200	-	3,000
16 to 20	Low strength rock	1,500	-	-	20,000	-
>20	Medium strength rock	2,500	-	-	-	-

(1) f_y varies linearly from 200kPa at top of pile to 900kPa at 2m below top of pile

Analysis of the above pile group example has been carried out using the computer program DEFPIG developed by the University of Sydney (Poulos, 1980). Two sets of analysis were carried out – (Case 1) without and (Case 2) with the application of geotechnical strength reduction factors to the shaft friction, end bearing pressure, and the lateral yield pressure. The results of the pile group analyses are presented in Table 2.

Table 2 – Pile Group Analysis Results

	Case 1 Without Strength Reduction	**Case 2 With Strength Reduction**
Single Pile Capacity	R_{ug} = 4.72MN	R_{ug}* =3.3MN
Maximum pile axial load	3.72MN	3.3MN
Pile group centre settlement	7mm	5mm
Lateral deflection	21mm	26mm
Pile group rotation	0.0017 rad.	0.003 rad.
Maximum pile bending moment	0.133MNm	0.118MNm
Maximum pile shear force	0.188MN	0.188MN

The analysis results are discussed below:

(a) Under the ultimate load case analysed, the pile group does not fail, and the computed deflections are within acceptable limits for both Cases 1 and 2. Deflections for the serviceability load case will be expected to be less, and it can be concluded that the Serviceability Limit State is therefore also satisfied. It should be pointed out, however, that if the serviceability analysis is required, only Case 1 needs to be performed.

(b) From Case 1, the maximum pile axial load occurs on the corner piles and is assessed to be 3.72MN. Some government authorities (e.g. Road and Traffic Authority of NSW for design of bridge foundations) would have required this value to be treated as the maximum design action effect S*, and would stipulate that a minimum R_{ug} value of 3.72MN/0.7 = 5.31MN is required to be demonstrated during dynamic testing. This interpretation is flawed and would cause the following problems:

- The theoretical R_{ug} is only 4.72MN, and would mean that the design would need to be modified by either the addition of more piles, or the use of larger size piles.
- If it is required to test the piles to achieve a higher capacity of 5.31MN, the piles will be exposed to a greater risk of damage due to excessive driving energy for the nominated pile size.

(c) On the other hand, it can be seen from Case 2 that even when the pile capacities are reduced using the nominated geotechnical strength reduction factor, Φ_g, the pile group (i.e. foundation system as a whole) has not reached collapse and is still performing adequately. This is how the Australian Piling Code (AS2159 – 1995) was originally intended.

(d) The maximum design ultimate bending moment on the piles computed from Case 1 is also higher than that computed from Case 2, and this is an important point to note with respect to structural design of the piles.

(e) In the authors' opinion, the correct design approach should be as follows:

- For geotechnical design, the application of a geotechnical reduction factor on the strength parameters may be used to assess the pile group's overall performance for the Ultimate Limit State condition, and S* may be taken from Case 2, with testing on single piles to demonstrate that Φ_g x $R_{ug} \geq S^*$ (i.e. ≥ 3.3MN for axial loading in this example).
- For structural design, S* should be taken from Case 1, and the piles should be designed with an ultimate structural strength, R_{us}, such that Φ_s x $R_{us} \geq S^*$ (i.e. ≥ 3.72MN for axial loading in this example) where Φ_s is the structural strength reduction factor. It is also advisable that the maximum design action effects on other quantities (e.g. bending moment and shear) from the higher of Cases 1 and 2 be adopted for structural design purposes.
- Effectively, the above method is a combined use of factored strength and factored resistance methods in Limit State Design that can result in more cost-effective designs.
- Where necessary, conduct serviceability analysis using parameters without the application of geotechnical strength reduction factors and the appropriate serviceability loading.

Although the method recommended in (e) above seems a little more complicated, and would need to have two different design values of S* that need to be specified in design and construction drawings for different purposes (i.e. a higher value for determining structural capacity of the pile material and a lower value for assessment of the ultimate geotechnical capacity in pile testing), the authors are of the opinion that it is essential for Geotechnical Engineers to communicate the underlying principles involved and to have the ability to produce more cost-effective designs for the industry.

4 CONCLUSIONS

From the examples given in this paper, the authors conclude that the use of Limit State Design in Geotechnical Engineering has the following benefits as long as the underlying design principles and soil-structure interaction effects are properly understood and communicated:

(a) Partial factors can be varied to take account the different levels of uncertainties in design.

(b) Strength and deflections criteria are clearly separated and if nothing else, should make the designer think more carefully about the meaning of factors of safety (i.e. are they for strength or deflection control?).

(c) Provide the ability to optimise the design of foundations based on satisfying both strength and serviceability criteria rather than lumping these together as used in the traditional working stress approach.

(d) For pile groups under combined axial, lateral and moment loading, the combined use of factored strength and factored resistance methods can be used effectively to optimise the design and testing of piles.

REFERENCES

Atkinson, J.H. (2007) *The Mechanics of Soils and Foundations,* 2nd Edition, Taylor and Francis (pub).

AS1170.1-1989 *SAA Loading Code - Dead and live loads,* Standards Australia International Ltd

AS2159 – 1978 *SAA Piling Code*, Standards Association of Australia

AS2159 – 1995 *Piling - Design and Installation*, Standards Australia International Ltd

Pells, P.J.N. Mostyn, G. and Walker, B.F. (1998) *Foundations on Sandstone and Shale in the Sydney Region,* Jnl and News of the Australian Geomechanics Society, Vol. 33, Part 3, 17-29.

Poulos, H.G. (1980) *User Guide to DEFPIG – Deformation Analysis of Pile Groups,* School of Civil Engineering, University of Sydney.