The mechanics of internal erosion and piping of embankment dams and their foundations

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ABSTRACT

Internal erosion and piping is the main cause of failure and incidents in embankment dams and their foundations. The failure process is best considered in four stages:

- Initiation of erosion which may occur within concentrated leaks such as cracks caused by differential settlement, desiccation or hydraulic fracture; backward erosion and suffusion.
- Continuation or whether filters will be effective in controlling the erosion.
- Progression. For erosion to progress the developing pipe must stay open, and flow limitation by upstream zones must be ineffective.
- Breach which may occur by gross enlargement and collapse of the top of the pipe, instability of the downstream slope, or settlement of the crest causing overtopping.

Over a number of years researchers at the University of New South Wales (UNSW) working with the author have developed an improved understanding of the physical processes involved in the internal erosion and piping process, particularly in regards to initiation of erosion and filter action. The paper provides an overview of the internal erosion and piping process, with a detailed discussion on the mechanics of initiation and progression of erosion.

1 INTRODUCTION

Internal erosion and piping is the main cause of failure and incidents in embankment dams and their foundations Research on the subject at UNSW began in 1996 with collection of data on dam failures and incidents and analysis of this data to develop an empirical method for assessing the likelihood of failure of embankment dams by internal erosion and piping (Foster et al, 2000a, b). A framework for considering the internal erosion process in a way which models the stages in the process was also developed (Foster, 1999, Foster and Fell, 1999). This uses expert judgement to assess the conditional probabilities within an event tree structure to assess the annual probability of failure.

The method was gradually improved by use in dam safety assessments throughout Australia, and as research was carried out to better understand the physical processes. Fell et al (2004) and Fell and Wan (2005) presented the most up-to date methods at that time. In parallel to the Australian development, the US Bureau of Reclamation (USBR) were using the method and feeding back their experiences to UNSW for use in the research process.

During 2006 and 2007 UNSW, URS, USBR and the US Army Corps of Engineers (USACE) have collaborated to prepare "A Unified Method for Estimating Probabilities of Failure of Embankment Dams by Internal Erosion and Piping". This is currently in draft form for Beta trialling within those organisations and will be available as URS; USBR, and USACE publications and through UNSW as Fell et al (2007b).

Over this time our understanding of the physical processes and mechanics of the internal erosion process have greatly improved, mainly as a result of research by a number of PhD and ME students, and post Doctoral fellows working under the direction of the author and colleagues at UNSW. The emphasis of research at UNSW has been on initiation and progression of internal erosion by erosion in cracks in cohesive soils and suffusion in cohesionless soils. We have also conducted a review of the backward erosion process in cohesionless soils, and related this to case data. Research was also carried out on the continuation stage with investigations of filters which do not meet filter design criteria (Foster 1999, Foster and Fell 2001).
This paper presents the results of this research and experience, with the emphasis on the mechanics of initiation and progression of internal erosion.

2 THE INTERNAL EROSION AND PIPING PROCESS

2.1 Terminology

The following definitions were agreed to at an International Workshop on Internal Erosion and Piping of Dams held in Aussois, France in 2005 (Fell and Fry 2007).

Internal erosion. Occurs when soil particles within an embankment dam or its foundation, are carried downstream by seepage flow. Internal erosion can initiate by concentrated leak erosion, backward erosion, suffusion and soil contact erosion.

Piping. Piping is the form of internal erosion which initiates by backward erosion, or erosion in a crack or high permeability zone, and results in the formation of a continuous tunnel called a 'pipe' between the upstream and the downstream side of the embankment or its foundation.

Backward erosion. Backward erosion involves the detachment of soils particles when the seepage exits to a free unfiltered surface, such as the ground surface downstream of a soil foundation or the downstream face of a homogeneous embankment or a coarse rockfill zone immediately downstream from the fine grained core. The detached particles are carried away by the seepage flow and the process gradually works its way towards the upstream side of the embankment or its foundation until a continuous pipe is formed (Figures 3A and C).

Concentrated leak erosion. Erosion in a concentrated leak may occur in a crack in an embankment or its foundation, caused by differential settlement, desiccation, freezing and thawing, and by hydraulic fracture; or it may occur in a continuous permeable zone containing coarse and/or poorly compacted materials which form an interconnecting voids system (Figure 3B). The concentration of flow causes erosion (sometimes called scour) of the walls of the crack or interconnected voids.

Suffusion and internal instability. Suffusion is a form of internal erosion which involves selective erosion of fine particles from the matrix of coarser particles (coarse particles are not floating in the fine particles). The fine particles are removed through the constrictions between the larger particles by seepage flow, leaving behind an intact soil skeleton formed by the coarser particles. Soils which are susceptible to suffusion are internally unstable. Coarse graded and gap graded soils, such as those shown schematically in Figure 1 are susceptible to suffusion. In these soils the volume of fines is less than the volume of voids between the coarse particles.

Self-filtering. In soils which self-filter, the coarse particles prevent the internal erosion of the medium particles, which in turn prevent erosion of the fine particles. Soils which potentially will not self-filter include those which are susceptible to suffusion (internally unstable), and very broadly graded soils such as those which fall into the grading envelope shown in Figure 2. (Sherard 1979). The soils had particle size distributions which plotted nearly as a straight line, were of glacial origin, and the dams from which the soils were taken had all exhibited signs of internal erosion. The soils have a volume of fine particles greater than the volume of voids between the coarse sand and gravel fraction and the coarser particles are "floating" in the finer particles. The figure is not meant to define the boundary of such soils, only examples.

Continuation. Continuation is the phase where the relationship of the particle size distribution between the base (core) material and the filter controls whether or not erosion will continue. Foster and Fell (1999, 2001) and Foster (1999) define four levels of severity of continuation from 'no erosion' to 'continuing erosion.

Progression. Progression is the third phase of internal erosion, where hydraulic shear stresses within the eroding soil may or may not lead to the enlargement of the pipe. Increases of pore pressure and seepage occur. The main issues are the likelihood of and rate of pipe enlargement and whether the pipe will collapse, whether upstream zones may control the erosion process by flow limitation and whether a pipe will extend through the low permeability zones of the embankment.

Breach. Breach is the final phase of internal erosion. It may occur by one of the following four phenomena (listed below in order of their observed frequency of occurrence). The breach phenomena are also shown graphically in Figure 3.

- Gross enlargement of the pipe (which may include the development of a sinkhole from the pipe to the crest of the embankment).
- Slope instability of the downstream slope.
- Unravelling of the downstream face.
- Overtopping (e.g. due to settlement of the crest from suffusion and/or due to the formation of a sinkhole from a pipe in the embankment).

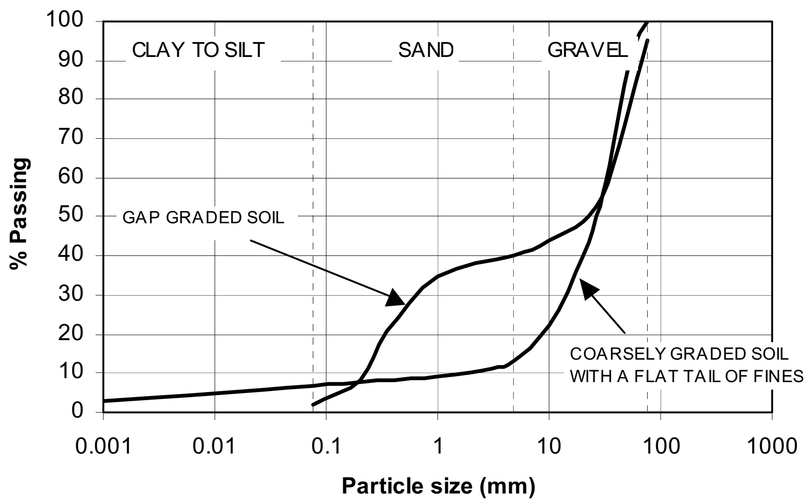


Figure 1 Soils gradation types which are susceptible to suffusion or internal instability

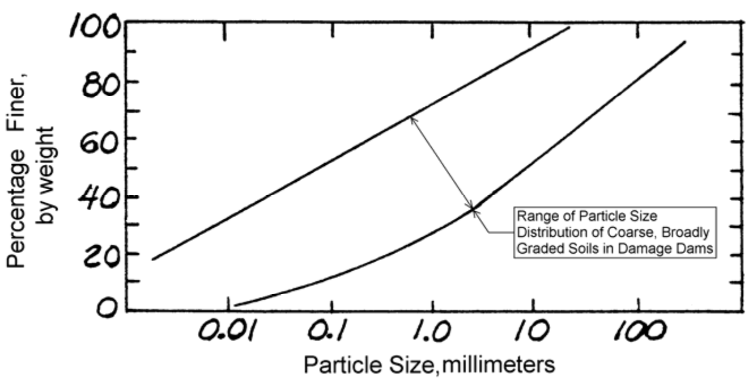
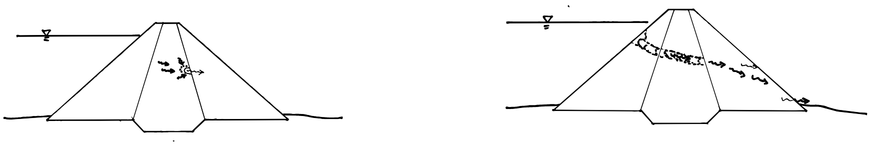


Figure 2 Grading envelopes of some broadly graded soils which did not self filter (Sherard, 1979)

2.2 The internal erosion and piping process

It is useful to consider the process of internal erosion and piping into four phases – initiation of erosion and continuation of erosion, progression to form a pipe, and formation of a breach. This is shown in Figure 3(A) and (B) for piping through the embankment initiated by a concentrated leak. Similar processes apply for piping through the foundation, and from the embankment to the foundation and are shown in Figure 3(C) and (D). Further details are given in Fell et al (2007a) and Fell and Fry (2007).



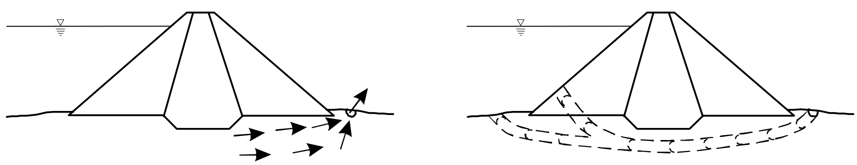
INITIATION → CONTINUATION → PROGRESSION → BREACH/FAILURE

INITIATION	CONTINUATION	PROGRESSION	BREACH/FAILURE
Leakage exits on d/s side of core and backward erosion initiates	Continuation of erosion	Backward erosion progresses back to the reservoir	Breach mechanism forms

(A) PIPING IN THE EMBANKMENT INITIATED BY BACKWARD EROSION

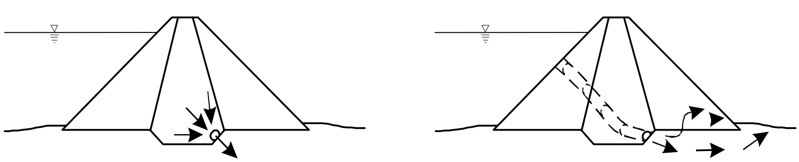
INITIATION	CONTINUATION	PROGRESSION	BREACH/FAILURE
Concentrated leak forms and erosion initiates along walls of crack	Continuation of erosion	Enlargement of concentrated leak	Breach mechanism forms

(B) PIPING IN THE EMBANKMENT INITIATED BY EROSION IN A CONCENTRATED LEAK



INITIATION	CONTINUATION	PROGRESSION	BREACH/FAILURE
Leakage exits from the foundation and backward erosion initiations	Continuation of erosion	Backward erosion in progresses to form a pipe	Breach mechanism forms

(C) PIPING IN THE FOUNDATION INITIATED BY BACKWARD EROSION



INITIATION	CONTINUATION	PROGRESSION	BREACH/FAILURE
Leakage exits the core into the foundation and backward erosion initiates as core erodes into the foundation	Continuation of erosion	Backward erosion progresses to form a pipe. Eroded soil is transported in the foundation	Breach mechanism forms

(D) PIPING FROM THE EMBANKMENT TO FOUNDATION INITIATED BY BACKWARD EROSION

Figure 3 - Models for the development of failure by piping (Foster and Fell 1999)

3 MECHANICS OF EROSION IN CONCENTRATED LEAKS

3.1 Overall process

Whether erosion will initiate in concentrated leaks in a dam or its foundation depends on:

- Whether there is a crack (or flaw) below the reservoir level formed by differential settlement during or after construction, hydraulic fracture, desiccation, or collapse of a poorly compacted layer of soil in the embankment or around a conduit sited to pass through the embankment.
- Given there is a crack, whether the forces imposed on the sides of the crack by the water flowing through the crack are sufficient to initiate erosion.

Whether in the absence of filters or where filters are too coarse, the erosion will progress to form a pipe depends on:

- Whether the water flowing through the crack will cause the soil on the sides of the crack to swell, closing the crack, or reducing the width so the forces imposed on the sides of the crack by the water flowing through the crack are insufficient to initiate erosion.
- Whether the soil will hold open the pipe (“will it hold a roof”).
- Whether upstream zones will limit the erosion process.

Also important is the rate of erosion and enlargement of the pipe because this influences whether the leak can be detected and intervention taken to lower the reservoir or stop the erosion process before breach occurs.

3.2 Assessing the likelihood of cracking or low stress zones subject to hydraulic fracture

3.2.1 Cracking and hydraulic fracture due to cross valley differential settlement

As an embankment dam is constructed the partially saturated compacted soil in the embankment consolidates and settlement occurs. Where the valley sides are steep and/or has steps in the profile, such as shown in Figure 4, differential settlements occur due to the variations in the height of the embankment, and these can lead to tensile or low stress zones in which cracks may form. This has been recognised for some time by Sherard (1973, 1985, 1986), Høeg et al (1998) and others. Hydraulic fracture through these low stressed zones as the dam is filled or under flood conditions is an associated phenomenon as discussed by Sherard et al (1972), and Sherard (1985,1986). For most dams, about 80% to 90% of the total settlement occurs during construction (Hunter, 2003, Hunter and Fell, 2003), so the stresses set up in the construction phase largely control the likelihood of low stress zones and cracking.

Bui et al (2004, 2005) carried out extensive numerical modelling to determine the conditions which are conducive to cracking and low stress zones due to cross valley differential settlements. The modelling was validated by carrying out numerical modelling of cases where cracking had been

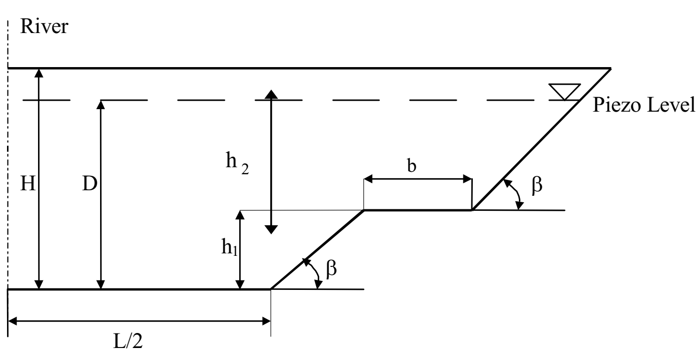


Figure 4 Definition of terms used to describe cross valley geometry (Bui et al 2004, 2005)

observed or internal erosion and piping had occurred. Based on an analysis of internal erosion and piping incidents Fell et al (2007b) assess the historic incidence of cracking, hydraulic fracture and flaws (open pathways for water to flow) in poorly compacted zones are as shown in Table 1.These are used to "anchor" quantitative estimates in risk analyses. Tables 2 and 3 were developed from the Bui et al (2004, 2005) analyses and using engineering judgement to give a guide on the likelihood of cracking and hydraulic fracture in the upper and lower parts of an embankment.

Table 1 - Estimated historical probabilities for a cracking, hydraulic fracture or flaw in a poorly compacted zone in embankment dams (Fell et al 2007b)

Location of cracking/High Permeability Zone	Estimated average probability of crack, hydraulic fracture or high permeability zone		
	First filling	Flood above historic high(1)	Normal operating
In embankment (dam body)	0.014	0.014	1×10^{-3} per annum
Associated with conduit	0.01	0.01	7×10^{-4} per annum
Associated with concrete wall or structure through embankment	0.004	0.004	3×10^{-4} per annum

Note: (1) Assume a flood at least about (0.3 m) above the historic high flood level.

Table 2. Factors influencing the likelihood of cracking, hydraulic fracture or a flaw in a poorly compacted zone in the upper part of embankment dams (Fell et al 2004, 2007a)

Factor	Relative Importance	Influence on Likelihood			
		Much More Likely	More Likely	Neutral	Less Likely
Cross valley profile under embank't core(1)	A	Wide bench near the crest in the abutment $b/h_2 > 1$; $0 < h_2/h_1 < 0.5$	Wide bench in upper half to one third of the abutment $b/h_2 > 1$; $0.5 < h_2/h_1 < 1$	Wide bench low in the abutment $b/h_2 > 1$; $h_2/h_1 > 1$ or Narrow bench in upper half to one third of the abutment.	Uniform abutment profile without benches
Slope of abutments under embank't core(1)	B	Very steep abutments, $\beta > 60^\circ$	Steep abutments $45^\circ < \beta < 60^\circ$	Moderate abutment slopes $30^\circ < \beta < 45^\circ$	Gentle abutment slope $\beta < 30^\circ$
Height of embank't	C	Dams> 60m high	Dams 30m to 60m high	Dams 15m to 30m high	Dams less than 15m high

Note: See Figure 1 for definitions of b, D, h_1, h_2, β.

If for example an embankment has characteristics which satisfy the "much more likely" and/or "more likely" descriptors, cracking or hydraulic fracture is assessed as more likely to be present than the average from Table 1.

Figures 5 and 6 show examples of the stresses calculated for valley sections with and without a step in the abutment profile. Also shown are the lateral displacements (as a ratio of embankment height) which cause the low stress zones. It should be noted that the magnitudes of the tensile stresses are not accurately modelled, but the extent of low stresses is a reasonable representation.

Figure 7 shows the stresses during the stages of construction of an embankment. It can be seen that low stress zones form over the step but these go back into compression as the embankment is raised. Based on cracking observed at Buffalo (Newman and Foster,2006) and Eppalock (Davidson et

al 2001) dams it is considered wise to assume cracking or low stress zones conducive to hydraulic fracture may persist down to the step in the foundation profile.
Bui et al (2004, 2005) have shown that the shapes of steps in the abutment profile are a secondary factor to the basic geometry.

Table 3. Factors influencing the likelihood of cracking, hydraulic fracturing or a flaw in a poorly compacted zone in the middle and lower part of embankment dams (Fell et al 2004, 2007a)

Factor	Relative Importance	Influence on Likelihood			
		Much More Likely	More Likely	Neutral	Less Likely
Cross valley profile under embank't core	B	Wide bench in lower half of the abutment $b/h_2 > 1$; $1 < h_2/h_1 < 1.5$	Bench in lower half of the abutment $b/h_2 > 0.5$; $1.5 < h_2/h_1 < 2$	Minor bench in lower half of the abutment, not persistent across the core	Uniform abutment profile without benches
Small scale irregular-ities in abutment profile	B	Steps, benches, depressions in rock foundation persistent across the core	Steps, benches, depressions in rock foundations, but not fully persistent across the core	Irregularities present, but only across less than 50% of the core width	Uniform abutment profile, or irregularities treated by slope modification
Slope of the abutments under embank't core	C	Very steep abutments, $\beta > 60^o$	Steep abutments, $45^o < \beta < 60^o$	Moderately abutment slopes, $30^o < \beta < 45^o$	Gentle abutment slope, $\beta < 30^o$
Height of embank't	C	Dams< 60m high	Dams 30m to 60m high	Dams 15m to 30m high	Dams less than 15m high

3.2.2 Cracking and hydraulic fracture due to cross valley arching

If the valley in which the dam is constructed is narrow and steep, cross valley arching can occur and the vertical stresses are shed onto the sides of the valley. This can lead to a situation where hydraulic fracture can occur. Fell et al (2007b) give some guidance on the conditions where this is likely to occur. Cross valley arching is most likely to occur if the width of the valley base is less than a quarter of the dam height, and the valley sides are steeper than about 60 degrees. It is unlikely to be an issue if the width of the valley base is greater than three quarters the dam height and the valley sides are flatter than 45 degrees.

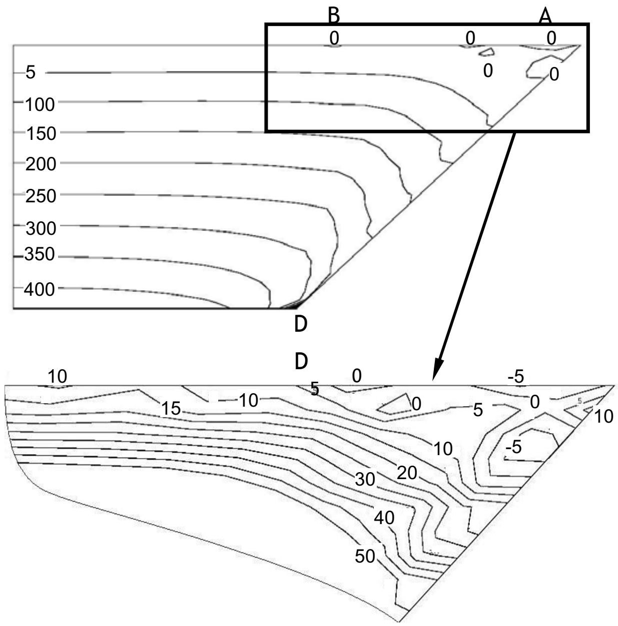
Figure 8 shows the results of cross valley numerical modelling by Bui et al (2004) of the 120 m high Mud Mountain dam which experienced severe internal erosion due to hydraulic fracture. The modelling estimates minor principal stresses and vertical stresses much lower than the stresses calculated from the depth of fill, and lower than the pore pressures which develop as the reservoir fills, so hydraulic fracture is predicted which is consistent with the field behaviour.

3.2.3 Cracking and hydraulic fracture due to differential settlement in the foundation

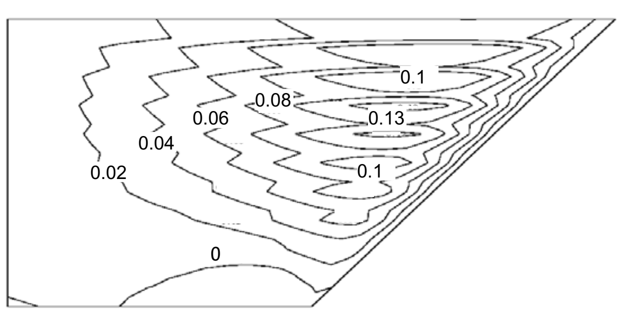
Figure 9 shows foundation conditions which are likely to lead to differential settlement and cracking or low stress zones conducive to hydraulic fracture. Fell et al (2007b) give some guidance on the geometric conditions where this is likely to occur. Zones which may lead to differential settlement greater than 0.5% of the dam height, with steep changes in the foundation profile are most likely to suffer cracking and hydraulic fracture. Differential settlements of less than 0.2% of the dam height spread over some distance are unlikely to lead to cracking and low stress zones.

3.2.4 Cracking and hydraulic fracture due to small scale irregularities in the foundation

Small scale irregularities in the foundation of the core can lead to cracking or low stresses conducive to hydraulic fracture. For this to happen the small scale irregularities need to be persistent over all or most of the distance across the core, and have steps greater than about 3% to 5% of the embankment height. Fell et al (2007b) give some further guidance on the conditions where this is likely to occur.



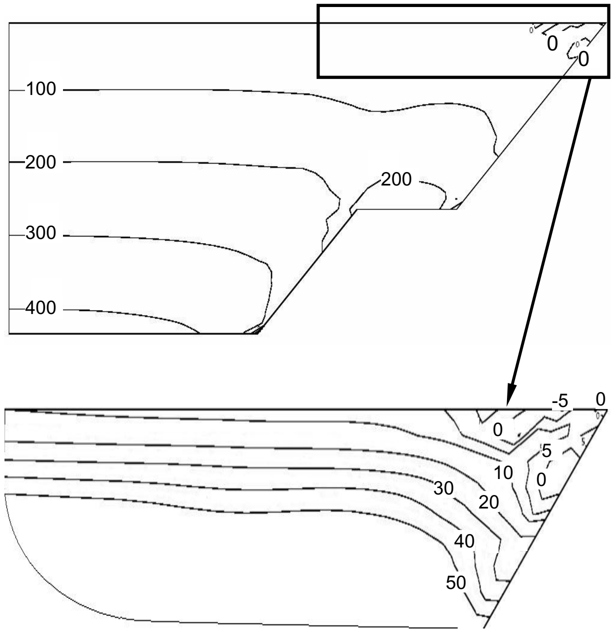
Minor principal stress (kPa)



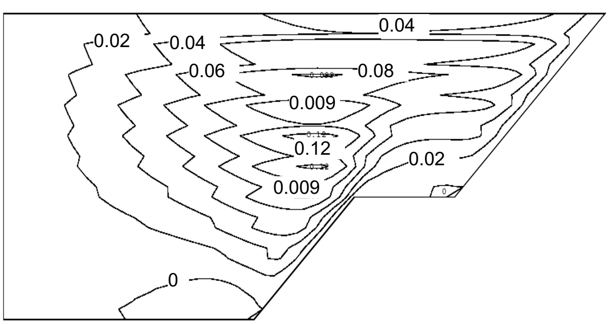
Lateral displacement / Height (%)

PROPERTIES	**GEOMETRY**
ν = 0.3	b = 0 m
E = 30 MPa	H = 50 m
ρ = 2.038 Tonnes/m^3	α = 45^0
ϕ' = 25^0	
c' = 10 kPa	

Figure 5. Results of Mohr Coulomb cross valley model without a step in the foundation profile (Bui et al 2004)



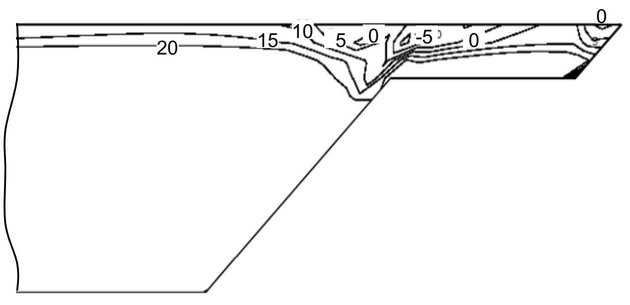
Minor principal stress (kPa)



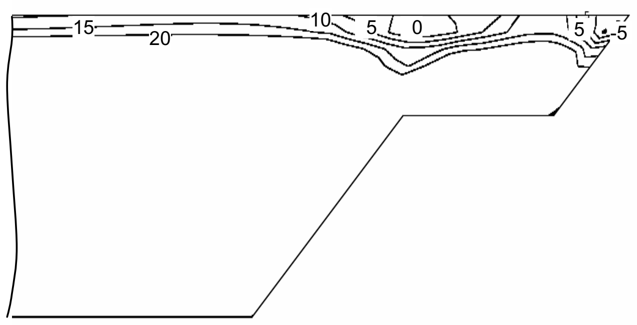
Lateral displacement / Height (%)

PROPERTIES	GEOMETRY
ν = 0.3	b = 20 m
E = 30 MPa	H = 50 m
ρ = 2.038 Tonnes/m^3	α = 45^0
ϕ' = 25^0	
c' = 10 kPa	

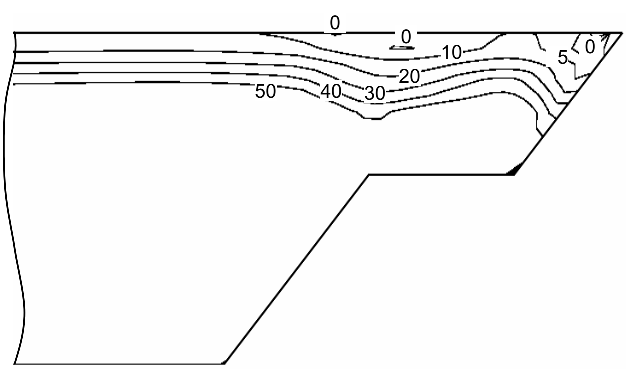
Figure 6. Results of Mohr Coulomb cross valley model with a step in the foundation profile (Bui et al 2004)



Stage 5



Stage 6



Stage 7

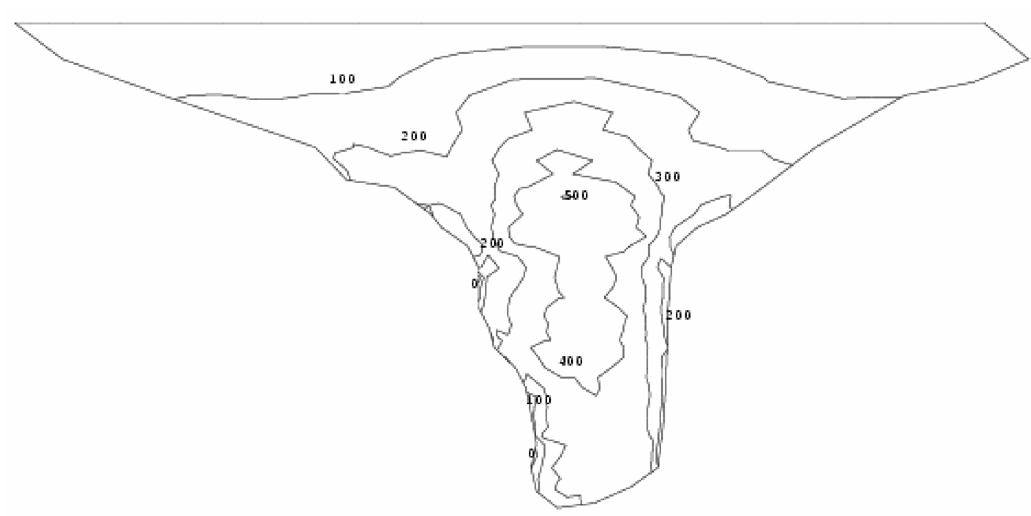
PROPERTIES

ν = 0.3
E = 30 MPa
ρ = 2.038 Tonnes/m^3
ϕ' = 25^0
c' = 10 kPa

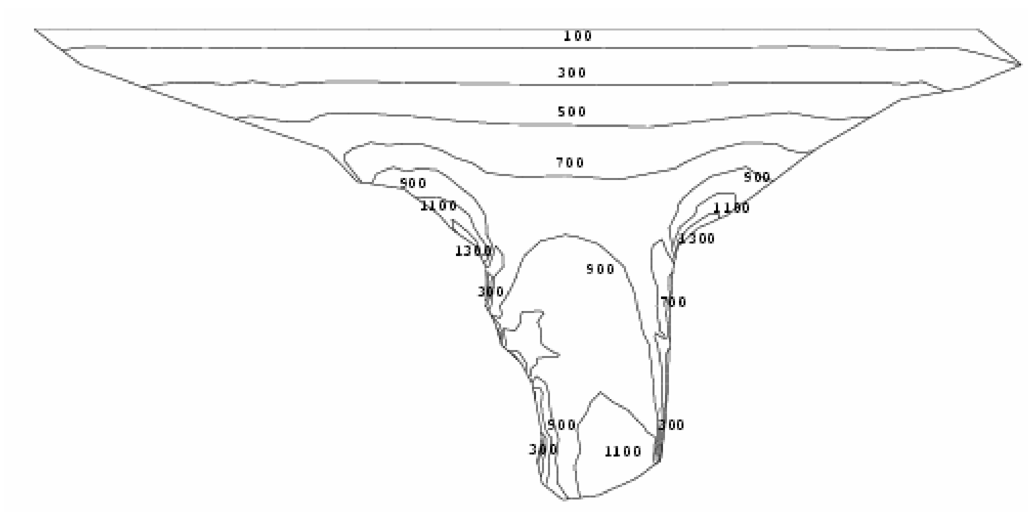
GEOMETRY

b = 20 m
H = 50 m
α = 45^0

Figure 7. Low stress zones formed in an embankment dam during construction (Bui et al 2004)



(a) Minor Principal Stress (kPa)



(b) Vertical Stress (kPa)

Figure 8. Mud Mountain dam – minor principal stress and vertical stress distributions (kPa) Bui et al (2005)

3.2.5 Cracking and hydraulic fracture due to arching of the core onto the shoulders of the embankment

Bui et al (2005) investigated this with a wide range of cross sections and material properties. This showed that for arching to occur there needs to be differential settlement between the core and the shoulders after construction. This may occur on first filling as the core is saturated or as the core consolidates after construction. It is most likely to be a problem for cores which are very narrow- core width less than 0.25 the embankment height, and for soils subject to collapse compression on saturation (poorly compacted soil placed dry of optimum moisture content). It is unlikely to be a problem for cores which are wider than 0.5 to 1.0 the embankment height and the core is well compacted at around optimum moisture content. Fell et al (2007b) give some further guidance on the conditions where this is likely to occur.

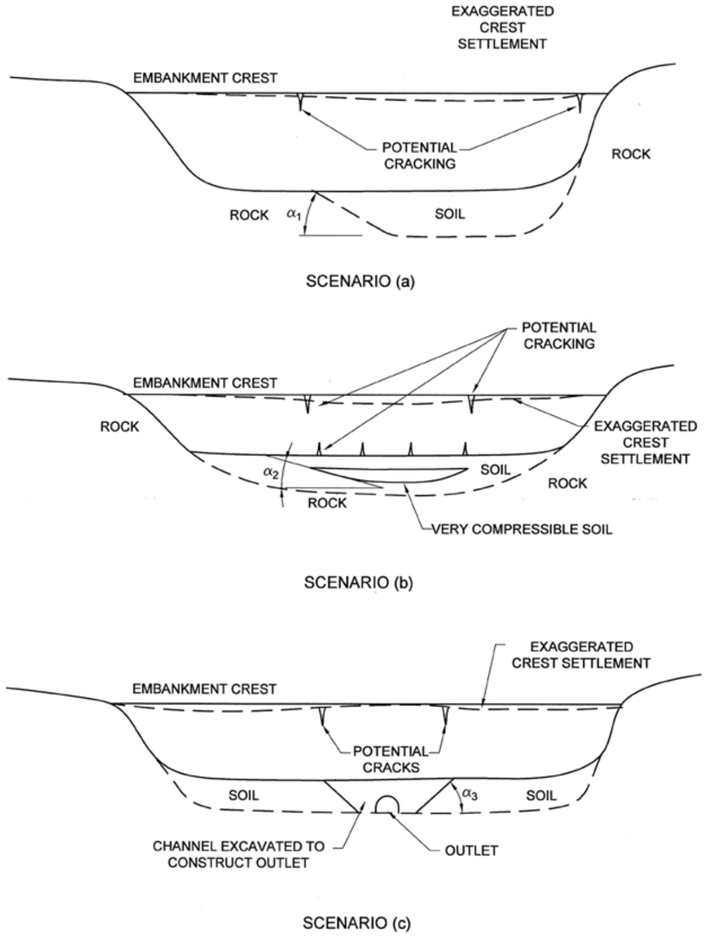


Figure 9 Typical Scenarios which may lead to differential settlement in the foundation of dams (Fell et al 2007b)

3.2.6 Cracking due to desiccation

Desiccation cracking is most likely to be an issue in climates with less than 250mm annual rainfall, in high plasticity cores, and where there is no surface layer over the core. Experience in excavating into the crest of embankments is that if there is a road pavement or a rockfill or other non-plastic layer at least 300mm thick cracking is not generally observed in eastern Australian climates. If the road is sealed cracking is not observed. Tandjiria et al (2005) describe a method for assessing the likely depth and width of cracking based on the measured moisture content profile in the embankment core, the moisture content versus matric suction, and properties which can be obtained from unconfined compression tests on soils compacted to the conditions in the embankment. The method has been validated against field trials.

3.2.7 Cracking adjacent to walls and associated with conduits

Cracking and low stress zones may occur adjacent to walls due to the core settling away from the wall, and around conduits through the embankment due to stress concentration effects. Guidance on assessing this is given in Fell et al (2007a, b)

3.3 Assessing the depth and width of cracking and hydraulic fracture

The preferred method is to do a numerical model with the profile of the valley of the dam in question, using Cam-Clay or Mohr-Coulomb models. Where this is impractical for cost reasons Table 4 which is based on the numerical modelling by Bui et al (2004) can be used as an approximate guide to the depths of zero stress zones in which cracking may occur and zones in which hydraulic fracture may occur. The depths of hydraulic fracture in Table 4 assume the reservoir water level reaches close to the crest of the dam. The depths do not allow for the tensile strength of the soil so may be on the conservative side.

It is not possible to accurately model the potential width of cracks because that requires knowledge of crack spacing as well as strains. Sherard (1973) describes cracks up to 75mm wide, and widths from 3mm to 20mm as common. Transverse cracking at Buffalo dam (Newman and Foster 2006) was up to 40mm wide, and diagonal cracking on the downstream slope of Tullaroop dam (Hunter et al 2007) was up to 60mm wide. The cracks become narrower at depth, and for practical purposes it may be assumed they have a linear taper from a maximum at the surface to zero at maximum depth. The evidence is this may be on the conservative side with cracks generally narrowing more rapidly with depth. Based on Sherard et al. (1972) it seems unlikely cracks due to hydraulic fracture will exceed 5mm to 10mm width but this is difficult to predict. Fell et al (2007b) give some further guidance on the likely depth and width of cracking.

Table 4 Summary of approximate depths of zero stress and potential hydraulic fracture as a ratio of embankment height

Abutment slope (β) degrees	Bench width/ embankment height (b/H)	Depth of zero stress/ embankment height	Depth of potential for hydraulic fracture/ embankment height
15	0.67	<0.01	0.05
25	No bench	0.02	0.05
45	No bench	0.12	0.3
45	0.2	0.12	0.3
45	0.4	0.09	0.25
45	1.0	0.10	0.25
60	No bench	0.35	0.5
60	0.4	0.25	0.4

3.4 Assessing the equivalent crack depth for collapse settlement of a poorly compacted layer of soil in an embankment or around a conduit through the embankment

It is well documented (e.g. Sherard 1973, Foster et al (2000a) that internal erosion and piping occurs in poorly compacted cohesive soils. This is particularly so for dispersive soils. The mechanism is potentially of two types:

- The soil behaves as a series of clods with openings between the clods in which water passes.
- The soil collapses on saturation forming a flaw (open pathway) in which the water flows.

This is most likely where there is poorly compacted soil against a conduit but is possible within layers of soil. Sherard (1973) gives examples of this. The author is aware of cases where a gap has been seen in test pits and / or zero Cone Penetration Test cone resistance value occurs just below the phreatic surface. This has been caused by settlement of the soil in the saturated zone and arching of the unsaturated soil over this. In these cases the gap may be a pathway for water at reservoir levels greater than the historic high.

To model these cases it is most practical to assume a crack or flaw is formed and to assess the likelihood of erosion initiating in the crack. There are examples where it appears a crack may form adjacent to a conduit due to freezing and differential movements even if the soil is well compacted.

For collapse settlement of a poorly compacted layer the procedure suggested by Fell et al (2007b) is:

- Assess the thickness of the layer of soil which is poorly compacted. (T_p). There may be a single layer or several layers.
- Estimate the amount by which the layer may collapse (C_F) using Table 5. Then estimate the height of the gap (flaw) which could result (G) from G = (T_p) x (C_F). This represents the scenario with the weight of the soil above being supported on non-collapsed soil adjacent.
- Assume G is the height of the flaw which is formed, and use the method outlined in Section 3.5 to assess the likelihood of initiation of erosion given this width flaw, the average gradient through the core at the level of the high permeability layer, and the soil properties.

This method should be also applied to silty sands, and silty sandy gravel soils which may be subject to collapse settlement even if the soils are non plastic, since they will erode rapidly in a flaw.

For poorly compacted soil around a conduit the suggested procedure is:

- Assess the thickness of the layer of soil which is poorly compacted. (T_p). There may be a single layer or several layers.
- Estimate the amount by which the layer may collapse (C_F) using Table 5 as a guide. Then estimate the height of the gap (flaw) which could result (G) from G = (T_p) x (C_F). To do this take account of the dimensions of the conduit and the trench in which it is placed.
- For cases where it appears that the soil around the pipe is well compacted and where flaw widths <5mm are calculated, assume a flaw width of 5mm to allow for possible shrinkage of the soil from the pipe during construction or in service.
- Assume G is the height of the flaw which is formed, and use the method outlined in Section 3.5 to assess the likelihood of initiation of erosion given this width flaw, the average gradient through the core at the level of the high permeability layer, and the soil properties.

3.5 Assessing whether erosion will initiate in a crack or flaw

3.5.1 The procedure

The procedure for assessing whether erosion will initiate in a crack or flaw is to:

- Estimate the hydraulic shear stresses in the crack for the reservoir level under consideration, taking account of the geometry of the core of the embankment and the assumed crack dimensions and location relative to the reservoir surface so the flow gradient can be determined.
- Compare this hydraulic shear stress to the critical shear stress which will initiate erosion for the soil in the core of the embankment (τ_c) at the degree of saturation of the soil on the sides of the crack. In doing this take account of the dispersivity of the soil and the chemistry of the seepage water.

There is always some uncertainty regarding the in-put parameters so the analysis should check the sensitivity to the assumptions made.

3.5.2 Estimating the hydraulic shear stresses in a cylindrical pipe or crack.

The following equations can be used to estimate the hydraulic shear stress (τ) on the surface of a cylindrical pipe, or parallel side transverse crack in an embankment.

Table 5 Amount of collapse compression which may occur on saturation versus compaction properties (Fell et al 2007b)

Description of the method and degree of compaction of the core or soil around the conduit		Amount of collapse compression as a proportion of the layer thickness
Soil placed with no formal compaction (e.g. by horse and cart in old dams), or by pushing into place by excavator or bulldozer or in very thick layers. No control of layer thickness. Well dry of optimum moisture content.	Layers very poorly compacted dry of standard optimum moisture content e.g. <90% standard dry density ratio, 3% dry of standard OMC	0.02 to 0.05
Soil placed and compacted by bulldozer, no compaction by rollers, or rolled in thick layers beyond the capability of the roller. Layer thickness at or beyond the limit of compaction equipment. Dry of optimum moisture content.	Layers very poorly compacted dry of standard optimum moisture content e.g. <93% standard dry density ratio,2% to 3% dry of standard OMC	0.01 to 0.02
Soil rolled in layers near the limit of the capability of the rollers, at moisture contents dry of standard OMC.	Compacted to e.g. e.g. 93% - 95% standard dry density ratio,2% to 3% dry of standard OMC	0.005
Soil compacted by suitable rollers in suitable layer thickness Around optimum moisture content.	Well compacted to e.g. e.g. 95% -98% standard dry density ratio,2% dry of standard OMC to 1% wet of standard OMC	Will not collapse
As above but with good documentation and records.	Very well compacted to e.g. e.g. >98% standard dry density ratio,2% dry of standard OMC to 1% wet of standard OMC	Will not collapse

The assumptions are:

- Linear head loss from upstream to downstream
- Steady uniform flow along the crack
- Zero pressure head at the downstream end
- Uniform frictional resistance along the surface of the crack or cylindrical pipe
- Driving force = frictional resistance.

(a) Cylindrical pipe:

$$\tau = \rho_w \frac{gH_f d}{4L}$$

where
τ = Hydraulic shear stress in N/m^2
ρ_w = Density of water in kg/m^3
g = Acceleration due to gravity = $9.8 m/s^2$
H_f = Head loss in pipe due to friction in metres
L = Length of pipe in metres
d = Diameter of the pipe in metres

(b) Vertical transverse crack

$$\tau = \frac{\rho_w \, g H_f^{\,2} W}{2(H_f + W)L}$$

where τ = Hydraulic shear stress in N/m^2
ρ_w = Density of water in kg/m^3
g = Acceleration due to gravity = 9.8m/s^2
H_f = Head loss in crack due to friction in metres
L = Length of crack base in metres
W = Width of crack in metres.

It is recognised that most cracks and pipes will not have uniform dimensions but the intention is to get an approximate estimate of the hydraulic shear stresses. There will be a range of possible crack or pipe dimensions possible so as a result there will be a range of possible hydraulic shear stresses.

3.5.3 Erosion properties of soils in the core of embankment dams - basic principles

Wan (2006), Wan and Fell (2002, 2004a, b) have developed two laboratory tests, the Slot Erosion Test (SET), and the Hole Erosion Test (HET), to measure the erosion properties of the soils used in embankment dams. The experimental investigations showed that the Slot Erosion Tests and the Hole Erosion Tests can successfully measure the erosion rate of a soil. The tests express erosion rate in the form of an Erosion Rate Index, I defined by:

$$I = -\log(C_e) \qquad (1)$$

$$\dot{\varepsilon}_t = C_e(\tau - \tau_c) \qquad (2)$$

where $\dot{\varepsilon}_t$ is the erosion rate per unit area (kg/sec/m^2)
C_e is the Coefficient of Soil Erosion (sec/m)
τ is the shear stress (N/m^2)
τ_c is the Critical Shear Stress for initiation of erosion (N/m^2)

It was shown that the Erosion Rate Index for the HET (I_{HET}) correlated strongly to that for the SET (I_{SET}) and given that the HET requires simpler equipment and is less costly, Wan and Fell adopted the HET as the measure of erosion rate. The representative erosion rate index $\tilde{I}_{HET}$ is the hole erosion index I_{HET} for soil compacted to a density ratio of 95% of standard maximum dry density at optimum moisture content. Where the critical Shear Stress is determined by varying the head in the HET Wan and Fell (2002, 2004a, b) called it the initial Shear Stress τ_o.

Soils can be classified into 6 groups according to their Representative Erosion Rate Index, $\tilde{I}_{HET}$. The 6 groups are as shown in Table 6. Note that this is a logarithmic scale and the rate of erosion of soils varies in practice by up to five orders of magnitude.

Table 6 Descriptors for erosion rates of soils (Wan 2006)

Group No.	Erosion Rate Index	Description
1	<2	Extremely rapid
2	2 – 3	Very rapid
3	3 – 4	Moderately rapid
4	4 – 5	Moderately slow
5	5 – 6	Very slow
6	>6	Extremely slow

Wan and Fell (2002) found that the Representative Erosion Rate Index ($\tilde{I}_{HET}$) could be related to soil properties and developed regression equations to do this. These equations are very sensitive to the in-puts and their use is not recommended. Wan and Fell (2002) also presented a table to relate $\tilde{I}_{HET}$

to soil classification. Table 7 has been developed from this and the original test data to give a first approximation to the likely range of $\tilde{I}_{HET}$ for different classifications of non-dispersive soils.

Wan and Fell (2002) found that there was a correlation between the initial Shear Stress (τ_{o}) and the representative erosion rate index ($\tilde{I}_{HET}$) for soils behaving in a non-dispersive manner either because they are non-dispersive, or they are dispersive but the eroding fluid suppresses the dispersion. Figure 10 presents the results of tests on a number of soils using Sydney tap water or water from the reservoir for the soil being tested. For some soils the water tends to inhibit dispersion although the results for samples from Ross River are partially reflecting dispersive behaviour. It should be noted there is a significant range in the correlation.

Table 7 Representative erosion rate index (I_{HET}) versus soil classification for non dispersive soils based on Wan and Fell (2002)

Soil Classification	Erosion Rate Index (I_{HET})		
	Likely Minimum	Best Estimate	Likely Maximum
SM with <30% fines	1	<2	2.5
SM with > 30% fines	<2	2 to 3	3.5
SC with < 30% fines	<2	2 to 3	3.5
SC with >30% fines	2	3	4
ML	2	2 to 3	3
CL-ML	2	3	4
CL	3	3 to 4	4.5
CL-CH	3	4	5
MH	3	3 to 4	4.5
CH with Liquid Limit <65%	3	4	5
CH with Liquid Limit > 65%	4	5	6

Notes. (1) Use best estimate value for best estimate probabilities. Check sensitivity if the outcome is strongly dependent on the results.

(2) For important decisions carry out Hole Erosion Tests, rather than relying on this table which is approximate

Table 8 Best estimates and likely range of initial shear stress (τ_0) versus Hole Erosion Index (I_{HET})

Hole Erosion Index (I_{HET})	Initial Shear Stress (τ_0) Pa			
	Non Dispersive Soil Behaviour		Dispersive Soil Behaviour	
	Best Estimate	Likely Range	Best Estimate	Likely Range
<2	2	1 to 5	1	0.5 to 2
2 to 3	2	1 to 5	1	0.5 to 2
3.5	5	2 to 20	2	1 to 5
4	25	10 to 50	5	2 to 10
5	60	25 to 100	5	2 to 10
6	100	60 to 140	5	2 to 10

Note. To be used with caution. For important decisions carry out Hole Erosion Tests to determine the initial shear stress (τ_0)

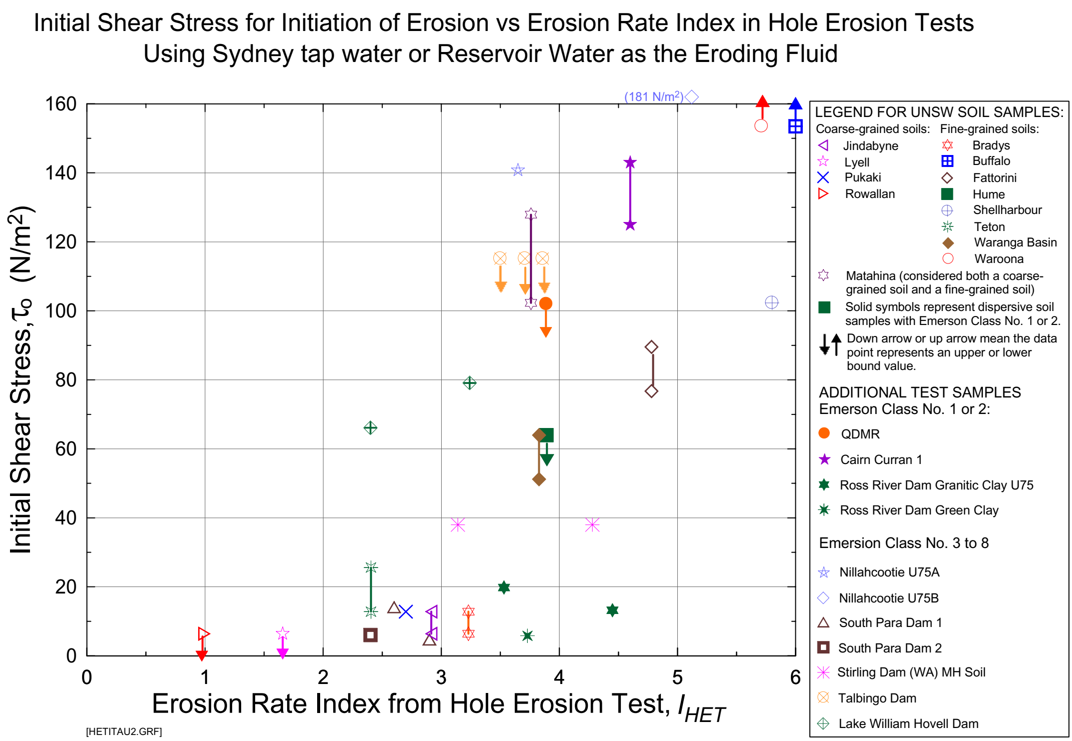


Figure 10 Initial shear stress (τ_o.) versus representative erosion rate index ($\tilde{I}_{HET}$) for soils which are non-dispersive, and for dispersive soils with eroding water suppressing dispersion. (Courtesy of C.F.Wan, GHD, 2006)

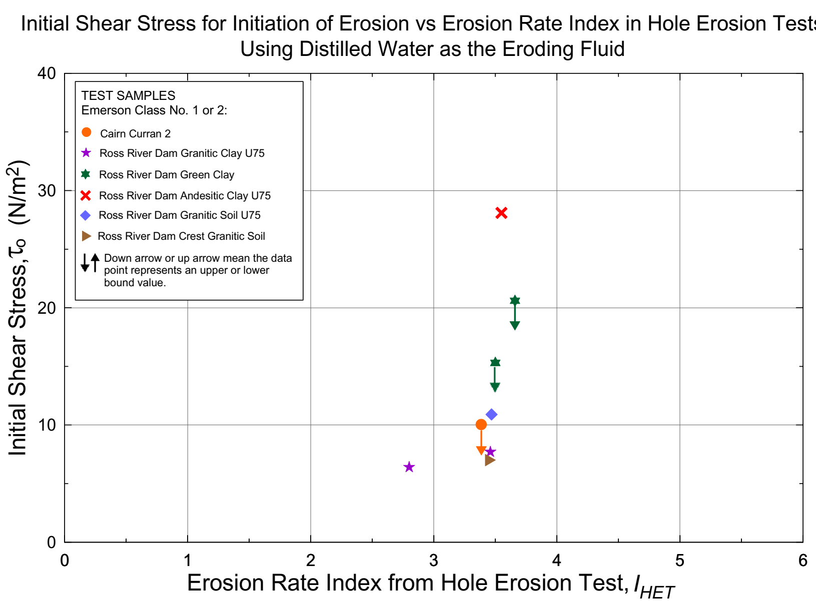


Figure 11 Initial shear stress (τ_o.) versus representative erosion rate index ($\tilde{I}_{HET}$) for dispersive soils using distilled water as the eroding fluid (Courtesy of C.F.Wan, GHD, 2006)

The initial shear stress for dispersive soils is significantly lower if the eroding fluid is sufficiently free of salts which suppress dispersion. Figure 11 shows some results. Note the different "Y" axis scale.

From these data Fell et al (2007b) have adopted the values shown in Table 8. It is emphasised that it is better to carry out a series of Hole Erosion Tests at varying heads to define the initial Shear Stress (τ_o) than to rely on these relationships.

3.5.4 Some more detailed discussion on the erosion properties of soils, cracking and the effect of swelling of the soil adjacent cracks

The following are some more detailed matters relating to the erosion properties of soils:

(a) Effect of degree of saturation of the soil

The discussion in Section 3.5.3 relates to the properties of soils at the representative compaction conditions. That is at 95% standard maximum dry density and standard optimum moisture content. Wan and Fell (2002, 2004a, b) and Lim (2006), Lim and Khalili (2007a, b) have found that the Erosion Rate Index for the Hole Erosion (I_{HET}) and Rotating Cylinder (I_{RCT}) tests are related to the degree of saturation of the soil. Figure 12 shows results of testing three soils. Soil S is a residual granitic soil from Serpentine Dam, Western Australia classifying as MH, Soil B is a sandy clay of low plasticity from Fairbairn Dam, Queensland, Soil F is a clay of low plasticity from Boggy Creek Dam in Oklahoma, USA, and 50% of kaolin is an artificial soil consisting of 50% kaolin and the remainder fine sand size particles.

It will be seen that there is a significant dependence of the erosion rate on degree of saturation for the two clay soils. There was however less dependence on the degree of saturation for the silty soil. Wan and Fell (2002) noted a similar trend with two other residual granitic soils showing little change in the erosion rate with increased degree of saturation.

Lim (2006) Lim and Khalili (2007a, b) noted that there was little change in the erosion rate for clay soils for degrees of saturation above 90%. Clay soils they tested which had $I_{RCT} \approx 3$ to 4 at the representative compaction and moisture content had $I_{RCT} \approx 4.5$ to 5.5 when 90% to 100% saturated. This is an important finding because it means that once the core of a dam constructed of clay soil is saturated, it will have a slower rate of erosion, and a higher initial shear stress. Just as important is this does not apply to silty sand cores such as decomposed and residual granites. Figure 12 shows the results which support this conclusion. Lim (2006) might well have shown the lines as bi-linear to better show the behaviour.

(b) Effect of dispersion on erosion properties

As discussed above, soils which show dispersive behaviour; soils classifying as Emerson Crumb Class 1 or 2, and Pinhole Dispersion D1 and D2, will have a very low initial shear stress if the eroding fluid does not inhibit the dispersion. That is if the eroding water has low salts content. However Lim (2006), Lim and Khalili (2007a, b) showed that for rotating cylinder tests the Erosion Rate Index is not greatly affected by whether the soil is dispersive after the initially rapid part of the erosion process.

(c) The importance of slaking in the erosion process

The term "slaking" or "soil slaking" is defined as "… disintegration of unconfined soil after exposure to the air and subsequent immersion in a fluid, usually water; no external confining pressure is assumed to act over the soil prior to immersion…" (Moriwaki and Mitchell, 1977). Lim (2006), Lim and Khalili (2007a, b) showed that slaking is the most important process involved in erosion in the RCT. They proved a strong correlation between the rates of slaking from a sample held statically in water to the erosion rate index from the RCT (I_{RCT}). They also showed that the slaking process was correlated strongly to the degree of saturation of the soil, with the slaking rate being up to 30 to 50 times lower between soils at 70% degree of saturation and those at 100% degree of saturation. This corresponds with the behaviour of the erosion rate index for clay soils.

This is a very important finding as it helps explain the actual mechanics of the erosion process being strongly linked to slaking.

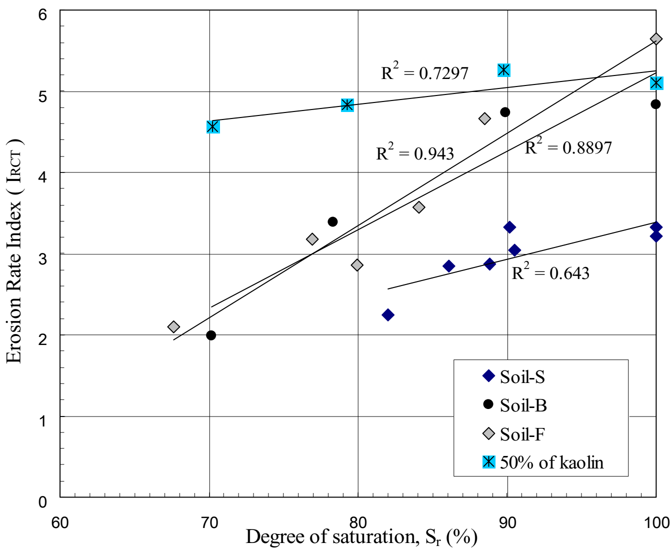


Figure 12 Relationship between degree of saturation and Erosion Rate Index (I_{RCT}) for non-dispersive soils (Lim 2006)

(d) Effect of testing method on erosion rate index

Wan and Fell (2002, 2004a, b) obtained a strong correlation between the erosion rate index from the Hole Erosion Test and the Slot Erosion Test. In the Hole Erosion Test erosion is from the sides of an initially 6mm diameter hole drilled into a 100mm long by 100 mm diameter cylindrical sample oriented horizontally. In the Slot Erosion Test erosion is from the sides of an initially 10mm by 2.2mm cross section by 1 metre long slot oriented horizontally.

Lim (2006), Lim and Khalili (2007a, b) investigated the erosion properties of soils using rotating cylinder and Hole Erosion Tests. In the rotating cylinder test the erosion is from the vertical sides of a 100mm diameter by 100mm high sample. Figure 13 shows a comparison between the erosion rate indices for these tests. It can be seen that for dispersive soils there is good correlation with the HET giving slightly larger index (slower rate of erosion). However there is a large difference for the non-dispersive soils with the rotating cylinder test giving rates 10 to 15 times those in the HET. Lim (2006) attributes this to the vertical face of the RCT, and to surface and/or body slaking occurring in the RCT which could not occur in the HET because of the relatively large size of the slaking particles and the small hole in the HET.

It is considered that the HET and SET better model erosion conditions in a crack and pipe in the early stages of development than does the rotating cylinder. However it could be that the HET and SET underestimate the rate of erosion as the pipe develops and the size allows the mechanisms modelled in the rotating cylinder to take effect. This could be allowed for by using a varying erosion rate index when calculating the rate of enlargement of a pipe.

(e) Effect of the testing method on the critical shear stress to initiate erosion (τ_c)

The critical shear stress to initiate erosion (τ_c) (or for the HET the initial shear stress, τ_0) is a difficult property to measure. Lim (2006), Lim and Khalili (2007a, b) have found the form of the hydraulic shear stress versus erosion rate plot varies for different soils, with some plotting as bi-linear plots, others with the intercept on the "X" and "Y" axes. They found that the correlation between critical shear stress and erosion rate index for the RCT was weak. This appears to be related to the complexity of the plots and interpretation. Wan and Fell (2002) had similar problems and this is why they adopted the procedure of varying heads in the HET to define the initial shear stress. However it appears that the extrapolation of the plots to the "X" axis for the RCT on non-dispersive soils gives critical shear stresses which are similar to the initial shear stresses shown in Figure 10 up to erosion indexes of 4. (Lim (2006), Lim and Khalili (2007a, b) did not test many soils with erosion rate indices greater than about 4 but those they did test seem to give somewhat lower critical shear stresses than the HET. This may reflect the occurrence of slaking of larger particles on the surface of the RCT samples.

(f) The effect of tensile strength of the soil on cracking

Win (2006) working with Dr Gareth Swarbrick and the author carried out extensive testing on the tensile strength of partially saturated soils compacted as they would be in the core of a dam embankment. This work was used by Tandjiria et al (2005) in the assessment of desiccation cracking.

(g) The effect of swelling of the soil adjacent to cracks

Some testing has been carried out at UNSW to assess the effects of swelling of the soil adjacent to cracks on the crack width, and hence the likelihood of erosion initiating in the crack. This work has not been assessed at this time but it is expected it will be a factor for some soils if the rate of reservoir rise is small.

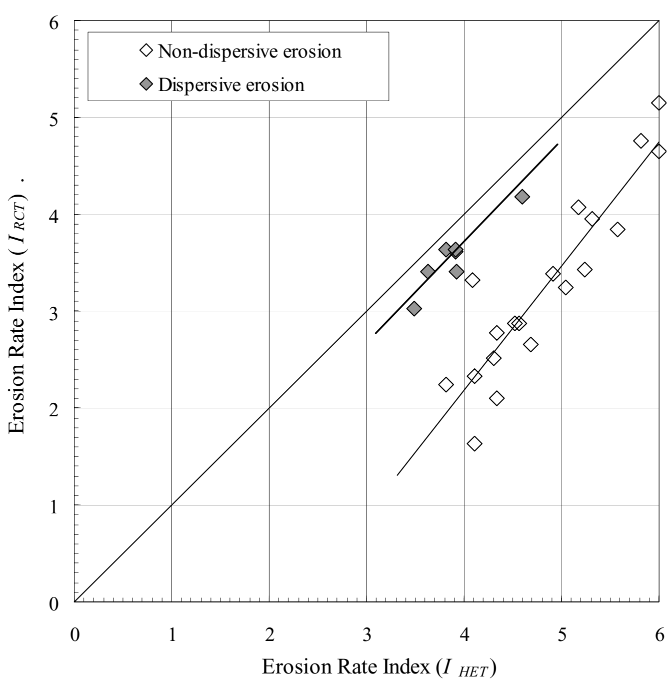


Figure 13 Correlation between erosion rate index from Rotating Cylinder and Hole Erosion Tests (Lim (2006)

(g) The time to set up a flow net

It is important to recognise that erosion in cracks or flaws is not a result of establishment of a flow net, but is due to flow in open cracks. This occurs quickly as the reservoir rises into the cracked zone. Transient flow net analyses are sometimes carried out but are irrelevant to this process.

3.5.5 Comparison of the hydraulic shear stress in the crack (τ) to the critical shear stress which will initiate erosion for the soil in the core of the embankment (τ_c)

To assess the likelihood erosion will initiate in a crack the estimated hydraulic shear stress is compared to the initial shear stress for the soil in the core taking account of the moisture content of the soil in the core, and the chemistry of the water in the reservoir. Table 9 is useful to assist in doing the assessment. It is based on the equations in Section 3.5.2. When doing this assessment allowance should be made for the uncertainty in the calculations and properties.

Table 9 – Estimated hydraulic shear stress (N/m^2) from water flowing in an open crack, versus crack width and flow gradient (Fell et al 2004)

Crack Width Millimetres	Flow Gradient in Crack					
	0.1	0.25	0.5	1.0	2.0	5.0
1	0.5	1.25	2.5	5	10	25
2	1	2.5	5	10	20	50
5	2.5	6	12	25	50	125
10	5	12	25	50	100	250
20	10	25	50	100	200	500
50	25	60	125	250	500	1250
100	50	125	250	500	1000	2500

It should be noted that under flood conditions the salts content of the water in the reservoir is likely to drop, so tests done in reservoir water may be un-conservative. If in doubt with dispersive soils it is best to assume the reservoir water will not inhibit dispersion and rely on the results of tests using distilled water.

3.6 Assessing the rate of development of the pipe

From the equations in Sections 3.5.2 and 3.5.3, the Hole Erosion Index of the eroding soil and average hydraulic gradient along the pipe it is possible to estimate the rate at which a piping hole will enlarge. This is important to know because it is a significant factor in assessing the likelihood of successful intervention to stop the piping process. The results are summarised in Figure 14 assuming (a) Unrestricted potential for erosion (i.e. no flow limitation, continuing erosion condition); (b) Initial pipe diameter of 25mm; (c) Zero critical shear stress which is conservative, particular for I_{HET} >3.5; (d) Reservoir level remains constant

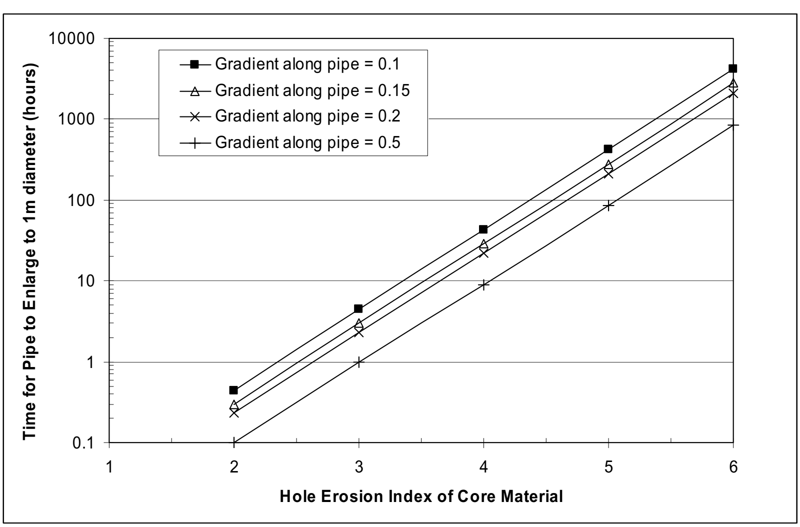


Figure 14: Approximate time for pipe to enlarge from 25mm to 1 m diameter. The time to erode to 2 m diameter is about 20% greater (Courtesy of M Foster, URS Australia)

The results in Figure 14 indicate the following;

- soils with I_{HET} of about 2 would erode very quickly if erosion is unrestricted (<1 hour).
- soils with I_{HET} of about 3 would erode quickly if erosion is unrestricted (2-4 hour).
- soils with I_{HET} of about 4 would erode more slowly if erosion is unrestricted (1 – 2 days).
- soils with I_{HET} of about 5 and 6 would erode slowly if erosion is unrestricted (9 to 18 days for I_{HET} = 5 and >3 months for I_{HET}=6).

The rates of development of the pipe are consistent with case studies and the method for assessing the time to progress from the first signs of a concentrated leak to breach as described in Fell et al (2001, 2003)

4 MECHANICS OF BACKWARDS EROSION IN COHESIONLESS SOILS

4.1 Some general concepts

Figure 15 shows a seepage flow net for an embankment on a soil foundation. It will be apparent that the seepage gradients are greatest at the toe of the embankment where the seepage from the foundation is flowing to the ground surface. If the seepage gradient is sufficiently high, a zero effective stress condition will occur. This is known as "heave", "liquefaction" or "blow-out".
The factor of safety against this occurring can be calculated from:

$$F_{UT} = \frac{\sigma_v}{u}$$

where σ_V = total vertical stress at any point in the foundation – kN/m^2
u = pore pressure at the same point – kN/m^2

or

$$F_{UT} = (h\gamma_{sat}) / (h_p \gamma_w)$$

where γ_{sat} = unit weight of saturated foundation soil – kN/m^3
γ_w = unit weight of water – kN/m^3
h_p = piezometric head – metres

The alternative method for estimating the factor of safety is to consider the gradient of the flow net. If the gradient approaches unity, liquefaction can be expected to occur. For seepage in the vertically upwards direction, the critical gradient i_c can be calculated from (Terzaghi 1960)

$$i_c = (G - 1)/(1 + e)$$

where G = specific gravity
e = void ratio

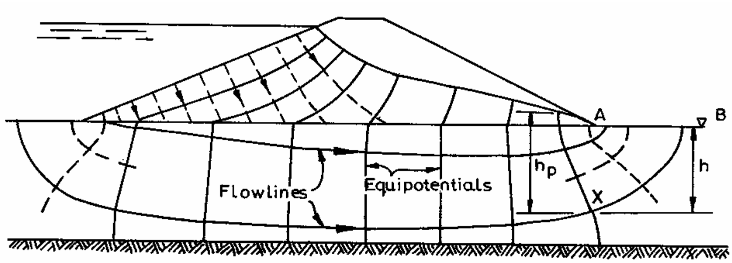


Figure 15. Seepage flow net for an embankment and its foundation

If there are low permeability strata overlying the soil subject to backward erosion there is a high likelihood of heave depending on the total head and the thickness of the low permeability strata.
It should be noted that heave is very unlikely beneath an embankment built on homogeneous isotropic soil. The gradients simply are not high enough for normal embankment slopes. Heave conditions may occur where the foundation soil is highly anisotropic e.g. with K_h/K_v> 10 or 20.
It is widely recognised that if heave occurs, backward erosion is likely to initiate. If this occurs it will usually manifest itself as sand boils. Heave does of course also potentially lead to instability of the land side slope of the embankment.

It has however commonly been assumed, at least implicitly, that if backward erosion initiates it will always progress to form a pipe, at least under repeated loading from successive floods. It has been demonstrated in laboratory flume tests by Schmertmann (2000), Sellmeijer and Koenders (1991), Koenders and Sellmeijer (1992), Weijers and Sellmeijer (1993) that this is not correct at least for a single reservoir loading or river stage, and that erosion may stabilize before progression to form a pipe back to the reservoir, but having stabilized for a given gradient, may progress further under higher gradients. Once it progresses beyond about 50% of the seepage path length, the gradients required for backward erosion to continue are lessened, so if the average gradient remains the same, progression to form a complete pipe is highly likely, and subsequent erosion and enlargement of the pipe is likely to be rapid. The rate of progression of the pipe is relatively uniform until the length approaches about 40% of the total seepage path. It then accelerates. The piping progressed about 6 metres in an hour in the largest of the Selmeijer and co-authors tests at the Road and Hydraulic Research Institute at Delft.

Sellmeijer and his co-authors, and Schmertmann (2000) have also shown that backward erosion may initiate and progress to form a pipe at average seepage gradients in an embankment foundation beneath the embankment as low as 0.1, but as high as 1.0. That is the average gradient beneath the embankment required to progress the backward erosion is potentially far less than the local gradient at the toe to cause heave.

Factors which influence the critical average gradient to initiate and progress backward erosion include the particle size (as measured by Cu, the uniformity coefficient (= d_{60}/d_{10}) by Schmertmann (2000), and the d_{70} size by Sellmeijer and his co-authors, and the geometry of the foundation strata.

The participants at the Aussois Workshop (Fell and Fry 2007) considered that at gradients likely to occur within a dam backward erosion was probably restricted to cohesionless soils or soils with only limited plasticity. However, the limits of plasticity at which backward erosion could occur were not known. Laboratory tests by Sun (1989) and Marot et al (2007) showed that backward erosion could occur in more cohesive soils, but initiated at very high gradients which were not likely to occur in dams or their foundations. For practical purposes Fell et al (2007b) have concluded that soils with any plasticity should be considered not subject to backward erosion at the gradients experienced in dams and their foundations. If for some particular reason the gradient is higher than about 4, the guidance given below for soils subject to suffusion could be adopted.

4.2 Sellmeijer and co-workers' method for modelling initiation and progression of backward erosion

Sellmeijer and co-workers from Delft Hydraulics and Delft Geotechnics Laboratories in The Netherlands carried out piping tests in flumes. These are reported in de Wit et al (1981), Weijers and Sellmeijer (1993). The results are summarized in Schmertmann (2000). The tests were mostly on fine to medium sands, with a few tests on medium to coarse sands. The sands were relatively uniform (Cu = 1.58 to 3.53). Early tests were on small scale models (base length 0.8m), but later tests were on very large models which were carried out in flumes 37 metres long by 6 metres wide and 6 metres deep.

Sellmeijer (1988), Sellmeijer and Koenders (1991), Koenders and Sellmeijer (1992) developed a mathematical model for backward erosion. They show that critical head occurs when the length of the tunnel (l) is about 0.4 to 0.5 of the flow path length L. As the piping tunnel extends further, the head required for backward erosion to progress is less There are various forms of the equation for the critical head, but the latest is given in Weijers and Sellmeijer (1993):

$$H_{crit} = \alpha c \frac{\gamma_p'}{\gamma_w} \tan\theta(0.68 - 0.1 \ln c)L$$

Where $\alpha = \left(\frac{D}{L}\right)^{\frac{0.28}{\left(\frac{D}{L}\right)^{2.8} - 1}}$

$$c = \eta\left(\frac{d_{70}^2}{K}\frac{d_{70}}{L}\right)^{0.33}$$

H_{crit}	=	critical value of head differential H(m)
γ_w	=	unit weight of water (kN/m^3)
γ_p'	=	submerged unit weight of soil particles (kN/m^3)
	=	(G-1) 9.8 kN/m^3
G	=	soil particle density (t/m^3)
θ	=	bedding angle (angle of repose of soil particles)
η	=	Whites drag coefficient
d_{70}	=	sieve size for which 70% by weight of the soil (m) is finer
D	=	thickness of sand layer under the embankment (m)
L	=	seepage length (= base length of the embankment) (m)
K	=	intrinsic permeability (m^2)

Where $K = \frac{\upsilon}{g}k$

υ	=	kinematic viscosity (m^2/sec)
g	=	gravity (m/sec^2)
k	=	hydraulic permeability (m/sec)

Weijers and Sellmeijer (1993) assume η = 0.3, θ = 41 degrees. For water at 20^o Celsius, K = 1.02 x 10^{-7} (k) where K is m^2, k is in m/sec. They indicate that the theory gave good correlation with their small and large scale tests. The test results in Weijer and Sellmeijer (1993) and Silvis (1991) show that backward erosion initiated at average gradients from upstream to downstream 35% to 70% of the critical average gradients to cause backward erosion to initiate and progress to form a pipe from upstream to downstream. More data is given in Technical Advisory Committee (1999).

4.3 Schmertmann (2000) method for modelling initiation and progression of backward erosion

Schmertmann (2000) also carried out piping tests in flumes at University of Florida. The tests were carried out on a range of soils from fine to medium sands, up to coarse sand/fine gravel mixes. The soils were mostly fairly uniform (Cu = 1.5 to 6.1). The head differential was increased progressively to keep the tunnel progressing upstream until the critical head was reached causing the tunnel to form a pipe to the upstream source. He found that the critical average gradient $\bar{i}_{pmt}$ was strongly related to the uniformity coefficient Cu (d_{60}/d_{10}) of the soils tested. He also plotted the Delft tests and found a similar correlation.

The test geometries used at University of Florida and Delft were not the same, so Schmertmann (2000) applied correction factors for geometry and was then able to plot all the results together to give Figure 16 with his recommended design line. Schmertmann (2000) indicates it is conservatively drawn. He made some interesting observations from the tests:

(a) The pipe path meanders over the area, forming a braided system of small channels, not a single pipe.

(b) Detachment of particles from the face from which the particles are eroding occurs at very much lower gradients than required to scour (erode) the surface of a water-soil interface (40 to 90 times for the soils tested).
Schmertmann explains this to there being seepage gradients locally at the piping face equal to the critical gradient giving the zero effective stress condition. He demonstrated that such gradients could occur due to flow concentrations towards the pipe in three dimensions. The piping process is observed to be a retrogressive sliding process on the pipe head.

(c) Vibrations tended to reduce the critical gradient. (This may have implications for structures which are marginal and subject to earthquake – authors note).

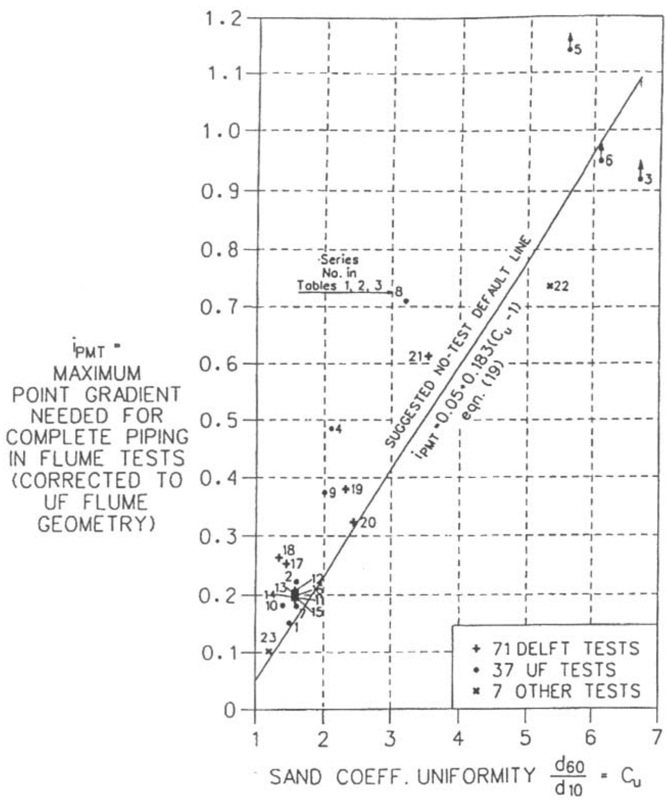


Figure 16. Maximum point gradient, i_{pmt}, needed for complete piping in University of Florida (UF), Delft and other flume tests versus uniformity coefficient of soil. (Note, UF tests L = 5.0 ft (1.52m) and D/L = 0.20) (Schmertmann 2000).

(d) There were no significant time effects in the Schmertmann (2000) experiments and he indicates others have found the same. When the head is increased, either erosion progresses, or it does not. Allowing additional time did not produce a delayed progression.

(e) The process seems to be independent of the effective confining stresses on the soil.

Schmertmann (2000) proposes a design method based around the results of the flume tests in Figure 16 which applies corrections for a number of factors to relate the laboratory tests to field conditions.

It is noted that the Delft tests had a slot at the toe of the bank to simulate cracking in the finer grained strata overlying the cohesionless soil, whilst the University of Florida tests had a sloping surface in the cohesionless soil. These differences may affect the average gradients at which backward erosion initiates but do not appear to influence the gradients at which the erosion progresses as the two sets of data seem to plot similarly on Figure 16. In practical terms this gradient may be estimated as $(H_2 - H_3)/L$ in Figure 17. This allows for the head losses through the clay layer.

4.4 Some field observations

The USACE have responsibility for managing the levee systems along major USA Rivers including the Mississippi. They have carried out extensive studies of backward erosion over many years. Some observations from these include:

(a) In any one flood the levee system in one USACE District, e.g. St Louis, may experience many hundreds of sand boils in the foundations. However there are few cases of breaching of the levees from piping. This is influenced by the “flood fighting” efforts of the Corps and the Levee District personnel. They build sand bags and in some cases sub-levees around the sand boils to stop the flow of sand (but not the flow of water) but it seems to support the laboratory tests which show erosion initiates at lower average gradients than is required to progress the erosion to form a continuous pipe from the river side to the land side.

(b) USACE (1956), Wolff (2002) and Shannon and Wilson (1995) record that sand boils often occur at average gradients defined as residual head at toe divided by thickness of the top strata (= $(H_3 - T)/(T)$ in Figure 17) between 0.5 to 0.8 compared to the theoretical critical gradient of 0.85. Shannon and Wilson (1995) record cases of sand boils with average gradients at the toe of 0.12 to 0.84. It appears from an initial review of these cases that most may have not been repeated sand boil activity so represent first time boils. The soils are very uniform fine grained sands so their behaviour is broadly consistent with what might be expected from the Schmertmann and Sellmeijer methods. More is to be done in the next few months to assess this better.

(c) Sills and Vroman (2007), Wolff (2002) and Glynn and Kusmaul (2004) report that there are cases of levees which have sand boil activity occurring at successively lower average gradients (lower river stages). This phenomenon does not appear to have been investigated by the laboratory flume tests.

(d) Glynn and Kusmaul (2004) show that greater sand boil activity occurred in the 1995 flood than the 1993 flood, even though the river stage was lower in 1995. It is not clear why this is so, but it may relate to the duration of the flood and the time it takes to set up a seepage flow net.

(e) USACE (1956) and Wolff (2002) show that local geology has an important influence on the occurrence of sand boils. Sand boils are more likely to occur where swales from point bar deposits cross the levee at an angle and concentrate seepage at the toe.

5 MECHANICS OF SUFFUSION IN COHESIONLESS SOILS

5.1 General description of the process

Suffusion is the process by which finer soil particles are moved through constrictions between larger soil particles by seepage forces. Soils susceptible to suffusion are usually described as internally unstable. Internally unstable soils are usually broadly-graded soils with particles from silt or clay to gravel size, whose particle size distribution curves are concave upward, or gap-graded soils.
For suffusion to occur, the following three criteria have to be satisfied:

Criterion 1: The size of the fine soil particles must be smaller than the size of the constrictions between the coarser particles, which form the basic skeleton of the soil.

Criterion 2: The amount of fine soil particles must be less than enough to fill the voids of the basic skeleton formed by the coarser particles. If there are more than enough fine soil particles for void filling, the coarser particles will be “floating” in the matrix of fine soil particles, instead of forming the basic soil skeleton.

Criterion 3: The velocity of flow through the soil matrix must be high enough to move the loose fine soil particles through the constrictions between the larger soil particles.

Suffusion occurring within an embankment core or the foundation of a dam will result in a coarser soil structure, leading to increased permeability and seepage, likely settlement of the embankment, and a higher likelihood of downstream slope instability which may result in failure of the dam. A filter constructed of internally unstable materials will have a potential for erosion of the finer

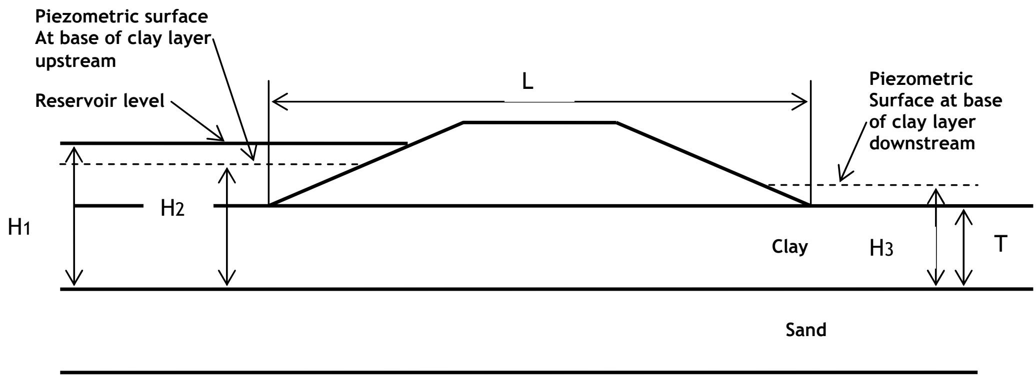


Figure 17. Section through embankment showing definition of terms

particles in the filter, rendering the filter coarser and less effective in protecting the core materials from erosion so piping failure may result.
It is the author's experience that most dam engineers in Australia and the USA had used the methods of Kenney and Lau (1985, 1986) or Sherard (1979) to assess whether the soils in a dam or its foundation are internally unstable, even if the soil is silt-sand-gravel, or clay-silt-sand-gravel. This is because there were no other well accepted methods. They did so not knowing if the methods they are using are conservative or otherwise. Wan and Fell (2004c, 2007) show that these methods are conservative for silty sandy gravel soils. In fact it is not surprising that the Sherard (1979) method does not predict well. It was empirically based and designed to identify soils which will not self-filter, which is a different process to internal instability as defined here.

5.2 Wan and Fell (2004c, 2007) method for assessing whether a soil is internally unstable

Wan and Fell found that the Burenkova (1993) method gave reasonable assessments of whether a soil was internally unstable. The Burenkova (1993) method is based on d_{90}/d_{60} and d_{90}/d_{15} ratios where d_{90} is the sieve size for which 90% of the sample by weight passes. The d_{90}/d_{60} ratio represents the slope of the coarse part of the particle size distribution plot. High values represents near single size coarse particles which will have large constriction spaces compared to a well graded soil. The d_{90}/d_{15} can be regarded as a measure of the filter action between the coarse fraction and the finer fraction.

The method does not give a clear-cut boundary between internally stable and unstable soils in the data set. To model this logistic regression was used by Wan and Fell (2004c, 2007) to define contours of equal probability of internal instability. Figures 18 and 19 show the contours and the logistic equations. Figure 18 is to be applied to silt-sand-gravel and clay-silt-sand-gravel mixtures with a plasticity index less than 13% and less than 10% clay size fraction (% passing 0.002 mm) and Figure 19 to sand-gravel soils with less than 10% non-plastic silt fines passing 0.075 mm.

5.3 Maximum fraction of erodible material in an internally unstable soil

Wan (2006) and Wan and Fell (2004c) give details of a method to determine what fraction of the soil which will be eroded. They have found that in practical terms it can be assumed that 50% of the finer fraction as defined by the point of inflection of broadly graded soils and the fine limit of the gap in gap-graded soils is eroded, and the particle size distribution re-plotted. This is based on the results of the laboratory testing. For a more conservative approach it may be assumed that all of the fine fraction is eroded.

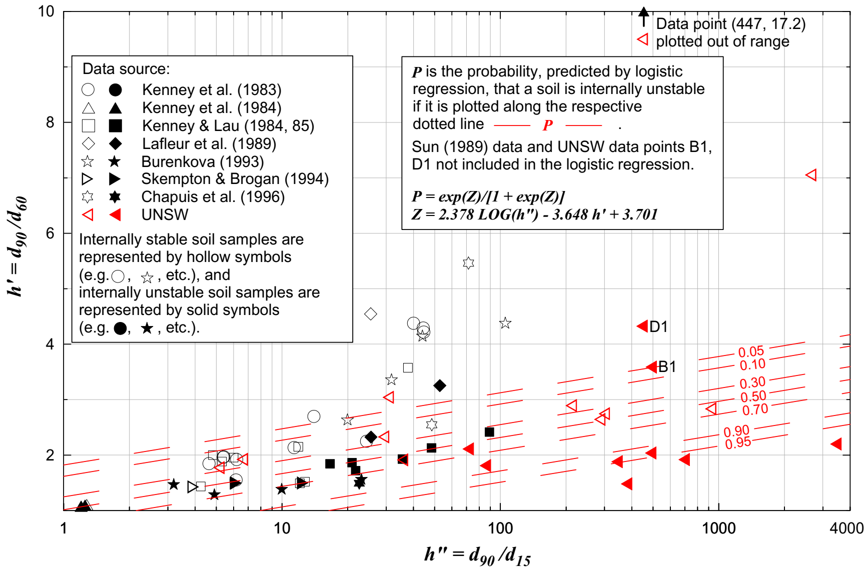


Figure 18 Contours of the probability of internal instability for silt-sand-gravel soils and clay-silt-sand-gravel soils of limited clay content and plasticity (Wan and Fell, 2004c, 2007).

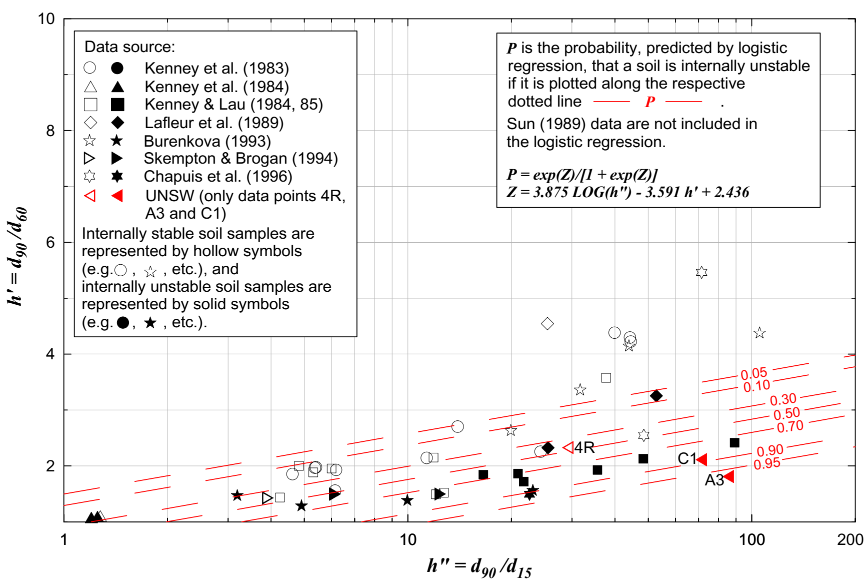


Figure 19 Contours of the probability of internal instability for sand-gravel soils with less than 10% non-plastic fines passing 0.075mm (Wan and Fell, 2004c, 2007).

5.4 Seepage gradient at which erosion will begin

Wan and Fell (2004c, 2007) laboratory tests and those by Skempton and Brogan (1994) show that erosion will begin in internally unstable cohesionless soils at exit seepage gradients lower than the critical or zero effective stress gradient. For the internally unstable soils tested, all began to erode with upward gradients of 0.8 or less, with several less than 0.3. There is a general trend that soils with a higher porosity begin to erode at lower hydraulic gradients. Loose, higher porosity soils tested began to erode at gradients less than 0.3. Soils with plastic fines required higher gradients to begin to erode. Gap-graded soils tended to begin to erode at lower gradients than non gap-graded soils with the same fines content.

6 CONCLUSIONS AND SOME PRACTICAL MATTERS

It is concluded that we are approaching the point where there are laboratory tests and analyses methods which can go a long way to explaining the physical processes of internal erosion and piping of dams and their foundations. Some matters of particular importance are:

- The main cause of internal erosion and piping in dam embankments is in concentrated leaks resulting from cracking and hydraulic fracture in low stress zones resulting from differential settlement. Most of this settlement occurs during construction and on first filling.
- The main causes of internal erosion and piping in dam soil foundations are backward erosion, often following heave of an overlying confining layer with subsequent development of sand boils. Suffusion is a less common cause and is more likely to result in a piping incident than breach. Erosion in cracks such as desiccation cracks left in the foundation may also occur.
- Backward erosion and suffusion are unlikely to occur in cohesive soils under the gradients which typically occur in embankment dams and their foundations. The cores of dams are mostly constructed of cohesive soils so these modes are seldom an issue within the embankment.
- Internal erosion and piping is common around conduits probably as a result of collapse compression of poorly compacted soil around the conduit. This mechanism can also cause piping in the embankment, particularly if the soil is poorly compacted at significantly dry of optimum moisture content.
- As recognised by Sherard (1973) cracking and low stress zones due to differential settlement are dependent on the geometry of the valley in which the dam is constructed, and will occur regardless of the method of compaction of the soil and the soil classification. They are most likely to occur where there are steps in the cross valley foundation profile, and / or if the valley slopes are steep. Cross valley numerical modelling of the stresses during construction is a good guide to whether such low stress zones will be present. These need to be based on the as-constructed foundation profile, not the design drawings, because the actual profile is often more irregular, and hence more likely to give low stress zones. The highest likelihood of cracking is usually in the upper parts of the embankment.
- In steep sided narrow valleys cross valley arching is likely to lead to low stresses low down in the embankment and conditions in which hydraulic fracture may occur. Arching in the embankment cross section is less likely unless the core is compacted dry of optimum moisture content leading to settlement on saturation; coupled with a very narrow core and stiff filters and/or rockfill shoulders.
- The Hole Erosion and Slot Erosion Tests are able to determine the hydraulic shear stress in cracks at which erosion will initiate, and the rate of erosion. They show there is a wide range of these properties in the soils used to construct the cores of dams. Some soils are particularly susceptible to initiation of erosion including dispersive soils and silty sands such as decomposed granite. Erosion in these soils will initiate in even narrow (e.g.1mm to 2mm) cracks under normal gradients in the core of dams. By contrast there are some soils which are very resistant to initiation of erosion. These are non-dispersive high plasticity clays, and medium plasticity clays which have an absence of any Smectite minerals. In these soils erosion is unlikely to initiate even if there is a 20mm wide crack.
- Testing by Lim (2006), Lim and Khalili (2007a, b) have shown that the main physical process occurring in erosion in cracks is slaking. The slaking process is enhanced by the presence of dispersive clay minerals in the soil, and is inhibited by saturation of the soil.
- The erosion properties of clay soils are very dependent on the degree of saturation. Saturation to above about 90% may result in the erosion rate being reduced by up to one or two orders of magnitude and the critical shear stress is also reduced making initiation of erosion less likely compared to the as-placed moisture condition. This means that it is far less likely that erosion will occur in the lower, saturated parts of the embankment than in the upper part above normal reservoir levels where the soils remain less saturated. Many older dams do not have any or well designed and constructed filters in the upper parts of the embankment, so these factors combine with the greater likelihood of cracking being in the upper part of the embankment to mean the most likely failure mode by internal erosion and piping is in the upper parts of the embankment under flood conditions.

- The rate of development of a pipe from the first sign of a concentrated leak to a pipe a metres diameter which would usually be followed by breach of the dam can be very rapid. It may be only a matter of minutes or hours for the most erodible soils such as silty sands and low plasticity clays and sandy clays. This is important when assessing the likelihood of intervention to prevent failure and the warning time for evacuating persons below the dam.
- Backward erosion may occur in cohesionless soils in the foundations provided there is a layer of more plastic soil overlying it (or the embankment is plastic) to form a roof for the pipe. The average gradient beneath the embankment at which backward erosion will initiate and progress is dependent on the uniformity coefficient, or the d_{70} size and permeability of the soil as well as the geometry of the foundation strata. For very uniform fine grained sands the average gradient to initiate and progress erosion can be as low as 0.1. For more well graded soils this average gradient approaches 1.0.
- Heave of an overlying low permeability strata is not a pre-requisite for backward erosion. The presence of boils is not a sign that backwards erosion will certainly progress to form a pipe from the upstream (river) side to the downstream (land side of an embankment (levee)) in a single reservoir or river loading. However there is evidence that repeated loading in successive high reservoir levels or floods can lead to initiation of boiling at successively lower levels indicating a deterioration effect which is not as yet quantifiable.
- Suffusion is not a common phenomenon in dam cores or foundations but should be considered. It is more likely to be an issue in filters, and if it occurs, the filters will be effectively coarser and may not be as effective against the embankment core. Whether a soil is subject to suffusion is dependent on the detail of the shape of the particle size distribution and there is some uncertainty in the available predictive methods.

It is now possible to assess in quantitative terms the likelihood of internal erosion and piping based largely on analysis and tests which model the physical process. There is however still a need to anchor the probabilities of cracking or hydraulic fracture through historic data.

It is important to apply engineering judgement and to not be overly reliant on the calculations outlined here. If the calculations give outcomes which are contrary to the judgement of experienced dam engineers then caution is appropriate.

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