Implementing anisotropy into a constitutive model for compacted unsaturated soils

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Keywords: Anisotropy, bounding surface plasticity, rotational hardening, suction hardening

ABSTRACT

Presented in this paper is a constitutive model for a compacted unsaturated soil that is formulated using bounding surface plasticity theory. Rotational hardening has been used to describe the evolution of the anisotropy caused by the compaction process and suction hardening has been used to describe the stiffening of the soil skeleton response and permit volumetric collapse upon wetting. The concept of effective stress is used and special attention is given to the effects of unsaturation and anisotropy in the definition of the critical state lines and compression lines. The model features are inspired by the stress-strain behavior observed in unsaturated speswhite kaolin compacted to two different densities and subjected to isotropic compression and conventional triaxial compression load paths while suction was held constant.

1 INTRODUCTION

Many compacted soils are unsaturated. During compaction the vertical stress the soil experiences may be different to the horizontal stress creating an anisotropic fabric. The subsequent stress-strain behaviour of the soil will then be influenced by this compaction induced anisotropy. The anisotropy may be altered during deformation as a limit state is approached and it is necessary to capture in the constitutive modelling of compacted unsaturated soils the evolution of the anisotropy if realistic simulations of stress-strain response are to be achieved. In this paper a constitutive model for a compacted unsaturated soil is detailed for conditions of axial symmetry. It is presented in a critical state framework using the concepts of effective stress, rotational hardening, suction hardening and bounding surface plasticity theory. A non associated flow rule is adopted.

2 EFFECTIVE STRESS AND STRESS-STRAIN RELATIONSHIPS

For axially symmetric conditions of the triaxial test conventional $p'-q$ notation is adopted. Compression is assumed positive and rate affects are ignored for simplicity. The mean effective stress p', the deviator stress q, the soil skeleton volumetric strain ε_p and the shear strain ε_q are correlated to axial and radial stresses and strains in the usual way. The mean effective stress (Bishop, 1959) is defined as $p' = p_n + \chi s$ where p_n is the total mean stress in excess of pore air pressure (u_a), the suction s is the difference between pore air and pore water pressures ($s = u_a - u_w$) and χ is the effective stress parameter. A well established relationship for χ is of the form $\chi = \left(s/s_e\right)^{-0.55}$ (Khalili and Khabbaz, 1998; Khalili *et al*, 2004), where s_e is the suction value separating saturated from unsaturated states, and is referred to as the air entry value when the soil undergoes drying or the air expulsion value when the soil undergoes wetting. The existence of χ and its dependence on material properties of the soil are not in contravention of continuum mechanics. In fact it arises from such principles due to the multi-phase and multi-scale nature of unsaturated soil mechanics (Khalili *et al*, 2000, Khalili *et al*, 2004). Volumetric strain is linked to specific volume (v) according to $\dot{\varepsilon}_p = -\dot{v}/v$, where $v = 1 + e$ and e is the voids ratio. The elastic-plastic stress-strain relationship is of the usual form:

$$\dot{\boldsymbol{\sigma}} = \left(\mathbf{D}^e - (\mathbf{D}^e\mathbf{m}\mathbf{n}^T\mathbf{D}^e)/(h + \mathbf{n}^T\mathbf{D}^e\mathbf{m})\right)\dot{\boldsymbol{\varepsilon}} \qquad (1)$$

where $\mathbf{n} = \left[n_p, n_q\right]^T$ is the unit normal vector at the current stress state controlling the direction of loading, $\mathbf{m} = \left[m_p, m_q\right]^T$ is the unit direction of plastic flow at the current stress state, h is the hardening modulus and $\mathbf{D}^e$ is the isotropic elastic stiffness matrix. The bulk modulus and shear modulus in $\mathbf{D}^e$ are defined in terms of Possion's ratio ν and κ in the usual way, such that isotropic elastic unloading and reloading would reveal linear relationships of slope κ in a semi-logarithmic compression (v - $\ln p'$) plane.

3 MODEL DEVELOPMENT

The model features are inspired by data presented in Sivakumar and Wheeler (2000) and Wheeler and Sivakumar (2000). The data were obtained from an experimental study of the stress-strain behaviour of compacted unsaturated speswhite kaolin involving a series of isotropic and triaxial compression tests performed at constant suction. However, the modelling framework presented is sufficiently general such that may be applied to compacted soils containing other clay minerals.

3.1 The critical state

Critical state conditions were approached in the triaxial tests and the data points for the end of the tests are plotted in the q - p' plane in Figure 1. A single value of $s_e = 85\text{kPa}$ was used to convert stresses to effective even though samples were prepared using a range of compaction methods. The data appear to obey a unique relationship in the q - p' plane providing evidence that s_e is a material constant at the critical state. The compaction induced anisotropy may influence the stress-strain behaviour well before the critical state is approached, but the compaction method has no influence on the stress-strain behaviour at the critical state. It is noted that a second type of anisotropy may be induced by the shearing process as the critical state is approached. This has been ignored here for simplicity and to focus on modelling compaction induced anisotropy.

The isotropic compression tests results provide evidence that suction influences the location of the isotropic compression lines and therefore critical state line (CSL) in the $v - \ln p'$ plane (Figure 2). Following the ideas of Loret and Khalili (2000, 2002) and Russell and Khalili (2006) this can be attributed to suction hardening, an isotropic hardening phenomenon that controls the size of a bounding surface in addition to plastic volumetric strains, and therefore the location of the CSL. A general definition of the CSL for saturated conditions in the $v - \ln p'$ plane is adopted here of the form $v = f_{cs}(p')$ where f_{cs} is a function unique for a given soil. A general definition of the CSL in the $v - \ln p'$ plane for unsaturated conditions is $v = f_{cs}(p', s)$. In the q - p' plane the CSL is linear and passes through the origin (Figure 1). The slope, denoted as M_{cs}, is linked to the critical state friction angle φ'_{cs} according to $M_{cs} = q/p' = 6\sin\varphi'_{cs}/(3t - \sin\varphi'_{cs})$ and is independent of suction. Note that t =+1 and $M_{cs} = M_c$ for compressive loading and t =-1 and $M_{cs} = M_e$ for extensive loading.

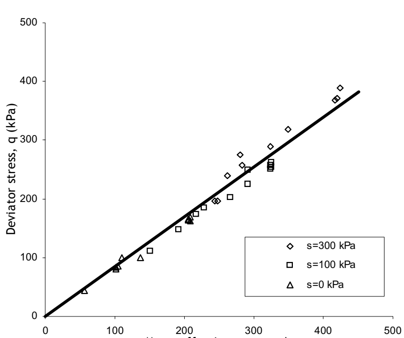

Figure 1: End points of a number of triaxial compression tests (Wheeler and Sivakumar, 2000)

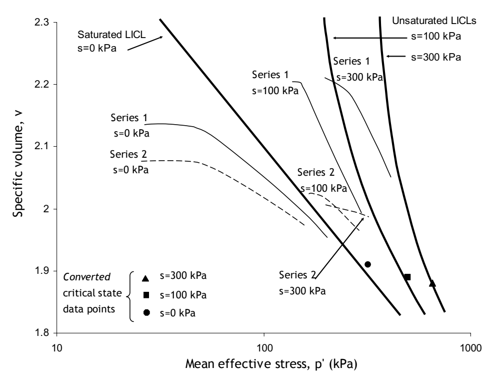

Figure 2: Isotropic compression tests, converted critical state data points (Sivakumar and Wheeler, 2000)) and LICLs as adopted in the model

3.2 Loading surface and bounding surface

During elastic-plastic deformation the current stress state is located on a loading surface. The loading surface is enclosed by a bounding surface. Simple and versatile functions for the loading and the bounding surfaces are proposed in the $q-p'$ plane, such that the two surfaces are of the same shape and homologous about the origin (Figure 3). An image point is located on the bounding surface and is denoted using $\bar{\boldsymbol{\sigma}} = [\bar{p}', \bar{q}]^T$ (where an overbar indicates the association with the bounding surface). Its location is controlled using a simple radial mapping rule such that a straight line through the centre of homology and $\boldsymbol{\sigma}$ intersects the bounding surface at $\bar{\boldsymbol{\sigma}}$. The unit normal vectors at $\boldsymbol{\sigma}$ and $\bar{\boldsymbol{\sigma}}$ are therefore the same. The specific functions for the loading and the bounding surfaces are respectively:

$$f = t\{q/p' - M_{cs}[\ln(p'_{ci}/p')/\ln(R)]^{1/N} - \beta\} = 0 \qquad F = t\{\bar{q}/\bar{p}' - M_{cs}[\ln(\bar{p}'_{ci}/\bar{p}')/\ln(R)]^{1/N} - \beta\} = 0 \qquad (2)$$

The shapes of the surfaces are similar to those proposed by Russell and Khalili (2006), but rotated and distorted by introducing β. Plastic anisotropy is associated with β. Changes to β, which in turn causes change to the amount of rotation and distortion, is referred to as *rotational hardening*. This is a well established concept and recent examples of its use include Gajo and Muir Wood (2001) and Dafalias *et al* (2006). On the bounding surface $\bar{p}'_{ci}$ represents an isotropic hardening variable while β represents a rotational hardening variable. A soil is said to be isotropic when $\beta = 0$. R and N are material constants, the first represents the ratio between $\bar{p}'$ at the intercept of the bounding surface with the M_{cs} line and $\bar{p}'_{ci}$ for $\beta = 0$, and the second controls the curvature of the surface. Notice that $M_{cs} = M_c$ when the stress ratio $q/p' > \beta$ and $t = +1$, $M_{cs} = M_e$ when the stress ratio $q/p' < \beta$ and $t = -1$.

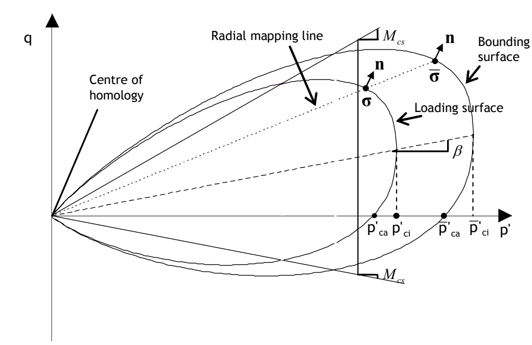

Figure 3: Loading surface, bounding surface and radial mapping line in the $q-p'$ plane

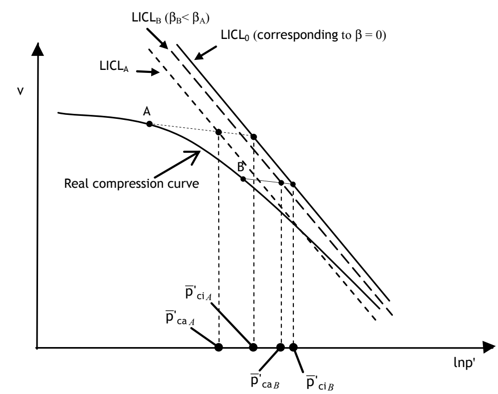

Figure 4: LICLs and a compression curve in the $v-\ln p'$ plane for constant suction

The unit normal vector at $\bar{\boldsymbol{\sigma}}$ defining the direction of the loading is then $\mathbf{n} = (\partial F/\partial \bar{\boldsymbol{\sigma}})/\|\partial F/\partial \bar{\boldsymbol{\sigma}}\| = (\partial f/\partial \boldsymbol{\sigma})/\|\partial f/\partial \boldsymbol{\sigma}\|$ and t is required to ensure that $\mathbf{n}$ always points outwards from the bounding surface during monotonic loading. The intercept of the bounding surface with the p' axis, denoted as $\bar{p}'_{ca}$, is linked to $\bar{p}'_{ci}$ by $\bar{p}'_{ci}/\bar{p}'_{ca} = \exp[(-\beta/M_e)^N \ln R] = \hat{R}$. $\bar{p}'_{ca}$ can be thought of as an hardening variable that combines anisotropic and isotropic mechanisms. However, in the formulations that follow, it is simpler to keep the isotropic hardening variables $\bar{p}'_{ci}$ and s separated from the anisotropic hardening variable β.

3.3 Limiting isotropic compression lines

An important feature of a bounding surface plasticity models is the existence of limiting isotropic compression lines (LICLs) in the $v-\ln p'$ plane. A particular LICL represents the upper envelope to load paths followed by isotropically compressed samples (that is compression while $q = 0$) of any initial density during which both suction and anisotropy are constant. In short, there are an infinite number of LICLs in the $v - \ln p'$ plane for different values of s and β. Only one of them is for the s = 0 and β = 0 condition. Russell and Khalili (2006) explain the roles of the LICLs in a model for unsaturated soils that does not account for anisotropy. Now the roles of LICLs in this anisotropic model will be explained by considering the load path followed by a sample loaded in compression with constant suction as shown in Figure 4. Consider first the sample at location A, that is when the mean effective stress, deviatoric stress, specific volume, suction and anisotropy are p'_A, q_A, v_A, s and β_A, respectively. As the load path passes through point A the parameter p'_{caA} which controls the size of the loading surface at A tends to approach $LICL_A$ (corresponding to s and β_A). More specifically, the intercept of a κ line passing through the coordinate (p'_A, v_A) in the $v - \ln p'$ plane with $LICL_A$ defines $\bar{p}'_{caA}$ and the size of the bounding surface. The intercept of the same κ line with $LICL_0$ (that is the LICL corresponding to $\beta = 0$) represents $\bar{p}'_{ciA}$. As loading continues to point B, with variables denoted as p'_B, q_B, v_B, s and β_B, the $LICL_B$ towards which p'_{caB} approaches is now defined by s and β_B. As loading continues beyond B and if anisotropy is gradually erased, β approaches 0 and the loading surface and bounding surface coincide ($p'_{ca} = \bar{p}'_{ca}$). Also, p'_{ca} finally reaches $LICL_0$. Suction influences the location of the $LICL_0$, as suction is an isotropic phenomenon.

For unsaturated and isotropic conditions the LICL can be expressed as $v = f_{cs}((p' - \gamma(s))/R) - \kappa \ln(Rp'/(p' - \gamma(s)))$ where $\gamma(s)$ represents the separation of the saturated and unsaturated LICLs along the κ line in the $v - p'$ plane (Russell and Khalili, 2006). For unsaturated and anisotropic conditions the LICL can be expressed as $v = f_{cs}\left((\hat{R}p' - \gamma(s))/R\right) - \kappa \ln\left(Rp'/(\hat{R}p' - \gamma(s))\right)$. As the location of the LICL is dependant on s and β, and when β becomes zero at the critical state, the location of the CSL is only a function of s. Assuming R is a material constant the unsaturated CSL is defined by $v = f_{cs}(p' - \gamma(s)/R) - \kappa \ln(p'/(p' - \gamma(s)/R))$.

3.4 Plastic potential, loading index and plastic modulus

The plastic potential defines the ratio between the incremental plastic volumetric strain and the plastic shear strain and the form used here is the same as that used by Russell and Khalili (2006):

$$d = \dot{\varepsilon}_p^p / \dot{\varepsilon}_q^p = g_{,p} / g_{,q} = \left(1.7\langle Rp'/\bar{p}'_c - 1\rangle + 1\right)M_c - \eta \qquad \mathbf{m} = (\partial g/\partial \boldsymbol{\sigma}) / \|\partial g/\partial \boldsymbol{\sigma}\| = \left[d/\sqrt{1+d^2}, 1/\sqrt{1+d^2}\right]^T \tag{3}$$

in which the symbol $\langle\ \rangle$ ensures that $\langle x \rangle = x$ when $x > 0$ and $\langle x \rangle = 0$ when $x \le 0$. As is normal in bounding surface plasticity theory the hardening modulus is split into two components $h = h_b + h_f$, where h_b is the modulus at $\bar{\boldsymbol{\sigma}}$ on the bounding surface and h_f is some arbitrary modulus at $\boldsymbol{\sigma}$, assumed to be of the form:

$$h_f = k_m \left(\frac{\bar{p}'_{ci} - p'_{ci}}{p'_{ci}} \right) \frac{p'}{\bar{p}'_{ci}} \left(\frac{\partial \bar{p}'_{ci}}{\partial \varepsilon_p^p} + \frac{\partial \bar{p}'_{ci}}{\partial s} \frac{\dot{s}}{\dot{\varepsilon}_p^p} \right) \tag{4}$$

where k_m is a scaling parameter controlling the steepness of the response in the $q - \varepsilon_q$ plane. For saturated anisotropic conditions h_b can be thought of as the sum of isotropic and rotational hardening:

$$h_b = -\left[F_{,\bar{p}'_{ci}} \bar{p}'_{ci,\varepsilon_p^p} m_p / \|\partial F/\partial \bar{\boldsymbol{\sigma}}\| + F_{,\bar{p}'_{ci}} \bar{p}'_{ci,\bar{p}'_{ca}} \bar{p}'_{ca,\beta} \left(\beta_{,\varepsilon_p^p} m_p + \beta_{,\varepsilon_q^p} m_q \right) / \|\partial F/\partial \bar{\boldsymbol{\sigma}}\| \right] \tag{5}$$

For unsaturated anisotropic conditions $\bar{p}'_{ci}$ undergoes isotropic hardening with changes in suction as well as plastic volumetric strains. It follows that:

$$h_b = -\left[F,_{\bar{p}'_{ci}} \left(\bar{p}'_{ci},_{\varepsilon_p^p} + \bar{p}'_{ci},_s \frac{\dot{s}}{\dot{\varepsilon}_p^p} \right) m_p / \left\| \partial F / \partial \bar{\boldsymbol{\sigma}} \right\| + F,_{\bar{p}'_{ci}} \bar{p}'_{ci},_{\bar{p}'_{ca}} \bar{p}'_{ca},_\beta \left(\beta,_{\varepsilon_p^p} m_p + \beta,_{\varepsilon_q^p} m_q \right) / \left\| \partial F / \partial \bar{\boldsymbol{\sigma}} \right\| \right] \quad (6)$$

The definition of β adopted here is of the form $\beta = \beta_0 / (1 + c\varepsilon_q)$ where β_0 represents the initial anisotropy induced by the compaction method and c is a material constant. This ensures that β approaches zero as the critical state is approached. Other more elaborate definitions for β could be adopted within the formulation presented above, such as one which involves a change of β with volumetric as well as shear strains, but a simple definition is preferred here.

4 MODEL SIMULATIONS OF TRIAXIAL TEST RESULTS

Figure 5 shows drained triaxial test results of Wheeler and Sivakumar (2000) in the $q - \varepsilon_1$ plane. The results correspond to two series of controlled suction triaxial tests, for which samples were created using two different compaction procedures. Series 1 involved static compaction to a vertical pressure of 400kPa at a water content of 25%. In series 2 the water content was also 25% but the static compaction pressure was 800kPa. Samples were then isotropically consolidated (that is while $q = 0$) to $p_n = 150$kPa and three suction values: $s = 0$, 100kPa and 300kPa. After isotropic consolidation, each sample was sheared in conventional triaxial compression while holding suction constant.

Model simulations are also shown in Figure 5. In generating the simulations it was assumed that the saturated CSL the $v - \ln p'$ plane may be defined by $v = \Gamma_0 - \lambda_0 \ln p'$ with $\lambda_0 = 0.18$ and $\Gamma_0 = 2.85$. Values of $s_e = 85$kPa and $\varphi'_{cs} = 21.9^\circ$ were found to define a unique CSL in the $q - p'$ for both saturated and unsaturated conditions. The parameters $\kappa = 0.015$, $\nu = 0.45$, $N = 1.4$ and $R = 1.6$ were also found to be appropriate. The suction dependant shift $\gamma(s)$ of the LICL was found to be $\gamma(s) = 0$, 170kPa and 340kPa for s = 0, 100kPa and 300kPa, respectively. The expression of Russell and Khalili (2006) for k_m in the hardening modulus was also adopted:

$$\log_{10} k_m = \left[1.45 \left(p'_0 R / \bar{p}'_{ci0} \right) - 0.32 \right] \quad (7)$$

where the subscript 0 indicates the initial condition of the subscripted variables. Initial states of anisotropy for the two compaction procedures were defined by $\beta_0 = 0.4$ for series 1 and $\beta_0 = 0.3$ for series 2. For both series the constant c = 30 was adopted which controls the rate of decay of the anisotropy.

Good agreement between experimental results and model simulations is observed. An improved fit may be obtained through a more elaborate definition of β which controls the evolving anisotropy. However, an experimental data set more complete than that used here would be needed to explore this further.

5 CONCLUSIONS

A constitutive model based on critical state concepts and bounding surface plasticity theory has been developed to simulate the evolving anisotropy in a compacted unsaturated soil. The model is an extension of the bounding surface plasticity model presented by Russell and Khalili (2006) for unsaturated soils; the extension consists of the introduction of the parameter β which controls the rotation, distortion and subsequent anisotropy of the stress-strain response. The model employs versatile functions for the loading surface and the bounding surface. The model formulation is simplified by separating isotropic hardening due to changes in suction and volumetric strain from rotational hardening due to changes in shear strain. The fit between model outputs and triaxial compression test results is good. Formulating the model in the multi axial stress space will require modifications, including more elaborate definitions of M_{cs} and β of which those presented above become special cases.

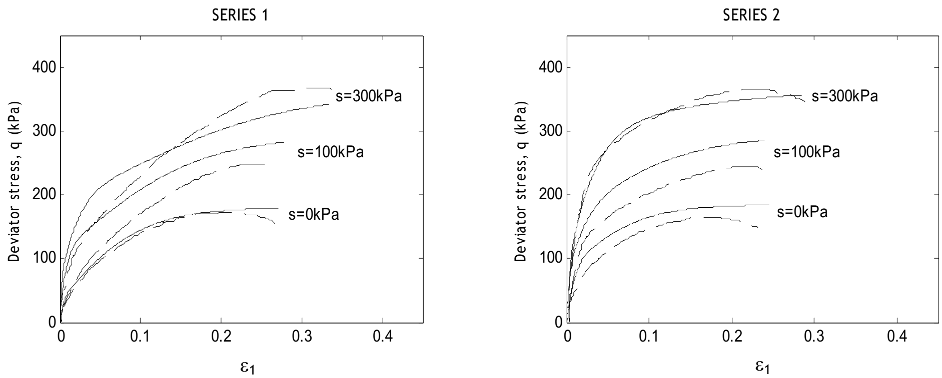

Figure 5: Experimental results (dashed lines) and model simulations (continuous lines) for constant suction triaxial compression tests in speswhite kaolin.

ACKNOWLEDGEMENTS

This research was funded by the UK's EPSRC, project title: Biological & Engineering Impacts of Climate Change on Slopes - BIONICS. The financial support of the EPSRC is gratefully acknowledged.

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