# Enhanced numerical analysis of ground behaviour influenced by tree root suction

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## ABSTRACT

Tree roots provide three stabilising functions: (a) reinforcement of the soil, (b) dissipation of excess pore pressures and (c) establishing sufficient matric suction to increase the shear strength. This paper looks at the way that vegetation influences soil moisture content distribution, and ground settlement. A theoretical model previously developed by the authors for the rate of tree root water uptake together with an associated numerical simulation is used in this study. Field measurements taken from literature published previously are compared with the authors' numerical model. The predicted results obtained from the numerical analysis, compared favourably with the field measurements, justifying the assumptions upon which the model was developed.

## 1 INTRODUCTION

Emergent public concern over slope appearance from mid 1990's has encouraged development of a systematic and integrated approach to the application of native vegetation especially in slope stabilisation. Initially, bioengineering applications including vegetation in civil engineering started in slope stabilisation and erosion control. One engineering structures whose behaviour is highly influenced by hydrological and environmental conditions is a railway line. Given the lengthy rail networks in coastal areas of Australia, numerous rail tracks have been built on clayey soil formations that are moisture sensitive. Therefore, bioengineering methods of ground improvement are becoming increasingly popular in Australia for stabilising railway corridors.

Although field measurement of soil moisture content and suction in the vicinity of vegetation has drawn the attention of soil scientists and engineers, analytical and numerical solutions of moisture flow equation have not been interesting to them. The reason is that there has not been any two or three dimensional root water uptake model which could capture the root water uptake, considering the soil suction, root distribution and atmospheric conditions as key factors. Furthermore, by introducing such a model into the unsaturated flow equation, a highly nonlinear partial differential equation results, which is difficult to solve analytically. The proper initial and boundary conditions of the model also introduce extra complexities. Moreover, coupled flow and deformation modelling is required to determine the real behaviour of the ground under transpiration effect, when the water flows through a porous media.

Fredlund and Hung (2001) conducted a numerical flow and deformation analysis employing a one dimensional root water uptake, changing linearly from the maximum value on soil surface under the tree trunk to the zero value at depth $z_{max}$. The solution of moisture flow equation as well as stress and displacement analysis was obtained using PDEase2D differential equation solver. Stress state variable method was used in their analysis to consider the volume change behaviour of an unsaturated soil. Although they have conducted a coupled flow and deformation analysis which is the first attempt in this area, a realistic root water uptake and root zone shape have not been considered. In fact, soil suction is a limiting factor for the root water uptake. Vertical and horizontal distribution of roots determine the root water uptake distribution, which need to be considered in the analysis. They have assumed that the root water uptake rate is time independent, which is not a realistic assumption.

## 2 THEORITICAL CONSIDERATION

The loss of moisture from the soil may be categorised as: (a) water used for metabolism in plant tissues and (b) water transpired to the atmosphere. However, as suggested by Radcliffe et al. (1980) the volume of water required for photosynthesis or metabolism in plant tissues compared to the total water uptake by roots is negligible. The total transpiration can then be assumed to be the same as the water uptake through the root zone. Therefore, the key variable for estimating the transpiration rate is the rate of root water uptake, which depends on the geological, hydrological and meteorological conditions. Figure 1 shows a schematic illustration of the soil-plant-atmosphere interaction. The rate of transpiration depends on the rate of root water uptake, hence:

$$T(t) = \int_{V(t)} S(x, y, z, t)\,dV \tag{1}$$

where, $T(t)$ is the transpiration rate at time $t$, $S(x, y, z, t)$ is the root water uptake at point $(x, y, z)$ at time t and, if $V(t)$ is the volume of root zone at time t, $dV$ denotes a small volumetric change.

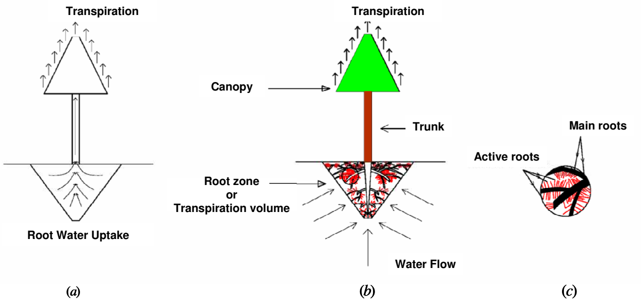

Figure 1: Schematic sketch of soil-plant-atmosphere system, (a) transpiration (b) soil-plant-atmosphere interaction and (c) active and main roots

The details of each single root and its interaction with the surrounding soil is required to identify the microscopic interaction between the soil and the root system (Figure 1). In this study a macroscopic approach is adopted, which considers the integrated properties of the entire root system, assuming that both soil and roots form a continuous medium. Therefore, the root water uptake is considered as a volumetric sink term in the flow continuity equation, which can be defined as the volume of water extracted per unit bulk volume of soil per unit time. The soil water flow differential equation, including the sink term, $S(x, y, z, t)$, can then be written as (Fatahi and Indraratna, 2006):

$$\frac{\partial \theta}{\partial t} = \nabla.(k \nabla \psi) - \frac{\partial k}{\partial z} - S(x, y, z, t) \tag{2}$$

where, $\theta = V_W / V$ is the volumetric moisture content, $V_w$ =volume of water, $V =$ total volume, $\nabla$ is the divergence vector, $\psi$ is the soil suction, $k$ is the hydraulic conductivity, and $z$ is the vertical coordinate (downward is positive).

Previous research by Indraratna et al. (2006) attempted to develop a mathematical model for the distribution of tree root water uptake within the root zone. This proposed model combining interactions of soil matric suction, root density and potential transpiration rate, has been used in this study. Accordingly, a mathematical model is formulated to represent the rate of tree root water uptake as:

$$S(x, y, z, t) = f(\psi).G(\beta).F(T_P) \tag{3}$$

where, $G(\beta)$ is the root density factor, $f(\psi)$ is the soil suction factor, and $F(T_P)$ is the potential transpiration factor. To calculate $f(\psi)$, different approaches have been recommended by various researchers. The equation suggested by Feddes et al. (1978) which is a simple and appropriate formula to determine soil suction effects, is used in this study (Equation (4)).

$$\left\{\begin{array}{ll} f(\psi)=0 & \psi<\psi_{an} \\ f(\psi)=1 & \psi_{an}\le\psi<\psi_{d} \\ f(\psi)=\dfrac{\psi_w-\psi}{\psi_w-\psi_d} & \psi_d\le\psi<\psi_w \\ f(\psi)=0 & \psi_w\le\psi \end{array}\right\} \tag{4}$$

where, $\psi_w$ is the soil suction at wilting point, $\psi_d$ is the highest value of $\psi$ and $\psi_{an}$ is the lowest value of $\psi$ at $S=S_{\max}$, where $S_{\max}$ is the maximum rate of root water uptake. The following two equations for the root density factor and potential transpiration factor, respectively, are suggested by the authors,

$$G(\beta)=\frac{\tanh(k_3\beta_{\max}e^{-k_1|z-z_0|-k_2|r-r_0|})}{\int_{V(t)}\tanh(k_3\beta_{\max}e^{-k_1|z-z_0|-k_2|r-r_0|})dV} \tag{5}$$

$$F(T_P)=\frac{T_P(1+k_4z_{\max}-k_4z)}{\int_{V(t)}G(\beta)(1+k_4z_{\max}-k_4z)dV} \tag{6}$$

where, $k_1$ and $k_2$ are two empirical coefficients depending on the tree root system and type, $k_3$ is an experimental coefficient, $z$ is the vertical coordinate, $r$ is the radial coordinate, $\beta_{\max}$ is the maximum density of root length located at the point $(r_0, z_0)$, $T_p$ is the rate of potential transpiration, $k_4$ is an experimental coefficient to involve depth on the potential transpiration distribution, and $V(t)$ is the volume of the root zone at time t. Borg and Grims (1986) reviewed the root growth data of 48 crop species and found that increase of root zone dimensions with time delineates a sigmoidal curve, which is a single sine function. Therefore,

$$z_{\max}(t)=z_{\max f}\{0.50+0.5\sin\{3.03(t/t_f)-1.47\}\} \tag{7}$$

$$r_{\max}(t)=r_{\max f}\{0.50+0.5\sin\{3.03(t/t_f)-1.47\}\} \tag{8}$$

where, $z_{\max}(t)$ is the maximum depth of root zone at time $t$, $r_{\max}(t)$ is the maximum lateral distance of root zone at time $t$, $z_{\max f}$ is the maximum possible root zone depth, $r_{\max.f}$ is the maximum possible lateral distance of root zone and $t_f$ is the time that tree's growth stops and root zone reaches to its maximum. Discussion about the influence of the above parameters on the rate of root water uptake (Equation 3) can be found in Fatahi and Indraratna (2006).

## 3 VALIDATION OF THE MODEL – CASE STUDY

This case study is related to the results of the field moisture content measured in the vicinity of a single, 20m high Poplar tree in Cambridge (U.K), as reported by Biddle (1998). The tree is located in an area of mown grass on Gault clay, and forms part of a double row of Poplars. According to Ng et al. (1998), the Gault clay is over consolidated and consists of stiff to hard grey silty clay of high plasticity. Table 1 shows the estimated parameters used in the finite element analysis, based on data available in literature. As reported by Biddle (1998), all soil moisture measurements were conducted using a neutron soil moisture gauge. Also, four access tubes were inserted at varying distances along a single radius, where possible. Radial distances from the tree were 4.8m, 9.5m, 19m, and 28.5m. A tube was inserted into the ground for comparison purposes, 57m away (more than twice the height of the tree), and is assumed to be unaffected by root suction.

This numerical analysis is based on the effective stress theory of unsaturated soils incorporated in the ABAQUS finite element code. The effective stress in the unsaturated soil is given by Bishop (1959):

$$\sigma'_{ij}=\sigma_{ij}-u_a\delta_{ij}+\chi(u_a-u_w)\delta_{ij} \tag{9}$$

where, $\sigma'_{ij}$ is the effective stress of a point on a solid skeleton, $\sigma_{ij}$ is the total stress in the porous medium at the point, $u_a$ is the pore air pressure, $u_w$ is the pore water pressure, $\delta_{ij}$ is Kronecker's delta ($\delta_{ij} = 1$ when $i = j$ and $\delta_{ij} = 0$ when $i \neq j$), and $\chi$ is the effective stress parameter attaining a value of unity for saturated soils and zero for dry soils.

Table 1: Parameters applied in the finite element analysis

| Parameter | Value | Reference/ Comments |
|---|---|---|
| $\psi_w$ | 1500 kPa | Feddes et al. (1978); $1500 < \psi_w < 2000$ kPa |
| $\psi_d$ | 40 kPa | Feddes et al. (1978); $40 < \psi_d < 80$ kPa |
| $\gamma$ | 20 kN/m$^3$ | Samuels (1975); Typical unit weight for Gault clay |
| $r_{\max\ f}$ | 20m | Biddle (1998); Estimated from field measurements $(17m < r_{\max} < 23m)$ |
| $z_{\max\ f}$ | 1.5m | Biddle (1998); Estimated from field measurements |
| $(k_s)_h$ | $5\times10^{-9}$ m/s | Terzaghi et al. (1996); Typical saturated permeability of unfissured clay |
| $PI$ | 41 | Biddle (1998); Measured plasticity index |
| $C_s$ | 0.023 | Ng (1998); Swelling index of heavily over consolidated clay |
| $k_3\beta_{f\cdot\max}$ | 2.18 m$^{-3}$ | Taken from the general shape suggested by Landsberg (1999) |
| $k_4$ | 0.014 | Coefficient of potential transpiration distribution |
| $k_{1f}$ | 5 | Coefficient of vertical root distribution |
| $k_{2f}$ | 0.50 | Coefficient of horizontal root distribution |
| $T_P$ | 45 l/day | Schneider et al. (2002) |
| Passing #200 | 95% | Typical value for Gault clay (5% fine sand) |

A two dimensional finite element analysis was used to predict the distribution of soil moisture content in the vicinity of the tree. The finite element mesh and specified boundary conditions is shown in Figure 2. Because of symmetry, a zero flux boundary was applied along the left boundary. As there was no meteorological data available for the site, similar to Indraratna et al. (2006) analysis, it is assumed that rainfall and evaporation can compensate each other, thus, a "no water in-flow" condition is applied on the soil surface. The root water uptake model was implemented in the numerical scheme via Visual Fortran sub-routines. The main sub-routine includes the rate of root water uptake as a moisture flux boundary applied to the top side of all elements within the root zone. In other words, Equation (3) incorporating Equations (4) - (8), has been implemented in the numerical model as boundary flux, which can determine the rate of root water uptake within the root zone at each increment of time. The mesh used in this simulation contains bi-linear strain quadrilateral elements (CPE4P) with 4 displacement and pore pressure nodes at the corner of each element. The entire FE mesh consists of 13,041 nodes and 12,800 elements.

The finite element analysis is conducted in two stages: (i) geostatic and (ii) consolidation. The first stage is to ensure that the analysis commences from a state of equilibrium under geostatic loading. The consolidation stage is to avoid non-physical oscillations and possible divergence problems caused by non-linearities. This stage includes a transient analysis of partially saturated soil under transpiration, starting with 1-day intervals and then continued for five months from the middle of spring until the end of September

According to the field measurements of soil moisture content reported by Biddle (1998), the initial matric suction has been assumed to be hydrostatic (Figure 2). The curve of the soil water characteristic used in this analysis has been based on a relationship suggested by Zapata et al. (2000) for different $w \times PI$, where $w$ is the fraction of soil passing sieve #200 (75 $\mu m$) as an index between 0 to 1, and $PI$ is the plasticity index. The coefficient of soil permeability ($k$) is described by Brooks and Corey (1964) as:

$$k = k_s(e) s_e^{\frac{2+3\lambda}{\lambda}} \tag{10}$$

where, $k_s(e)$ is the saturated coefficient of permeability estimated based on Kozeney-Carman equation, $S_e$ is the effective degree of saturation, and $\lambda$ $(= \Delta \log S_e / \Delta \log \psi)$ is the slope of the soil water characteristic curve on a log-log plot.

Deformation in the soil profile due to the root water uptake was predicted through a coupled flow-deformation analysis. Ground settlement at various depths after five months of continuous transpiration is shown in Figure 3. The ground settlement presented, decreases rapidly up to 12m and settlement also decreases rapidly with depth. As mentioned by Schneider et al. (2002), transpiration shifts the ground water table toward the surface, as a consequence, decreases the effective stresses in deeper layers of soil. Therefore, as Figure 3 demonstrates, swelling of the soil is predicted below 2.5m. In the top 2.5m, the maximum deformation is at $r = r_0$, while below that, it is between $r = 0$ and $r = r_0$. At the surface, 8.8mm of vertical settlement at the tree is observed, which increases to 9.2mm, 3m away, and then decreases sharply to 2mm, 13.5m away.

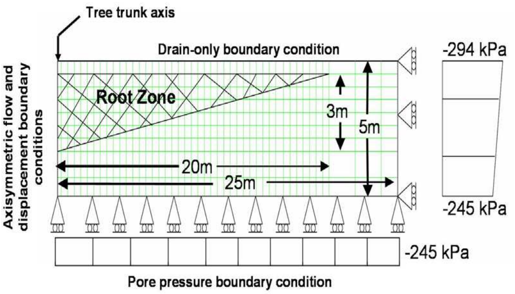

Figure 2: The geometry and boundary conditions of the FE model (After Fatahi and Indraratna, 2006)

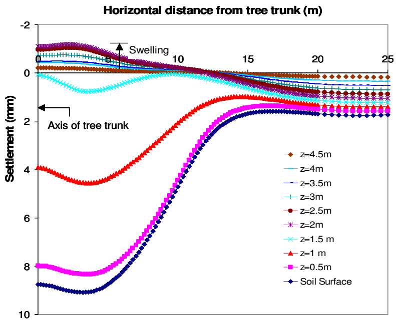

Figure 3: Ground settlement profile at various depths

Figure 4 shows the comparison between the field measurements and numerical predictions for a reduction in moisture content. Numerical analysis predictions based on the authors' model for root water uptake are in acceptable agreement with the field measurements in September reported by Biddle (1998). It is important to note that in the numerical analysis, root water uptake as a sink term were considered in the flow equation, but the effect of individual root was not considered. As the main roots penetrate the soil, there may be a gap between them which can lead to water collecting in the gap. Since the woody roots are in a denser pattern under the trunk and in close proximity, a disparity between the field measurements and predictions in this area seems more likely. Furthermore, the actual field data is probably influenced by the soil heterogeneity. In the numerical model, only the matric suction was considered and osmotic suction was neglected.

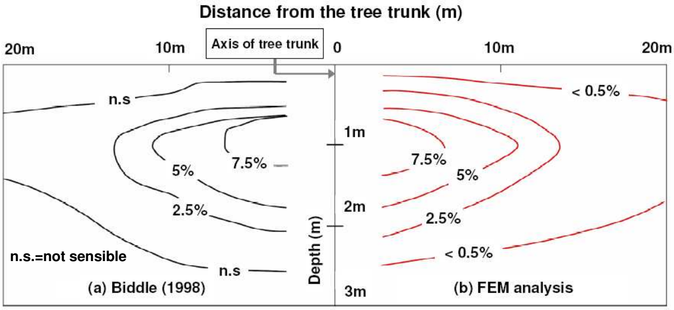

Figure 4: Contours of moisture content reduction (%) in the vicinity of a Poplar tree (a) Biddle (1998) (b) FEM analysis (After Fatahi and Indraratna, 2006)

## 4 CONCLUSIONS

An evaluation of the model suggested by Indraratna et al. (2006) was successfully carried out by comparing the numerical results and field measurements of the moisture content in the vicinity of a Poplar tree, as reported by Biddle (1998). In spite of uncertainties in the assumption of soil parameters, the actual distribution of tree roots, and the atmospheric parameters, there was an acceptable agreement between the measurements and predicted moisture distribution. Trees can provide suction up to the wilting point of a soil-root system and therefore, ground consolidation associated with transpiration increases the soil strength and stiffness. This process may be compared with improving soft soil via prefabricated vertical drains and vacuum preloading. The moisture reduction by transpiration increases the effective stresses, which in turn increases the settlement and stiffness of unsaturated soil. Considering various soil conditions, the type of vegetation and atmospheric conditions, the proposed mathematical model for biostabilsation is most useful to predict the ground behaviour in vicinity of structures such as rail track.

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