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Analysis of cone penetration test in cohesive soils

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ABSTRACT

This paper presents a numerical model for the analysis of cone penetration test in cohesive soils based on the finite element method. The constitutive behaviour of the soil is modelled using an elastic-perfectly plastic model obeying Von-Mises yield criterion. Since this is a problem involving large soil deformations, the analysis is carried out using an Arbitrary Lagrangian Eulerian method where the quality of the mesh is preserved during the penetration process. The variation of the cone resistance is examined with various parameters such as rigidity index of the soil, in situ stress anisotropy and roughness at the cone-soil interface, which influence the penetration resistance of the cone. A theoretical correlation has been developed incorporating these parameters and compared with previous correlations based on the cavity expansion theory, finite element method and strain path method.

1 INTRODUCTION

The cone penetration test is widely used in geotechnical engineering practice to investigate the engineering properties of soil. Since the test can be performed under existing stresses and boundary conditions in the field, they can provide reliable results for in-situ soil compared to laboratory tests. Also the test can be used to obtain continuous profiling of soil properties with depth. The main problem associated with the cone penetration test is that it does not measure any soil property directly. During the penetration, cone tip resistance and sleeve friction are typically measured. These measurements are used to infer engineering properties of soil. Hence, development of correlations between measured parameters and engineering properties of the soil is very useful.

Several theoretical relations have been proposed to interpret the cone penetration in cohesive soils. These relations are based on a variety of different approaches that include bearing capacity theory, cavity expansion theory, strain path method, calibration chamber testing and the finite element method. Compared to other methods, the finite element method offers a number of advantages: geometry of the cone can be modelled accurately, different soil constitutive models can be utilized, failure modes do not have to be assumed, initial stress state can be defined and both equilibrium equations and yield criteria are satisfied (Susila and Hryciw, 2003). However, cone penetration in cohesive soils involves large soil deformations and highly nonlinear material and interface behaviour. Therefore conventional small-strain finite element analysis is not suitable for the analysis of this problem and a large strain finite element formulation is essential.

Many previous large strain finite element analyses treated the cone penetration as a collapse problem (Yu and Mitchell, 1998), where the interface between the cone and soil is modelled using zero-thickness interface elements. In these models, the new location of the boundary nodes are decided after each calculation step and the procedure becomes complicated when the cone-soil interface friction is incorporated to the analysis (van den Berg, 1994).

The finite element method offers two approaches to deal with problems involving large soil deformations, namely, Lagrangian and Eulerian formulations. In the pure Lagrangian formulation, mesh points coincide with the same set of material points throughout the analysis. This results in a set of distorted elements in the finite element mesh. In the Eulerian formulation, material is allowed to move through a fixed mesh avoiding any mesh distortions. In solid mechanics, less attention has been paid to Eulerian formulations compared with Lagrangian formulations, although Van den Berg (1994) applied the Eulerian method to analyse deep penetration of a cone.

To avoid, excessive element distortions associated with Lagrangian formulations, finite element method offers two approaches. The first approach is the Arbitrary Lagrangian-Eulerian formulation, where the extent to which material flows through the fixed finite element mesh (Eulerian), or the mesh moves with the material (Lagrangian) may be varied arbitrarily to avoid excessive mesh distortions. In the second approach, a Lagrangian formulation is used until the mesh distortions are large. Then it is adapted using a remeshing technique. The RITSS approach developed by Hu and Randolph (1998) is somewhat similar to the second approach. In this approach, a series of small strain analysis increments are carried out until the mesh distortions are significant. Then a complete remeshing step is carried out and stresses and other material properties are transferred from the old mesh to the new mesh. Lu et. al. (2004) applied this method to simulate cone penetration test in an elastic, perfectly plastic soil.

In this paper, the commercial finite element software ABAQUS is used to simulate the deep penetration of a cone. A high quality mesh is maintained throughout the analysis using the Arbitrary-Lagrangian-Eulerian method, which consists of two steps. In the first step a new mesh is created and in the second step solution variables are mapped from the old mesh to the new mesh using a second order advection technique. Soil behaviour is modelled assuming elastic perfectly-plastic behaviour with a Von Mises yield criteria. The effect of rigidity index of the soil, cone and shaft roughness and in situ stress state on the penetration resistance has been investigated. Results are compared with those derived by other researchers.

2 NUMERICAL MODEL

The finite element mesh used for the analysis is shown in Figure 1. It is created as a free mesh using the advancing front algorithm. Cone and the shaft of the cone penetrometer are modelled using two analytical rigid surfaces. They are pushed into the ground at the same rate in such a way that they are in contact throughout the analysis and this will facilitate the calculation of shaft friction and cone resistance separately. In ABAQUS/Explicit, only linear elements can be used for the large deformation analysis. Therefore, soil is modelled using four-node rectangular elements with reduced integration (CAX4R). The soil surrounding the cone is extended 15 cone diameters in both radial and vertical directions and initially the cone is placed 2 m below the ground surface.

Cone penetration is simulated using the Arbitrary Lagrangian-Eulerian (ALE) method available in ABAQUS/Explicit, which combines the features of pure Lagrangian analysis and Eulerian analysis. ALE method is computationally efficient for modelling high speed dynamic events and slow quasi-static events involving large deformations. During each time increment of the analysis, a volume smoothing technique is used to improve the quality of the finite element mesh, and ten mesh sweeps are performed after each increment. Although the quality of the mesh is preserved during the analysis, the same topology is used throughout the analysis. This procedure will not change the mesh density near areas with high stress intensity and this is one of the limitations in ALE formulation available in ABAQUS.

The interface between the cone and the soil is modelled using the contact algorithm in ABAQUS, which is based on the concept of a master surface and a slave surface. In this problem, the master surface is the rigid cone and the slave surface is the soil surface in contact with the cone. During cone penetration, the master surface pushes into the slave surface and contact forces are generated to prevent penetration of the slave nodes into the master surface. The contact properties of the interface are defined using the penalty method, which calculates frictional force at the cone-soil interface as the product of the frictional coefficient between two surfaces and the normal force transmitted between the surfaces in contact. The shear stress at the interface is the frictional force applied over a unit surface area and the maximum shear stress that can be transferred between the surfaces in a von Mises material is given by:

$$\tau_{\max} = \frac{2s_u}{\sqrt{3}} \qquad (1)$$

Where s_u is the undrained shear strength of the soil under triaxial conditions.

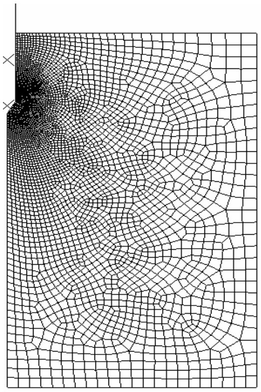
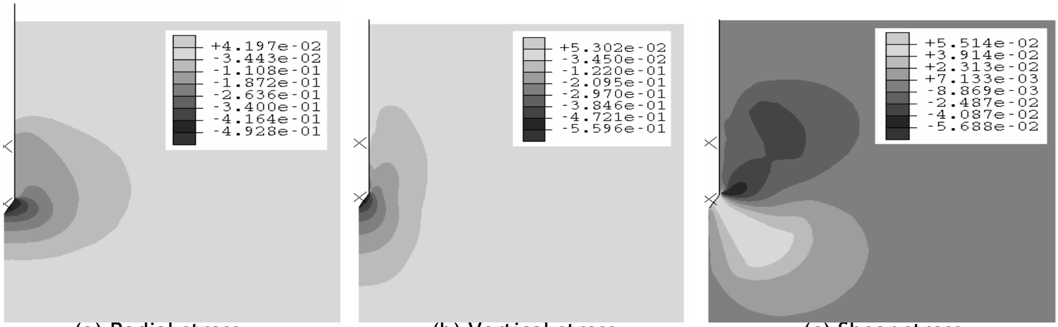


Figure 1. Finite element mesh used to simulate cone penetration.

3 STRESSES AROUND CONE PENETROMETER

Stresses within the soil are significantly increased during the cone penetration. Radial and vertical stress distributions around the cone and the shaft are shown in Figures 2(a) and 2(b), respectively. The maximum radial stress occurs at the shoulder of the cone and with the progression of penetration radial stress bulbs expand in the radial direction. The maximum vertical stress occurs beneath the cone tip and the vertical stress bulbs expand in the vertical direction beneath the cone tip. In the shear stress distribution shown in Figure 2(c), the positive and negative shear stress bulbs develop around the cone tip and the cone shoulder. These two active (negative) and passive (positive) zones expand as the penetration progresses.



(a) Radial stress (b) Vertical stress (c) Shear stress

Figure 2. Stress distributions around the cone at steady state.

4 THEORETICAL CORRELATIONS BETWEEN CONE FACTOR AND SOIL PROPERTIES

In engineering practice, the cone factor, N_c, is defined based on the bearing capacity theory. For clay soils, it is a function of the cone resistance, total overburden pressure and the undrained shear strength of the soil as follows:

$$N_c = \frac{q_c - \sigma_{vo}}{s_u} \tag{2}$$

where q_c is the cone tip resistance and σ_{vo} is the total overburden pressure at the level of the cone tip. Over the years, several theoretical models have been derived based on different approaches to relate cone factor with rigidity index of the soil, G/s_u, and the roughness of the cone, where G is the shear modulus of the soil. Based on cavity expansion theory, Vesic (1972) gave expressions for the cone factor as follows:

$$N_c = 1.33\left(1 + ln\frac{G}{s_u}\right) \quad \text{Spherical cavity expansion} \tag{3(a)}$$

$$N_c = 1.00\left(1 + ln\frac{G}{s_u}\right) \qquad \text{Cylindrical cavity expansion} \qquad 3(b)$$

Baligh (1985) and Whittle (1992) used the strain path method to obtain the soil stress and strain fields around a driven cone penetrometer. As presented by van den Berg (1994), the relation between the cone factor and the rigidity index is given by:

$$N_c = 1.51 + 2\,ln\frac{G}{s_u} \qquad (4)$$

A major drawback of the strain path method is that it cannot model accurately the cone geometry and the roughness of the cone-soil interface. In addition, the equilibrium equations are not always satisfied in the approach (Whittle, 1992). Teh and Houlsby (1991) carried out an analysis based on the strain path method incorporating the actual geometry of the cone penetrometer. They also incorporated the influence of roughness between cone-soil interface and in situ soil stresses on the cone factor and gave the following equation for the cone factor:

$$N_c = 1.25 + 1.84\,ln\frac{G}{s_u} + 2\alpha_f - 2\Delta \qquad (5)$$

where $\Delta = \frac{\sigma_{vo} - \sigma_{ho}}{2s_u}$ is a measure of the anisotropy of the initial stress state of the soil and $\alpha_f = \frac{\sqrt{3}\tau_f}{2s_u}$, which is the roughness factor, varies between 0 (smooth) and 1 (rough). However, it was again found that the final stresses obtained were in error and they did not satisfy the equilibrium equations. Yu et al. (2000) used a finite element procedure based on the steady state deformation approach, and the following expression was given for the cone factor incorporating the influence of rigidity index, cone-soil interface friction and in situ stress ratio:

$$N_c = 0.33 + 2\,ln\frac{G}{s_u} + 2.37\frac{\delta}{\varphi'_{cs}} - 1.83\Delta \qquad (6)$$

where δ is the friction angle for the cone-soil interface and φ'_{cs} is the critical state friction angle for the soil. Recently, Lu et al. (2004) also conducted a large strain finite element study of cone penetration in clay and proposed the following formula for N_c:

$$N_c = 3.4 + 1.6\,ln\frac{G}{s_u} + 1.3\alpha_f - 1.9\Delta \qquad (7)$$

5 RESULTS AND DISCUSSION

Figure 3 shows the values of N_c predicted from Teh and Houlsby (1991), Yu (2000), Lu et al. (2004) and the Arbitrary Lagrangian-Eulerian approach used in this study for $\Delta = 0$ and assuming a smooth contact between the cone-soil interface. The variation of cone factor with soil rigidity index obtained from this study can be described by the following equation:

$$N_c = 1.0 + 1.825\,ln\frac{G}{s_u} \qquad (8)$$

Equation (8) is comparable to Equation (5), which was derived by Teh and Houlsby (1991) for the assumed conditions. Figure 4 shows the influence of friction at the cone-soil interface on cone factor. The friction coefficient α_c has been defined as:

$$\alpha_c = \frac{\tau_{int}}{\tau_{max}} \qquad (9)$$

where τ_{int} is the interface friction acting on the cone-soil interface. Results of this study show that the gradient of each line is a function of the rigidity index of the soil and the influence of α_c can be incorporated into Equation 8 as shown below:

$$N_c = 1.0 + 1.825\,ln\frac{G}{s_u} + \left(0.33\,ln\frac{G}{s_u} + 0.5\right)\alpha_c \qquad (10)$$

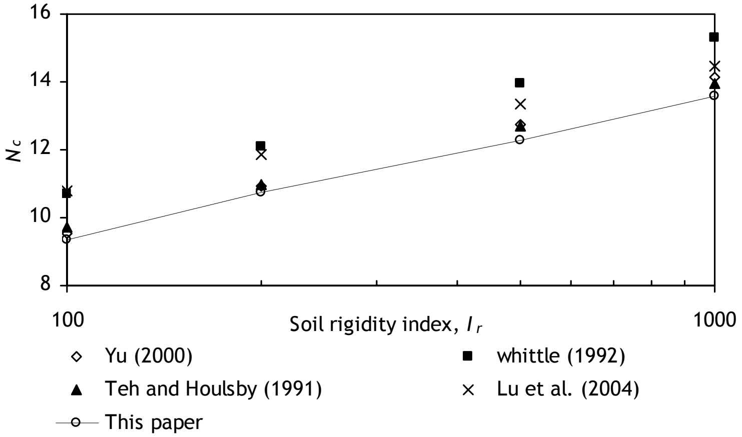


Figure 3. Variation of cone factor with rigidity index of soil (smooth cone and isotropic stresses).

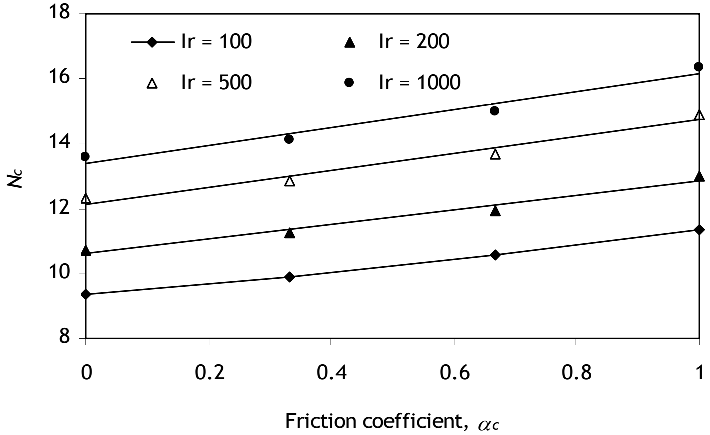


Figure 4. Variation of cone factor with friction coefficient between cone and soil (isotropic stresses).

The initial stress state within the soil has a significant influence on the cone penetration resistance. Teh and Houlsby (1991) defined in situ stress parameter, Δ, to study the in situ stress state on cone factor. This parameter can range between –1 (heavily over-consolidated soil) and 1 (normally consolidated soil). Figure 5 shows the influence of in situ stresses on cone factor and the cone factor has been defined using vertical stress as in Equation 2, horizontal stress (N_{ch}) and mean stress (N_{cp}) as shown below:

$$N_{ch} = \frac{q_c - \sigma_{ho}}{s_u} \qquad N_{cp} = \frac{q_c - p}{s_u} \qquad (11)$$

where *p* is the mean stress. It can be seen that the cone factor based on the vertical stress is more sensitive to the in situ stress ratio than the cone factors calculated based on the horizontal and mean stresses. This shows that the cone resistance is more sensitive to the horizontal and mean stresses than to the vertical stress. Usually horizontal stress is not known accurately and for this reason it is more convenient to use the cone factor based on the vertical stress (Teh and Houlsby, 1991) to define the cone penetration resistance. However, we can further modify Equation 10 to take into account the effect of the in situ stress ratio on cone factor, as shown below:

$$N_c = 1.0 + 1.825\, ln\frac{G}{s_u} + \left(0.33\, ln\frac{G}{s_u} + 0.5\right)\alpha_c - 3.43\Delta \qquad (12)$$

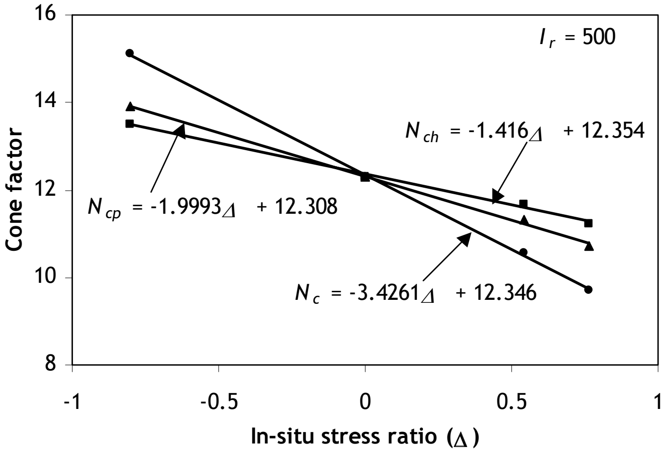


Figure 5. Variation of cone factor with in situ stress ratio.

6 CONCLUSION

Using a finite element program based on the Arbitrary Lagrangian-Eulerian method, cone penetration in cohesive soils has been studied numerically. Results from the present study have been compared with published correlations between cone factor and soil properties derived based on the strain path method, an adaptive finite element procedure and a finite element procedure based on the steady state deformation approach. Although cone factors predicted based on the pure strain path method (Whittle, 1992) is about 13% higher than the present study, the strain path results given by Teh and Houlsby (1991) incorporating geometry and roughness of the cone penetrometer agree well with the present study.

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