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Application of limit analysis to soil-structure interaction problems

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ABSTRACT

Finite element limit analysis is becoming an established computational means by which upper and lower bound strength estimates can be obtained for complex geotechnical problems. Advantages are that only soil strength parameters and details of the mesh geometry are required in order to obtain solutions, which also have clear status (in contrast to the solutions obtained using incremental elastic-plastic methods for example). Whilst several finite element limit analysis studies of soil-structure interaction problems are already described in the literature (e.g. for bearing capacity and anchor pullout problems), interest has typically focussed on soil performance only. For particular classes of problems it is desirable to perform a coupled analysis of the soil and adjacent structural elements. This paper describes how such problems can be formulated and explores some of the challenges and issues involved, including that of relative soil and structure strength mobilisation. Illustrative example problems are described, including backfilled masonry arch bridge and retaining wall problems, and results are compared with those from simpler analyses and large scale experiments.

1 INTRODUCTION

Since its original development in the last century, plastic limit analysis has been widely applied to geotechnical analysis and design problems. It is normally challenging to generate complete limiting solutions for problems involving complex geometry and/or loading conditions by hand but fortunately over the last few decades computational methods have been developed to enable the collapse state to be modelled. To date there have been two main approaches taken: (i) incremental elasto-plastic analysis techniques; (ii) numerical limit analysis techniques. Although flexible and powerful, category (i) methods suffer the drawback of typically requiring large numbers of input parameters and significant operator expertise to use effectively, and may be seen as ‘overkill’ if only the limit load and corresponding collapse mechanism are required. In contrast category (ii) methods rely on the limit theorems of plasticity to directly compute the limit load, most commonly involving finite elements and the use of mathematical programming techniques to obtain solutions. Thus so called ‘finite element limit analysis’ has become increasingly attractive to researchers seeking to solve complex limit analysis problems involving soil.

The aim of this paper is to demonstrate that finite element limit analysis can readily be used in conjunction with the limit analysis techniques already used by structural analysts to enable soil-structure interaction problems to be treated. The interaction of soil with two types of masonry structure will be used to illustrate some of the relevant issues.

2 FINITE ELEMENT LIMIT ANALYSIS

For plane strain soil mechanics problems, Lysmer (1970) was perhaps the first to propose a finite element limit analysis problem formulation. In such a formulation dealing with the yield function is of particular interest because of its non-linear nature. For example, Anderheggen and Knopfel (1972) chose to linearise the yield surface in a piecewise manner so that linear programming could be employed; more recently workers such as Krabbenhoft and Damkilde (2003) have shown that interior-point algorithms can be applied to problems involving smooth convex non-linear yield functions. However, whilst finite element limit analysis has been extensively applied to various geotechnical problems by researchers such as Sloan (1988) over the last two decades, it can be observed that interest has typically been confined to modelling failure of soil elements only. Exceptions to this include the recent study by Krabbenhoft and Damkilde (2005) of sheet pile – soil interaction using lower bound limit analysis, and the upper bound limit analysis study of masonry bridge soil-structure interaction by Cavicchi and Gambarotta (2005). However, in this latter study the masonry was represented using 1D beam elements (rather than using more realistic 2D masonry blocks). Furthermore, relatively inaccurate constant strain elements were employed in the finite element model of the soil; in this paper a key aim is to address both these shortcomings.

Considering now the modelling of structural masonry elements, Livesley (1978) was the first to develop a computational limit analysis model for masonry structures. He discretised the structure into an assemblage of rigid blocks which were free to slide and rock relative to each other. By assuming an associative sliding friction model he was able to use linear programming to obtain a solution. This model can readily be extended to model crushing at hinge points and, rather less straightforwardly, be extended to model non-associative frictional sliding. One notable application of rigid block analysis is the RING masonry arch bridge analysis software program (Gilbert, 2001); masonry bridges are often backfilled with soil and form the backbone of much of the transport infrastructure in the developed world. However, in the current version of RING the soil is modelled in a simplistic manner, with uniaxial struts being used to represent the passive restraint offered by the soil. To address this upper and lower bound limit analysis formulations of the combined soil-structure problem have now been developed and are outlined briefly in this paper.

3 NUMERICAL MODEL OF STRUCTURAL ELEMENTS

3.1 Model of masonry structure.

For simple masonry block limit analysis problems both equilibrium and kinematic formulations lead to identical solutions, which may be considered 'exact' according to plasticity theory (providing all physical blocks are included in the model). Considering an equilibrium formulation, contact and block forces, dimensions and frictional properties are shown on Figure 1. The problem variables are the contact forces: n_i, s_i, m_i (where $n_i \geq 0$; s_i, m_i are unrestricted 'free' variables), and the unknown load factor λ.

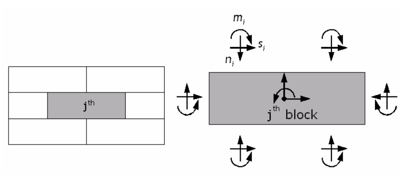

Figure 1: Typical block assemblage

Assuming there are b blocks and c contact surfaces, the problem constraints are as follows:
Equilibrium constraints:

$$\mathbf{Bq} - \lambda \mathbf{f}_{\mathrm{L}} = \mathbf{f}_{\mathrm{D}} \tag{1}$$

Yield constraints:

$$\mathbf{C}^{\mathrm{T}}\mathbf{q} \leq 0 \tag{2}$$

where $\mathbf{B}$ is a suitable ($3b \times 3c$) equilibrium matrix containing direction cosines and $\mathbf{q}$ and $\mathbf{f}$ are respectively vectors of contact forces and block loads. Thus $\mathbf{q}^{\mathrm{T}} = \{n_1, s_1, m_1, n_2, s_2, m_2, \ldots n_c, s_c, m_c\}$; $\mathbf{f} = \mathbf{f}_{\mathrm{D}} + \lambda \mathbf{f}_{\mathrm{L}}$ where $\mathbf{f}_{\mathrm{D}}$ and $\mathbf{f}_{\mathrm{L}}$ are respectively vectors of dead and live loads. Finally $\mathbf{C}$ defines the geometry of the active yield functions, e.g. of the familiar sliding yield constraint: $|s_i| \leq n_i \tan\phi_i$ and, if crushing is not being modelled, the rocking yield constraint $|m_i| \leq 0.5 n_i t_i$, for each contact i possessing thickness t_i and friction angle ϕ_i.

3.2 Model of soil

3.2.1 Lower bound formulation

The soil is discretised using three-noded linear-stress elements separated by discontinuities (use of three-noded elements rather than elements with higher numbers of nodes has been found to avoid problems maintaining non-violation of yield). Each node in a triangular element therefore has three unknown stresses, σ_x, σ_y, τ_{xy}, which are constrained so as to satisfy (linear) equilibrium and (non-linear) yield constraints. Linear programming (LP) can however be used to obtain a solution if the

Mohr Coulomb failure envelope is approximated by a polygon; if an exterior polygon is used then this can be adaptively refined as part of an efficient iterative LP solution scheme which terminates when no stresses violate yield.

3.2.2 Model of soil-structure interface

For each soil-masonry contact i, the requisite yield constraint can be written as:

$$\tan\phi_i^{sm} n_i + |s_i| \le c_i^{sm} \tag{3}$$

where c_i^{sm} and ϕ_i^{sm} respectively represent the cohesion and angle of friction of soil–masonry interface i. The requisite equilibrium constraint can be written as:

$$\begin{bmatrix} l_i/2 & l_i/2 & & \\ & & l_i/2 & l_i/2 \\ l_i^2/6 & -l_i^2/6 & & \end{bmatrix} \begin{bmatrix} \sigma_i^A \\ \sigma_i^B \\ \tau_i^A \\ \tau_i^B \end{bmatrix} - \begin{bmatrix} n_i \\ s_i \\ m_i \end{bmatrix} = 0 \tag{4}$$

where l_i is the length of a contact i between soil and masonry elements and where σ_i^A, σ_i^B, τ_i^A, τ_i^B are normal and shear stresses acting at end points A and B of the contact.

3.2.3 Upper bound formulation

The soil is discretised using six-noded linear strain elements with straight sides. The apex of each triangular element is associated with a specified number of plastic multipliers. This allows the soil behaviour to be modelled more accurately than when using lower order elements, e.g. the three-noded constant strain elements used by Cavicchi and Gambarotta (2005). It also avoids the locking problem discussed by Nagtegaal et al. (1974) without the need to resort to special arrangements of elements within the mesh. Each node is unique to its element permitting velocity jumps to be modelled at inter-element boundaries. A kinematically admissible velocity field will be obtained provided the associative flow rule is enforced both within elements and along discontinuities (Makrodimopoulos and Martin 2006); the velocity boundary condition should also be enforced. The upper bound collapse load can then be obtained by minimising the internal energy dissipation, set to be equal the work done by external applied loads.

4 APPLICATION OF NUMERICAL MODEL TO SOIL-MASONRY STRUCTURE INTERACTION PROBLEMS

4.1 Analysis of brickwork retaining wall

Table 1: Material properties

	c (kN/m^2)	ϕ (°)	γ (kN/m^3)
Soil	5	40	14
Masonry	0	31	20
Soil-masonry	0	26.6	-

This example illustrates the application of the model to the analysis of a masonry block retaining wall with a surface surcharge. Material properties are provided in Table 1 and the geometry is shown on Figure 2. Results are shown in Figures 3 to 7. Lower and upper bound predictions of the surface load required to cause collapse were 58.6 kN/m and 65.4 kN/m respectively. To obtain these values the finite element mesh was manually refined around the base of the surface surcharge, though further improved predictions could be achieved with a finer mesh (here a single coarse mesh comprising 3831 elements was used for both lower and upper bound analyses). Figure 3 shows how the computed stresses vary in relation to the yield stress (dark areas being closest to yield). The velocity field obtained from the upper bound analysis shown in Figure 4 indicates the presence of a crack between the wall and soil and also that the wall fails by rotating near its base. Note that in Figure 6, the predicted vertical stresses 6m behind the front of the wall vary approximately linearly with depth, the values being close to γh as expected. In contrast, the vertical stresses immediately behind the wall are significantly above this level below a depth of approx. 2.5m. The low horizontal stresses 1m behind the wall (Figure 7) indicate the potential presence of cracking down to a depth of ≈1.6m. The wall fails because of the increasing horizontal stresses below this depth, acting to overturn the wall.

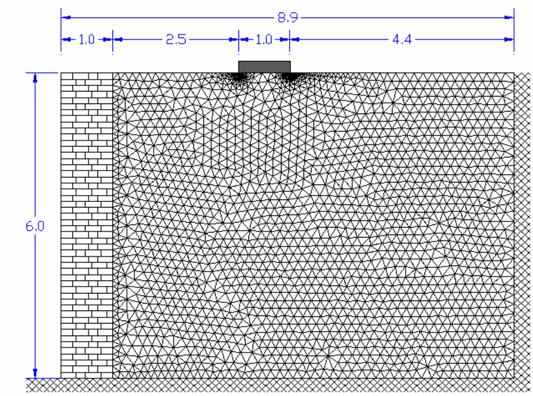

Figure 2: Retaining wall geometry (all dimensions are in metres)

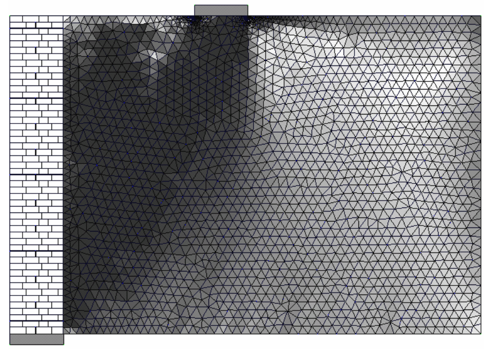

Figure 3: Variation in maximum shear stress relative to the yield stress (darker areas being closest to yield)

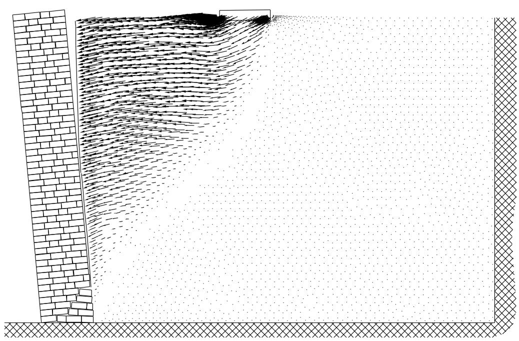

Figure 4: Soil velocity field and wall deformation

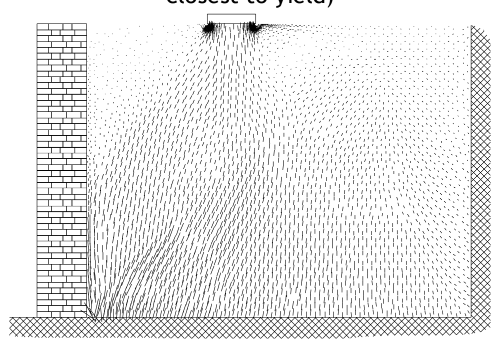

Figure 5: Maximum (compressive) principal stress vectors

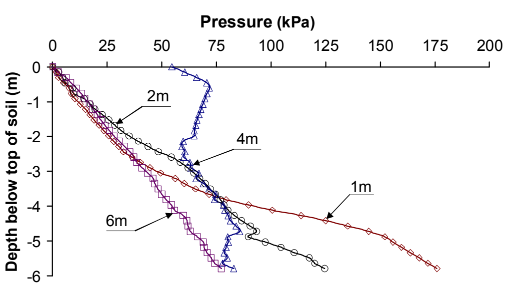

Figure 6: Vertical stresses measured at various offsets from the front of the wall

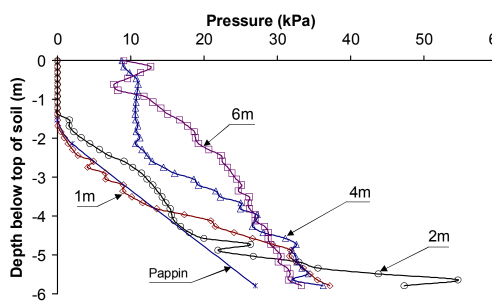

Figure 7: Horizontal stresses measured at various offsets from the front of the wall

No benchmarks are available in the literature for this type of problem. However the horizontal stress distribution on the wall due to the surface load can be compared with that derived using a procedure due to Pappin et al (1986); it is evident from Figure 7 that the results correlate reasonably well, at least at shallow depths. Note that for this example it has been implicitly assumed that peak soil and structural strength will be mobilised simultaneously; this may not always hold true, as will be demonstrated in the next example. Note also that in non-yielding areas, the given stress field solution is non-unique. It simply has to satisfy equilibrium and not violate yield, resulting, for example, in non-zero surface horizontal stresses 6m away from the wall.

4.2 Analysis of Masonry Arch Bridges

As well as developing numerical soil-arch interaction models, the authors are in parallel carrying out physical tests on full-scale 3m span soil-filled masonry arch bridges (refer to Smith et al. (2006) for geometrical and other relevant data), which can be used to calibrate the numerical models. High quality materials data is available (Soil: c' = 3.3 kN/m^2, ϕ' = 54.5°, γ = 19.1 kN/m^3; Masonry: ϕ =31°, γ = 23.7 kN/m^3), which, when used in the models generated the results given in Figures 8, 9 and 10 (N.B. the soil-masonry interface angle of friction has not yet been measured experimentally and was taken as 1/3ϕ', taking into account the nature of the masonry surface and the likely critical state angle of shearing resistance of the soil). Similar though not identical meshes comprising 8132 and 8121 elements were used for the upper and lower bound analyses respectively. The meshes were manually refined around the base of the surface load and a relatively fine mesh was used around the arch barrel to capture the essential features of the soil-arch interaction.

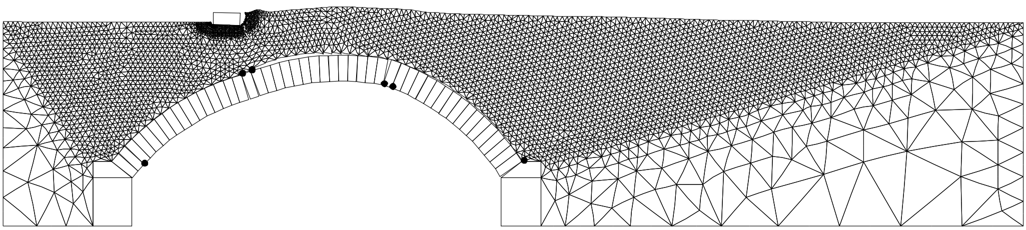

Figure 8: Deformed shape of soil and arch

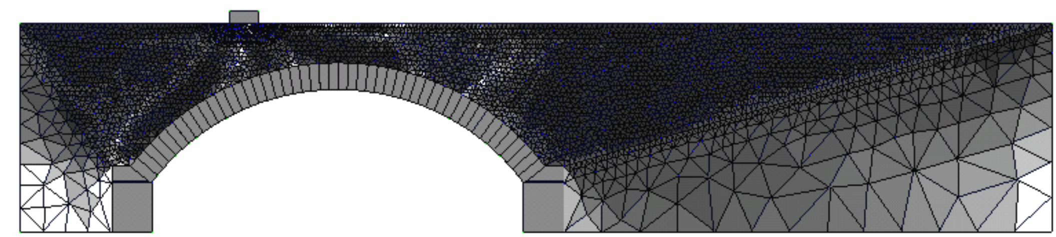

Figure 9: Variation in maximum shear stress relative to the yield stress (darker areas being closest to yield)

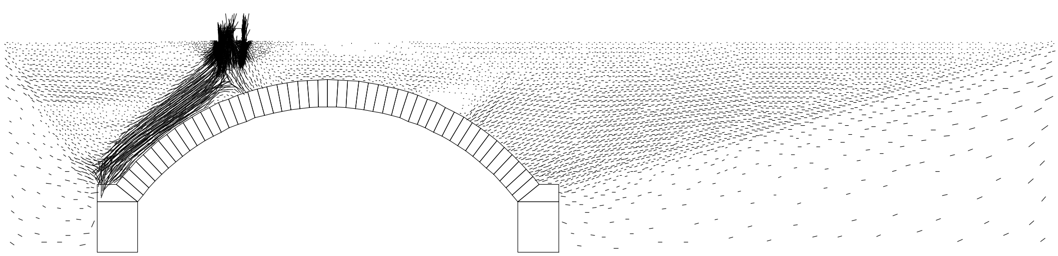

Figure 10: Maximum (compressive) principal stress vectors

In both model and experiment, the arch collapsed in a classic 4 hinge mechanism, as shown in Figure 8 (though in some cases 'diffused' hinges were in evidence). Figure 9 indicates the presence of yielding soil (dark shaded areas) both under the load and in the soil around the 'passive' right hand side of the arch, where the arch barrel sways into the soil mass. Figure 10 shows the flow of stress from the load down to the left abutment.

However, Figure 11 shows that direct application of the limit analysis model leads to predicted collapse loads which are significantly higher than those obtained experimentally. To explain this, consider first the structure: as the arch starts to deform in a 4-hinge mechanism it immediately starts to lose strength, and this loss of strength increases with further displacement. In contrast the soil will still be mobilising increasing strength well beyond any initial displacement. Inclusion of a strength mobilisation factor of $M \approx 0.25$ (i.e. $\phi'_{mobilised} = 0.25\phi'$, $c'_{mobilised} = 0.25c'$), based on inferred average soil strains on the passive side at around 1mm radial displacement (analogous to an approach employed in BS8002) results in a significantly improved prediction, as shown in Figure 11. This is clearly an important issue which deserves more in-depth study.

5 CONCLUSIONS

- Upper and lower bound computational limit analysis models have been developed for soil-structure interaction problems. Reasonably close bounds on the 'exact' collapse load have been obtained for the examples investigated (a masonry retaining wall and an arch bridge).
- Strength mobilisation has been found to be an important issue when predicting the limit loads associated with certain types of soil-structure interaction problems. Neglecting this may lead to significant over-prediction of the ultimate collapse load. Application of strength mobilisation factors, based on anticipated soil strains, appears to address this issue though further investigations are warranted.

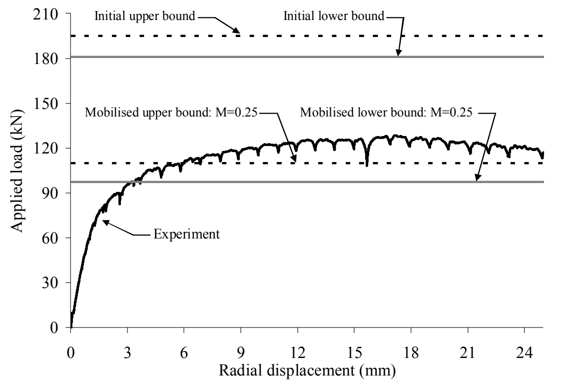

Figure 11: Predicted and experimental responses for masonry bridge with compacted limestone backfill

6 ACKNOWLEDGEMENTS

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