Limit analysis solutions for the bearing capacity of rock masses using the generalised Hoek-Brown criterion

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ABSTRACT

The ultimate bearing capacity is an important design consideration for dams, roads, bridges and other engineering structures, particularly when large rock masses are the foundation materials. With the exception of some very soft rocks and heavily jointed media, the majority of rock masses provide an excellent foundation material. However, there is a need to accurately estimate the ultimate bearing capacity for structures with high foundation loads such as tall buildings and dams. This paper applies numerical limit analyses to evaluate the ultimate bearing capacity of a surface footing resting on a rock mass whose strength can be described by the generalised Hoek-Brown failure criterion. Results from the limit theorems have been presented in the form of bearing capacity factors for a range of material properties. Where possible, a comparison is made between existing numerical analyses, empirical and semi-empirical solutions. This revealed that estimating the ultimate bearing capacity of a rock mass using equivalent Mohr-Coulomb parameters was found to overestimate the bearing capacity by as much as 157% for very good quality rock masses

1 INTRODUCTION

The ultimate bearing capacity is an important design consideration for dams, roads, bridges and other engineering structures, particularly when large rock masses are the foundation materials. Most geotechnical practitioners know that, with the exception of some very soft rocks, the majority of rock masses provide an excellent trouble–free foundation material. However, there is a need to accurately estimate the ultimate bearing capacity for structures with high foundation loads such as tall buildings and dams.

Rigorous theoretical solutions to the problem of foundations resting on rock masses do not appear to exist in the literature. This may be attributed to the fact that rock masses are inhomogeneous, discontinuous media composed of rock material and naturally occurring discontinuities such as joints, fractures and bedding planes. This makes the derivation of simple theoretical solutions based on limit equilibrium methods very difficult. In addition, fractures and discontinuities occurring naturally in rock masses are difficult to model using the displacement finite element method without the addition of special interface or joint elements. The upper and lower bound formulations of Lyamin and Sloan (2002a,b) are ideally suited to analysing jointed or fissured materials due to the existence of discontinuities throughout the mesh. These discontinuities allow an abrupt change in stresses in the lower bound formulation and in velocities in the upper bound formulation.

The purpose of this paper is to take advantage of the ability of the limit theorems to bracket the actual collapse load by computing both lower and upper bounds for the bearing capacity of strip footings on a broken rock mass. These solutions are obtained using the numerical techniques developed by Lyamin and Sloan (2002a,b) which have been modified to incorporate the well known Hoek–Brown yield criterion (Hoek et al 2002).

2 THE GENERALISED HOEK-BROWN FAILURE CRITERION

2.1 Applicability

It is well known that the strength of jointed rock masses is notoriously difficult to assess. The behaviour of a rock mass is complicated greatly because deformations and sliding along naturally occurring discontinuities can occur in addition to deformations and failure in the intact parts (blocks) of the rock mass. Unfortunately, laboratory tests on specific core samples is often not of a rock mass on a large field scale, while in-situ strength testing of the rock mass is seldom practically or economically feasible. Nonetheless, engineers and geologists are required to predict the strength of large scale rock masses when designing such things as drifts, foundations, slopes, tunnels and caverns.

Many criteria have been developed that seek to capture the important elements of measured rock strengths or seek to modify theoretical approaches to accommodate experimental evidence. The currently more or less accepted approach to estimating rock mass strength is to use the Hoek-Brown failure criterion where the required parameters are estimated with the help of a rock mass classification system. The Hoek-Brown failure criterion is an empirical criterion developed through curve-fitting of triaxial test data. The original Hoek-Brown empirical failure criterion (Hoek & Brown, 1980) was developed in the early 1980s for intact rock and jointed rock masses, and has been subject to continual refinement (Hoek et al 2002) . It is widely accepted that the Hoek-Brown criterion is virtually the only non-linear criterion used by practicing engineers (Mostyn and Douglas 2000). It is therefore appropriate to use this yield criterion when estimating the bearing capacity of surface foundations on rock.

It is important to remember that the Hoek-Brown failure criterion, which assumes isotropic rock and rock mass behaviour, should only be applied to those rock masses in which there are a sufficient number of closely spaced discontinuities, with similar surface characteristics, that isotropic behaviour involving failure on discontinuities can be assumed.

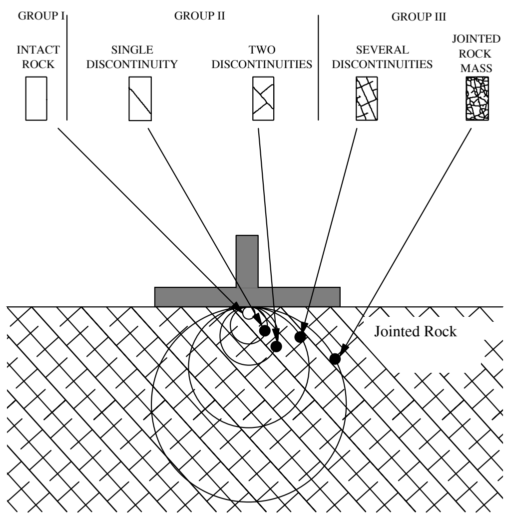

Figure 1: Applicability of the Hoek-Brown yield criterion for shallow foundations

With reference to the bearing capacity problem considered herein, the applicability of the Hoek–Brown criterion is best described by referring to Figure 1. After Hoek(1983) it appears three main structural groups can be differentiated for rock masses, namely GROUP I, GROUP II, and GROUP III. Figure 1 shows the transition from an isotropic intact rock(GROUP I), through a highly anisotropic rock mass (GROUP II), to an isotropic heavily jointed rock mass (GROUP III), with increasing sample size for a surface foundation on a hypothetical rock mass. In this paper it has been assumed that the underlying rock mass is either; 1) intact or; 2) heavily jointed with "small spacing" between discontinuities so that, on the scale of the problem, it can be regarded as an isotropic assembly of interlocking particles. Consequently, the results presented are valid for "intact rock" (GROUP I), "several discontinuities" and "jointed rock mass" (GROUP III) conditions, respectively.

2.2 Limit analysis implementation

One of the strong features of the Lyamin and Sloan (2002a,b) formulations is that they can deal with general yield criteria including multi-surface ones where several convex domains are combined to constrain the stresses at each node of the mesh. These combinations can be different for different parts of the discretised body. Because they are employed in their native form, a wide range of yield criteria can be used in the analysis. Each of the surfaces must be convex and smooth but the resulting composite surface, though convex, is generally non-smooth.

The Hoek-Brown failure criterion for rock masses has been updated in 1983, 1988, 1992, 1995, 1997, 2001 and 2002. A brief history of the development of the Hoek-Brown criterion can be found in Hoek(2004). The generalised 2002 criterion that has been used is given as

$$\sigma'_1 = \sigma'_3 + \sigma_{ci}\left(m_b \frac{\sigma'_3}{\sigma'_{ci}} + s\right)^{\alpha} \tag{1}$$

The relationships between m_b / m_i, s and α and the Geological Strength Index GSI are as follows

$$m_b = m_i exp\left(\frac{GSI-100}{28-14D}\right) \quad , \quad s = exp\left(\frac{GSI-100}{9-3D}\right) \quad , \quad \alpha = \frac{1}{2}+\frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right)$$

The GSI ranges from about 10, for extremely poor rock masses, to 100 for intact rock. The parameter D is a factor that depends on the degree of disturbance. The suggested value of the disturbance factor is $D=0$ for undisturbed in situ rock masses and $D=1$ for disturbed rock mass properties. For the analyses presented here, a value of $D=0$ has been adopted.

Similar to Mohr-Coulomb failure envelope, the Hoek-Brown yield surface has apex and corner singularities in stress space. The direct computation of the derivatives at these locations, which are required for the non-linear programming (NLP) solver, becomes impossible. To resolve the issue, three different approaches can be considered, namely, global smoothing, local smoothing and multi–surface representation. As the current study is limited to the case of plain strain conditions, the corners are automatically avoided and the only singularity which needs to be dealt with is the apex of the yield surface. From an implementation point of view, the easiest option in this situation are the cut-off (multi–surface technique) and quasi-hyperbolic approximation (global smoothing). The authors decided to adopt the later approach. A description of the implementation procedure can be found in Merifield et al (2006).

2.3 Equivalent Mohr-Coulomb parameters

Since most geotechnical software is still written in terms of the Mohr-Coulomb failure criterion, it is often necessary for practising engineers to determine equivalent angles of friction and cohesive strengths for each rock mass and stress range. In the context of this paper, estimating the equivalent Mohr–Coulomb parameters will enable a comparison to be made with the finite element limit analysis solutions that use the non–linear Hoek–Brown criterion.

The choice of method to use for determining equivalent cohesion and friction angle is largely a matter of taste and experience. An equivalent cohesion and friction angle at a specified normal stress or minor principal stress, as determined by an elastic analysis, may give locally accurate values for a small stress variation. Alternatively, average values applicable to a wider range of stress conditions may be obtained by using a regression procedure. Nonetheless, a regression approach appears to be the most widely accepted method and is typically performed by fitting a linear relationship to the curve generated by equation (1) for a range of minor principal stress values defined by $\sigma_t < \sigma_3 < \sigma'_{3max}$. This has been performed recently by Hoek et al (2002) where the fitting process involves balancing the areas above and below the Mohr-Coulomb relation.

Note that the value of σ'_{3max}, the upper limit of confining stress over which the relationship between the Hoek-Brown and the Mohr-Coulomb criteria is considered, has to be determined for each individual case. It is likely that the stresses will vary greatly throughout the rock mass which will make it difficult to select a representative value of σ'_{3max}. From experience and trial and error, Hoek & Brown (1997) suggest a value of $\sigma'_{3max} = 0.25\sigma_{ci}$ will provide consistent results.

Table 1 presents the results obtained for three different quality rock masses. The equivalent Mohr-Coulomb parameters were obtained over two separate ranges of the minor principal stress σ'_3;

namely, $0 < \sigma_3 < 0.25\sigma_{ci}$ and $0 < \sigma_3 < 0.75\sigma_{ci}$. This Table indicates just how sensitive the interpreted values of c' and ϕ' are to the value of σ'_{3max}.

Table 1: Equivalent Mohr-Coulomb parameters for different rock qualities

Rock Quality	σ_{ci}	m_i	GSI	$0 < \sigma_3 < 0.25\sigma_{ci}$		$0 < \sigma_3 < 0.75\sigma_{ci}$	
				c'	ϕ'	c'	ϕ'
Very poor	20	8	30	0.65	22.8	1.3	15.9
Average	80	12	50	4.2	32.1	8.55	23.4
Very good	150	25	75	14.1	45.8	28.6	36.6

3 PROBLEM DEFINITION

The plane strain bearing capacity problem to be considered is illustrated in Figure 2. A strip footing of width *B* rests upon a jointed rock mass with an intact uniaxial compressive strength σ_{ci}, geological strength index GSI, rock mass unit weight γ, and intact rock yield parameter m_i. The ultimate capacity can be written as

$$q_u = \sigma_{ci} N_\sigma \quad (2)$$

Where N_σ is defined as the bearing capacity factor. For a weightless rock mass ($\gamma = 0$), the above expression is valid but the bearing capacity factor N_σ is replaced with $N_{\sigma o}$. The form of equation (2) is a convenient way of expressing the ultimate bearing capacity as a "fraction" of the uniaxial compressive strength and is historically consistent with previous bearing capacity representations.

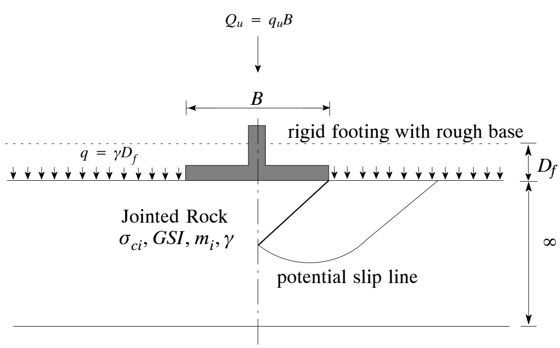

Figure 2: Problem Definition

4 RESULTS AND DISCUSSION

The computed upper and lower bound estimates of the bearing capacity factor N_σ for both the weightless and ponderable rock analyses were found to be within 5% of each other. This indicates that, for practical design purposes, the true collapse load has been bracketed to within $\pm 2.5\%$ or better. As a consequence, average values of the upper and lower bound bearing capacity factor have been calculated and will be used in the following discussions.

The average upper and lower bound estimates of N_σ and $N_{\sigma o}$ (weightless) are shown in Figure 3 for several GSI and m_i values. The effect of rock weight and rock strength has been incorporated in the analyses using the non-dimensional factor $\sigma_{ci} / \gamma B$ which varies between 125 and 10000. As expected, for a given GSI, increasing m_i leads to an increase in the ultimate bearing capacity. For a given GSI, as m_i increases, so does the extent of the observed velocity field and zone of plastic yielding. This is expected as an increase in m_i will, in essence, increase the strength of the rock and the equivalent Mohr–Coulomb parameters.

Referring to Figure 3, the effects of including self-weight in the analyses may be explained as follows. For any given rock mass (σ_{ci}, GSI, m_i) and foundation width B (i.e $\sigma_{ci} / B =$constant), the addition of self-weight γ (i.e decrease in the ratio $\sigma_{ci} / \gamma B$) will lead to an increase in the bearing capacity factor N_σ and thus the ultimate bearing capacity. That is, the bearing capacity factor for a weightless rock $N_{\sigma o}$ is always less than the bearing capacity factor N_σ for a rock with unit weight $(N_\sigma \geq N_{\sigma o})$. The effect of a small increase or decrease in the estimated rock weight γ is only likely to have a small effect on the bearing capacity factor N_σ. The effect of the rock weight γ was found to decrease with increasing GSI. This is shown clearly in Figure 3 where the lines represented by the ratio $\sigma_{ci} / \gamma B$ begin to converge towards the weightless case where $\sigma_{ci} / \gamma B = \infty$.

As a preliminary comparison, several limit analyses were performed for weightless rock masses using equivalent Mohr–Coulomb parameters as given in Table 1. Table 2 presents the results obtained for three different quality rock masses. It can be seen that, for both ranges of the minor principal stress σ_3, the ultimate bearing capacity has been overestimated significantly ($46\% - 157\%$) when we adopt equivalent Mohr–Coulomb strength parameters. Although the inclusion of rock weight γ is likely to increase the ultimate values shown in Table 2 by up to 25%, the overall predictions will still be poor for these rock qualities.

Table 2: Equivalent Mohr-Coulomb parameters for different rock qualities

Rock Quality	Hoek-Brown $q_u MPa$	$0 < \sigma_3 < 0.25\sigma_{ci}$	$0 < \sigma_3 < 0.75\sigma_{ci}$
		Mohr-Coulomb $q_u MPa$	$q_u MPa$
Very poor	6.7	12.0 (+46%)	15.3 (+87%)
Average	98.5	156.4 (+59%)	161.0 (+63%)
Very good	886.0	2279.4 (+157%)	1614.6 (+82%)

5 CONCLUSIONS

The bearing capacity of a surface strip footing resting on a rock mass whose strength can be described by the generalised Hoek–Brown failure criterion has been investigated. Using powerful formulations of the upper and lower bound limit theorems, rigorous bounds on the bearing capacity for a wide range of material properties have been obtained. The results have been presented in terms of a bearing capacity factor N_σ in graphical form to facilitate their use in solving practical design problems.

The following conclusions can be made based on the limit analysis results :

- The upper and lower bound estimates of the bearing capacity factors for either weightless or ponderable rock foundations, were found to be within 5% of each other.
- The effect of ignoring rock weight can lead to a very conservative estimate of the ultimate bearing capacity. This is particularly the case for poorer quality rock types with GSI values less than approximately 30, where the ultimate bearing capacity can be as much as 60% below the actual capacity when rock weight is included.
- Estimating the ultimate bearing capacity of a rock mass using equivalent Mohr–Coulomb parameters was found to significantly overestimate the bearing capacity. This overestimate was found to be as high as 157% for very good quality rock masses.

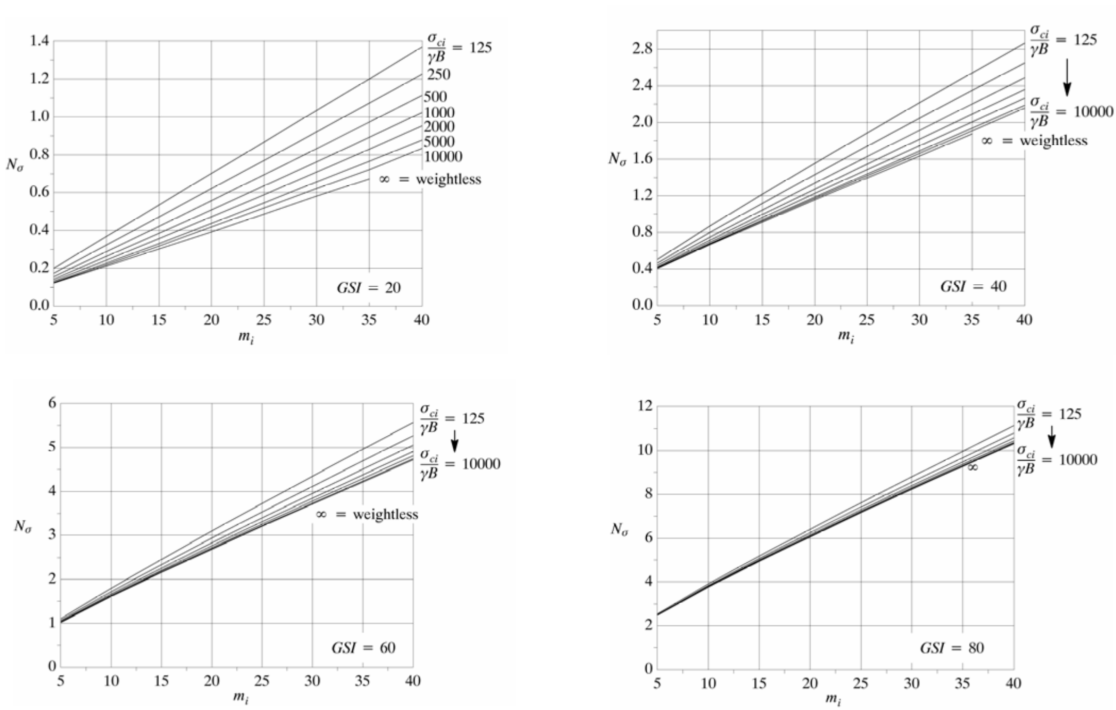

Figure 3: Average finite element limit analysis values of the bearing capacity factor N_σ

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