INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING

A method of analysis of piled rafts

John Small
Professor, Dept of Civil Engineering, The University of Sydney, Sydney, NSW, Australia
Harry Poulos
Senior Principal, Coffey Geosciences, 8/12 Mars Rd., Lane Cove West, NSW, Australia

Keywords: piled raft, piles, case study, non-linear

ABSTRACT

In this paper, the principles of a piled raft analysis, employing a combination of finite element and boundary element analyses, are described. This analysis is implemented via a computer program GARP (Geotechnical Analysis of Rafts with Piles). In this program, the raft is analysed via a finite element formulation for a plate, the piles are considered as non-linear interacting springs, and the soil is represented as a layered elastic system. Non-linear behaviour of the piles may also be considered. A comparison of computed results with those measured for a foundation constructed in Gdansk, Poland is presented.

1 INTRODUCTION

The use of piled raft foundations has become more popular in recent years, as both the raft and the piles can provide bearing support as well as contributing towards settlement reduction. The most suitable ground conditions for the use of piled rafts are those where the soil is reasonably stiff at the surface. Soft surface layers will allow transfer of the load mainly onto the piles, and the raft will be ineffective (see Badelow et al. 2006).

Analysis of piled rafts is complex as there is interaction between the raft and the piles as well as pile to pile interaction. Various investigators have devised methods of analysis, including the finite layer methods of Small and Zhang (2002), and Chow and Small (2005), and the method of Clancy and Randolph (1993) that combined finite element analysis for the raft and analytic solutions for the piles. There is also the approach of Kitiyodom and Matsumoto (2003) who calculate interactions using Mindlin’s equations (with an averaging technique for layered soil) and consider both axial and lateral loads.

Finite element analyses have been presented by Katzenbach and Ruel (1997) who incorporated non-linear soil behaviour for the soil into the analysis. Prakoso and Kulhawy (2001) used a two-dimensional finite element model to predict piled raft performance. Reul and Randolph (2003) have also presented a three-dimensional analysis of piled rafts in the overconsolidated Frankfurt clays, and made comparisons with measured behaviour.

Finite element analysis does allow complex geometries and soil behaviour to be modelled. However, the amount of effort required to set up meshes, especially when there are large numbers of piles, is quite time consuming, and large amounts of computer memory is required. It is certainly not suitable for practical design purposes, where the number of piles and their location may have to be altered several times. Combinations of finite element analysis of the raft and analytical or semi-analytical methods for the piles and soil are attractive because of the speed of solution. The computer program GARP (Geotechnical Analysis of Rafts with Piles) that is described in this paper falls into this category, combining finite element analysis for the raft, and boundary element analysis for the piles.

2 METHOD OF ANALYSIS

The contact pressure between the raft and the soil is assumed to be made up of uniform blocks of pressure following a technique originally proposed by Zhemochkin and Sinitsyn (1962). The uniform blocks of pressure are chosen to correspond to the elements of the raft that is divided into eight noded isoparametric thin shell elements. These elements are chosen to be rectangular in shape, and so the uniform blocks of pressure are also rectangular.

The piles are located at the nodes of the finite element mesh for the raft and are assumed to provide a point load acting upward at the node. In order to solve the interaction problem, the unknown loads acting on each of the piles, and the unknown contact pressures acting on each raft element, need to be obtained.

This is done by applying a unit pressure to each rectangular area corresponding to raft element on the surface of the soil and the deflection of the surface is found at the centre of each pressure block, and at the location of each pile. A unit load is also applied to each pile and the deflection at the centre of each pressure block and at each pile location found. The deflections of the piles are found by using an elastic solution based on integrating Mindlin's equation for a point load over segments of the pile shaft, as described by Poulos and Davis (1980). This technique can also be used to find the effect of pile deflection on the surface deflection of the soil. The effect of loading a rectangular region on the surface of the soil can be computed by integrating the Boussinesq solution for a point load over the area of the rectangle. Through this technique, the influence matrix [I_s] may be found for the foundation.

We can therefore write

$$\delta_s = \left[I_s\right] p \tag{1}$$

where

I_s	is the influence matrix for the foundation
δ_s	are the deflections at the centres of each rectangular pressure block, and at the pile locations
p	is a vector containing the unknown contact stresses q acting over each block, and the unknown loads P acting on the piles i.e. $p = (q_1, q_2, \ldots\ldots q_n, P_1, P_2, \ldots\ldots P_m)^T$ if there are n elements and m piles.

The same can be done for the raft to form an influence matrix. A unit load is applied to each raft element in turn and the deflection that this causes at the centre of all raft elements and at the location of each of the piles is found. The deflections so determined form the columns of the influence matrix [I_r] for the raft.

However, the unit loads cannot be applied to the raft unless it is fixed in some way to avoid rigid body rotations and translations. The raft is therefore fixed at one node against rotation in the ***x*** and ***y*** directions and against vertical movement.

We can then write the deflection of the raft as

$$\delta_r = \left[I_r\right] - p + a\Delta + b\theta_x + c\theta_y + \delta_{app} \tag{2}$$

where

I_r	is the influence matrix of the raft
δ_r	are the deflections at the centres of each raft element, and at the pile locations
Δ	is the unknown rigid body translation of the raft
θ_x	is the unknown rigid body rotation of the raft about the x-axis
θ_y	is the unknown rigid body rotation of the raft about the y-axis
δ_{app}	is the deflection caused at the element and pile locations due to the applied loads.

The raft must also be in equilibrium, so vertical force and moment equilibrium about both the x and y axes must be satisfied.

We can therefore write

$$\begin{aligned} \alpha^T p &= P_{Tot} \\ B^T p &= M_x \\ y^T p &= M_y \end{aligned} \qquad (3)$$

where $\alpha^T = (A_1, A_2, \ldots\ldots\ldots A_n, 1, 1, \ldots\ldots\ldots 1)^T$
$B^T = (A_1x_1, A_2x_2, \ldots\ldots\ldots A_nx_n, x_1, x_2, \ldots\ldots\ldots x_m)^T$
$y^T = (A_1y_1, A_2y_2, \ldots\ldots\ldots A_ny_n, y_1, y_2, \ldots\ldots\ldots y_m)^T$

and A_j are the areas of each raft element of which there are n, m is the number of piles, P_{Tot} is the total vertical load applied to the raft, M_x is the total moment about the pin in the x-direction due to loads applied to the raft and M_y is the total moment in the y-direction if x and y are the axis directions in the plane of the raft.

Combining equations 1,2 and 3 gives the final set of equations to be solved for the unknown contact pressures and loads on each pile.

$$\begin{bmatrix} I_s + I_r & -a & -b & -c \\ -a & 0 & 0 & 0 \\ -B & 0 & 0 & 0 \\ -y & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ \Delta \\ \theta_x \\ \theta_y \end{bmatrix} = \begin{bmatrix} \delta_{app} \\ -P_{Tot} \\ -M_x \\ -M_y \end{bmatrix} \qquad (4)$$

By solving equation 4, we have the contact pressures under each element, the pile loads, and the rigid body translations and rotations of the raft. By applying the loads back to the pinned raft, the moments, deflections and shears in the raft can be found.

Non-linear behaviour of the piles can be modelled by reducing the stiffness of the pile in an incremental fashion as load is increased. This is done approximately by using a hyperbolic function to fit the pile load-deflection behaviour up to failure. A reduced pile stiffness means that the deflections in the interaction matrix [I_s] are larger for the piles, and so the diagonal term of the influence matrix at the location of the piles is altered according to the hyperbolic function. The off-diagonal terms are not altered, implying that the interactions do not change.

3 COMPUTER PROGRAM GARP

The above theory has been implemented in a computer program called GARP (Geotechnical Analysis of Rafts on Piles). The program may be run under the Windows operating system, and is graphics based, so that the mesh may be generated and loads and piles can be located with a mouse. Load and pile locations can also be input from coordinates read in from a file.

Uniform loads, point loads, and point moments in the x and y directions can be applied to the raft. Loads can be specified as components (e.g. dead, live, wind, earthquake etc) and combinations of these applied using partial factors on each of the load components.

The elements used for the raft are 8-noded isoparametric shell elements, so that moments and shears in the raft can be calculated. Plots are available for the displacements, rotations, moments, and shears in the raft, and for loads carried by the piles. Plots can also be made of all of these quantities on sections through the raft.

Plots can be sent directly to a printer or saved using Postscript. Section plots can be sent directly to Excel, so that they can be changed to suit the user's taste, and any annotations and graphics added.

4 EXAMPLE

An example of the use of the GARP program, is the analysis of the piled raft foundation constructed for a liquid gas teminal in Gdansk, Poland. The details of the foundation have been presented in a report by ISSMGE Technical Committee No. 18 (Van Impe 2001), with information drawn from Tejchman (2000).

The foundation consists of a raft 71.0m x 61.2m in plan and 0.9m thick that is supported on 180 piles laid out in a rectangular pattern. The piles were bored piles and are of two sizes, 1.0m diameter and 0.62m diameter. The position of the piles is shown in Figure 1, where the smaller diameter piles are shown at the top, bottom and right hand sides of the raft.

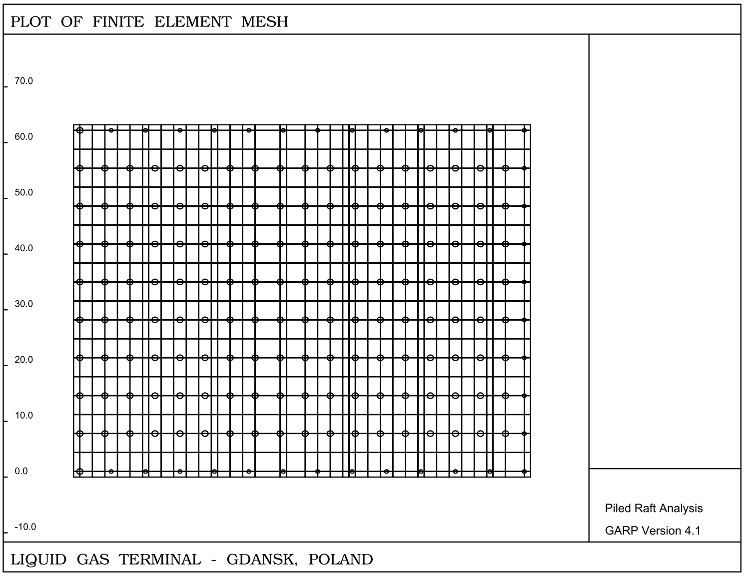

Figure 1: Layout of piles underneath the raft

Table 1: Properties of soil layers used in analysis

Number of layer	Lower level of layer	Type of soil	Modulus E_s (MPa)	Poisson's ratio ν
I	2.9	Fine/Medium sand	32	0.28
II	0	Fine/Medium sand	52	0.28
III	-2.5	Sandy alluvium	0.9	0.37
IV	-7.0	Fine/Medium sand	52	0.28
V	-9.7	Medium sand	76.4	0.25
VI	-11.9	Alluvium	1.3	0.37
VII	-18.0	Medium sand	76.4	0.25
VIII	-22.3	Alluvium	1.3	0.37
IX	-23.6	Medium sand	76.4	0.25
X	-24.0	Alluvium	36.4	0.32
XI		Medium sand	76.4	0.25

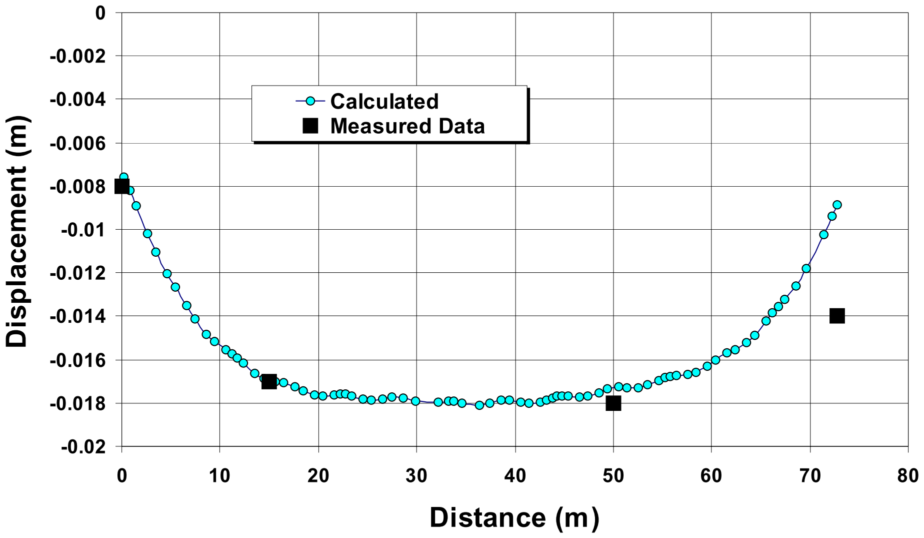

Figure 2: Comparison of calculated and measured settlements along the edge of the raft

The piles were 26.5m long and were driven into a soil profile consisting of interbedded soft alluvium and sand layers. The properties of the various soil layers provided by Tejchman (2000) are shown in Table 1, and these values were used in the GARP analysis.

Figure 2 shows the deflections that were calculated along the edge of the raft compared with the measured vertical deflections (shown as black squares). The prediction matches fairly closely with the measured values, except at the right hand side where the measured value is about 5mm larger.

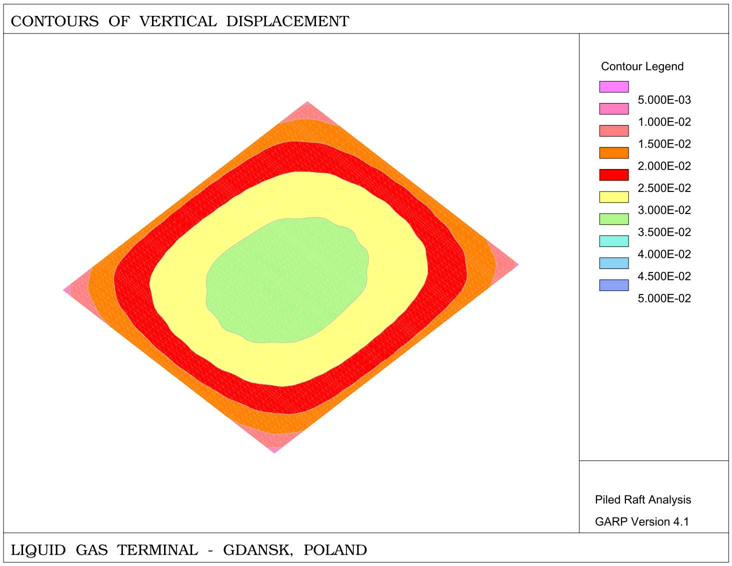

Figure 3: Displacement contours calculated for the piled raft

Contours of displacement for the raft are shown in Figure 3. These show that the maximum displacement is at the centre of the raft and is a little above 30mm. This is similar to the magnitude of predictions made by other investigators in the report by (Van Impe 2001).

5 SUMMARY AND CONCLUSIONS

The theory behind the computer program GARP has been presented. The program is able to deal with complex pile and raft geometries and layered soil profiles. It may also be used to take into account non-linear behaviour of the piles. A case study of the foundation for a liquid gas storage tank constructed in Gdansk, Poland showed that reasonably good prediction of foundation settlement could be obtained with the program.

REFERENCES

Badelow, F., Poulos, H.G., Small, J.C. and Moyes, P. (2006) *Economic Foundation Design for Tall Buildings*. Proceedings of 10th International Conference on Piling and Deep Foundations (DFI – Amsterdam, 31 May-2 June, 2006), Eds. J.Lindenberg, M.Bottiau, A.F. van Tol, 200-209.

Chow, H.S.W. and Small, J.C. (2005) *Finite layer analysis of combined pile-raft foundations with piles of different lengths*. Proceedings of the 11th IACMAG (International Conference on Computer Methods and Advances in Geomechanics), June 19-24, Turin, Italy, 2005, Editors Giovanni Barla and Marco Barla, Patron Editore (pub.). 2, 429-436.

Clancy, P. and Randolph, M.F. (1993) *An approximate analysis procedure for piled raft foundations*. Int. Jl. For Numerical and Analytical Methods in Geomechanics. 17(12), 849-869.

Katzenbach, R. and Reul, O. (1997) *Design and performance of piled rafts*. Proc. of XIVth ICSMFE 97, Hamburg. 4, 2253-2256.

Kitiyodom, P. and Matsumoto, T. (2003) *A simplified analysis method for piled raft foundations in non-homogeneous soils*. Int. Jl. Numerical and Analytical Methods in Geomechanics. 27, 85-109.

Pakso, W.A. and Kulhawy, F.H. (2001) *Contribution to piled raft foundation design*. Jl. Geotechnical and GeoEnvironmental Engng., *ASCE*. 127(1), 17-24.

Poulos, H.G. and Davis, E.H. (1980) *Pile foundation analysis and design*. Wiley, New York.

Reul, O. and Randolph, M.F. (2003) *Piled rafts in overconsolidated clay: comparison of in situ measurements and numerical analyses*. Géotechnique. 53(3), 301-315.

Small, J.C. and Zhang, H.H. (2002) *Behaviour of piled raft foundations under lateral and vertical loading*. International Journal of Geomechanics. 2(1), 29-45.

Tejchman, A. (2000) *Calculations and measurements for three piled foundations*. Internal ISSMGE TC-18 Memorandum, October, Technical University of Gdansk.

Van Impe, W. F. (2001) *Case histories of the performance of pile-assisted rafts*. A report by Technical Committee No. 18, W.F. Van Impe, Chair.

Zhemochkin, B.N.and Sinitsyn, A.P. (1962) *Practical methods of designing foundation beams and slabs resting on an elastic foundation*. 2nd Ed. Moscow: State Publishing House for Literature on Structures, Architectural and Structural Materials.