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Applicability of a new strength criterion in comparison to failure criteria

H. Bineshian[1], Abdolhadi Ghazvinian[2], Z. Bineshian[3]

[1]School of Civil Engineering, The University of Western Australia; email: Bineshian@gmail.com
[2]Tarbiat Modares University, Tehran, Iran; email: abdolhadi@yahoo.com
[3]Perlite Co, Tehran, Iran; email: leilabineshian@gmail.com

ABSTRACT

A comparison between a proposed strength criterion by the authors and four most popular failure criteria has been done in this research. The results of evaluation of the criteria show better correlation for the proposed strength criterion with tri-axial data. The calculated coefficient of determination for the proposed criterion is near to unity. Also accordance of the criteria for tension has been assessed, which shows better accordance between the proposed criterion and tensile data. The proposed strength criterion describes well the available data of geo-materials. The fit is good for both compression and tension, and the availability of fit is better than other known criteria. The proposed criterion includes three parameters, which vary for different materials. These parameters are inherent characteristics and adjustment factor that are depending on the geo-materials mechanical properties. These parameters have been presented in this paper for a variety of geo-materials with different range of strengths as constant values to use in the proposed strength criterion to predict the unconfined and tri-axial compressive and tensile strength of geo-materials in practical geomechanical and geotechnical projects and purposes.

Keywords: strength/failure criterion, compression, tension, parameters

1 INTRODUCTION

Nonlinearity is the common behaviour of geo-materials (von Karman 1911, Griggs 1936 and Mogi 1965). Griffith (1921) developed a nonlinear criterion based on the initiation of crack propagation in brittle materials. Murrel (1963) developed a nonlinear compressive strength criterion, which has been revised by Bieniawski (1974) to improve its applicability. Bieniawski criterion predicts the strength for positive confining pressures reliably (Bineshian 2000). Hoek and Brown (1980) based on McClintock and Walsh modifications (1962) to Griffith theory (1921), proposed a new criterion for both tension and compression. Hoek-Brown criterion is widely using in practical projects and calculations of geo-materials' strength. Ramamurthy et al (1985) also proposed a nonlinear strength criterion for only compression that predicts the compressive strength of geo-materials appropriately.

Not only the existing criteria are not adequately comprehensive, applicable under different conditions, and flexible at practical circumstances, but also their precision and accuracy will be lost when they are generally applied to different geo-materials (Bineshian 2012) and they cannot be applied for all geo-materials. The proposed compressive-tensile strength criterion is based on theoretical and experimental study on failure of soil, rock, coal, and concrete including data on 1743 samples of intact rocks. The proposed criterion introduces an exact linear-nonlinear Mohr envelope on tri-axial data depending on natural trend of (σ_3, σ_1) data pairs. The calculations of unconfined compressive/tensile strengths are precisely possible by the proposed criterion. Authors to evaluate the applicability of the proposed criterion compared it with four most applicable criteria: Coulomb-Mohr, Bieniawski (1974), Hoek-Brown (1980) and Ramamurthy et al (1985), for more than 250 intact rocks. Afterwards constant values for the proposed criterion have been suggested.

2 STRENGTH CRITERIA

Coulomb-Mohr criterion (eq.1) is a linear compressive-tensile criterion. Bieniawski criterion (eq.2) is the normalized form of Murrel criterion (1963) for intact rocks, which is applicable only for compression. Hoek-Brown strength criterion (eq.3) is a compressive-tensile criterion for intact rocks based on the McClintock and Walsh modifications (1962). Ramamurthy et al strength criterion (eq.4) is a compressive criterion that uniaxial compressive strength cannot be calculated by this criterion.

$$\sigma_1 = C + B\sigma_3 \tag{1}$$

$$\frac{\sigma_1}{\sigma'_c} = 1 + B\left(\frac{\sigma_3}{\sigma'_c}\right)^{\alpha} \tag{2}$$

$$\frac{\sigma_1}{\sigma'_c} = \frac{\sigma_3}{\sigma'_c} + \sqrt{\frac{m\sigma_3}{\sigma'_c} + 1} \tag{3}$$

$$\frac{\sigma_1 - \sigma_3}{\sigma_3} = B\left(\frac{\sigma'_c}{\sigma_3}\right)^{\alpha} \tag{4}$$

The proposed strength criterion (eq.5) is a compressive-tensile criterion that has been obtained after assessing different functions and analysing the results of more than 8'700 fittings.

$$\sigma_1 = \sigma_3 + \sigma_c\left(\frac{\sigma_c + \lambda\sigma_3}{\sigma_c + \zeta\sigma_3}\right) \quad , \quad \sigma_c = \beta C \tag{5}$$

Hereafter, σ_1 and σ_3 are the major and minor principal stresses in failure, σ'_c and σ_t are unconfined compressive and tensile strengths, B, α, and m are the parameters of the criteria, σ_c is regulated unconfined compressive strength (RUCS), C is apparent unconfined compressive strength (AUCS), which may be obtained from Coulomb–Mohr criterion, λ and ζ are rock constants, and β is an adjustment factor for AUCS. We emphasise that C cannot be assumed as the real value of unconfined compressive strength because it is obtained from the linear envelope on nonlinear tri-axial data. Thus, parameter β in eq.5, adjusts C as per nonlinear natural trend of tri-axial data, and yields the closest value to the real unconfined compressive strength as RUCS. Hence, σ_c and σ_t are calculated from eq. 5 as follows:

$$\sigma_3 = 0 \quad \rightarrow \quad \sigma_c = \beta C \tag{6}$$

$$\sigma_1 = 0 \quad \rightarrow \quad \sigma_t = \frac{-\lambda - 1 + \sqrt{\lambda^2 + 2\lambda - 4\zeta - 1}}{2\zeta}\beta C \tag{7}$$

The proposed criterion shows a highly satisfactory correlation with tri-axial data and absolutely abides by the natural trend of data and estimation of strength by this criterion is very tending to certainty in comparison to the selected criteria (next section).
Figure 1 shows fitting the proposed criterion on tensile tri-axial data of Johnston (1985); selected criteria except Coulomb-Mohr cannot be fitted on this data group.

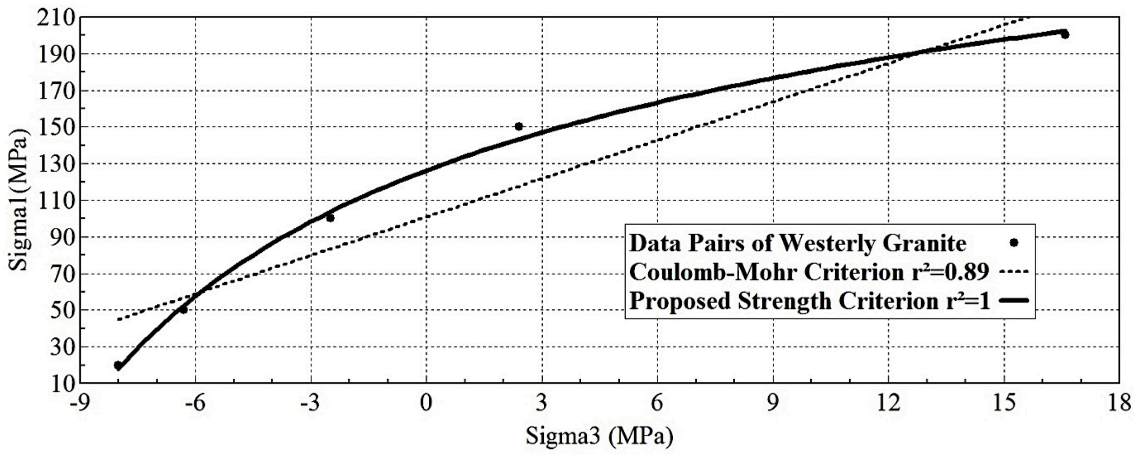

Figure 1. Fitting the proposed strength criterion on the Westerly granite at 600˚C

3 QUALIFICATIONS

Authors applied seven essential qualifications to data groups to exclude unsuitable data from analyses:

1. Each data group should contain at least five data pairs,
2. All data groups should include both principal stresses in failure and UCS data,
3. All data pairs must satisfy Mogi's (1966) transition limit,
4. Each data group must contain results of tri-axial tests with $\sigma_3>0$,
5. Data groups should contain range of low to partly high confining pressure (Hossaini 1993),
6. Data groups with $\sigma_3 - \sigma_1$ upward concave, should be excepted,
7. Data pairs with $\sigma_1 < \mathrm{UCS}$, must be excluded throughout the data pairs.

After applying the qualification criteria, more than 250 data groups of tri-axial compressive tests consist of 1'743 data pairs – (σ_3, σ_1) – have been qualified. Qualified data groups contain a variety of intact igneous, sedimentary and metamorphic rocks and coals. Data groups in this research collected from Schwartz (1964), Horino & Ellikson (1970), Ouyang & Elsworth (1991), Hossaini (1993), Sheorey (1997), Mahab Ghodss (2003), Bineshian (2000), and Bineshian & Perlite (2008). To determine the criteria's parameters, we have used linear regression method for Coulomb-Mohr, and nonlinear regression method (Nelder and Mead, Netter et al 1988) for the proposed and other selected criteria by considering $\mathrm{X_i} = \sigma_3$ and $\mathrm{Y_i} = \sigma_1$. Coefficient of determination (r^2) and accordance coefficient (ψ^2) have been calculated to assess correlation and accordance of the criteria with data. There is a good correlation between criterion and tri-axial data if $\mathrm{r}^2 \to 1$ and accordance of function with tri-axial data is increased when $\psi^2 \to 0$.

4 EVALUATION OF THE STRENGTH CRITERIA

The coefficient of determination has been calculated for more than 250 reliable tri-axial data groups of geo-materials to assess correlation of the selected criteria with tri-axial data to compare their applicability. Table 1 indicates the excellent correlation between the proposed criterion and actual data, which can be ranked the first. Figure 2 shows fitting the selected criteria on data of Betourney et al (1991) for Strathcona Gneiss, which compressive criteria because of existence of tensile data pairs, cannot be regressed.

Table 1: Evaluation of correlation between criteria and tri-axial data

Criteria	Coefficient of Determination (r^2)			
	=1.00	≥0.98	≥0.96	≥0.94
The Proposed	47%	87%	93%	98%
Bieniawski	22%	53%	67%	78%
Hoek-Brown	09%	44%	63%	73%
Ramamurthy et al	11%	40%	59%	66%
Coulomb-Mohr	02%	22%	43%	58%

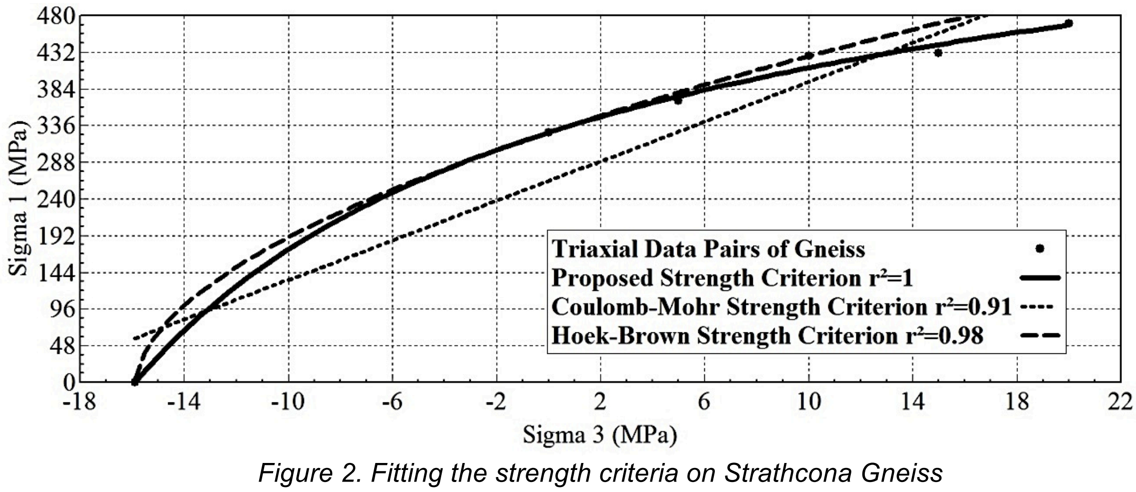

Figure 2. Fitting the strength criteria on Strathcona Gneiss

Evaluation of compatibility of the selected criteria with tri-axial data (from the aspect of accordance with natural trend of data) has been done, by calculating the accordance coefficient (Table 2) for 12 data groups of Chamshir Limestone (Bineshian 2000). From Table 2, the proposed criterion shows minimum regression fitness errors; hence it has a satisfactory compatibility with natural trend of tri-axial data, and therefore can present better estimation of tri-axial strengths in comparison to others.

Table 2: Compatibility of strength criteria with tri-axial data of Limestone (Bineshian 2000)

Criteria	Accordance Coefficient (ψ^2)				
	≤0.001	≤0.005	≤0.010	≤0.050	≤0.150
The Proposed	9%	25%	42%	75%	92%
Bieniawski	0%	9%	17%	75%	83%
Hoek-Brown	0%	0%	0%	67%	83%
Ramamurthy et al	0%	0%	17%	50%	83%
Coulomb-Mohr	0%	0%	0%	67%	75%

Uniaxial tensile strength (σ_t) by the criteria for 22 reliable tri-axial data groups of coals (Hobbs 1964) has been compared with uniaxial tensile strength obtained by Brazilian tests (UTS) to verify tensile applicability of the compressive-tensile criteria (The Proposed, Hoek-Brown and Coulomb-Mohr). The proposed strength criterion in 82% and 100% of all cases compared to Hoek-Brown and Coulomb-Mohr respectively yields better estimations for σ_t (Table 3). Also it can be concluded that:

$$\sigma_{\text{t Proposed}} = 1.20\times\text{UTS} \tag{8}$$

$$\sigma_{\text{t Hoek−Brown}} = 2.20\times\text{UTS} \tag{9}$$

$$\sigma_{\text{t Coulomb−Mohr}} = 5.20\times\text{UTS} \tag{10}$$

That means the estimation of tensile strength by Hoek-Brown and Coulomb-Mohr criteria are not accurate and reliable. However, the proposed criterion’s estimation for tensile strength has an acceptable accuracy.

Figure 3 presents a summary of accordance of the criteria with aforementioned 22 data groups for tension, which shows the best compatibility of the proposed criterion with tensile data in comparison to the selected criteria.

Table 3: Comparison between σ_t and UTS for Hobbs (1964) Coal data groups

Groups	UTS (Mpa)	σ_t (Mpa)		
		The Proposed Criterion	Hoek-Brown	Coulomb-Mohr
1	-0.83	-1.06	-0.90	-2.85
2	-0.76	-1.09	-1.89	-5.17
3	-3.41	-3.69	-9.99	-14.55
4	-1.79	-2.08	-2.99	-6.46
5	-2.07	-2.39	-3.54	-7.39
6	-2.45	-2.69	-3.51	-7.16
7	-0.69	-0.70	-1.10	-1.88
8	-2.69	-3.57	-2.10	-4.79
9	-0.69	-1.33	-0.64	-2.96
10	-0.55	-1.18	-0.38	-2.75
11	-0.62	-0.66	-3.49	-5.64
12	-0.76	-0.79	-3.18	-7.00
13	-1.58	-1.75	-0.96	-8.01
14	-0.89	-1.22	-1.46	-3.78
15	-1.10	-1.71	-2.96	-7.28
16	-1.10	-1.18	-2.60	-5.94
17	-1.10	-1.54	-3.08	-6.94
18	-0.69	-0.90	-1.33	-2.99
19	-0.55	-0.95	-1.35	-3.90
20	-0.76	-0.88	-3.29	-5.87
21	-1.10	-1.64	-2.11	-5.32
22	-0.89	-1.15	-1.86	-4.75

5 CONSTANT VALUES

Calculation/prediction of the strength of geo-materials by the proposed criterion can be done by two methods:

- Performing required tri-axial tests, fitting the criterion on the obtained data, calculating the parameters and finally applying the obtained parameters to the criterion,
- Selecting suitable suggested values for the parameters and then applying them to the criterion.

The first method is the most accurate way, because it is done especially for the certain geo-material; however the second method is very popular in application. Authors examined the proposed criterion to determine the most accurate constant values to make it more credible using the second method by many tests and reliable data. Nonlinear regression has been used to calculate the parameters of the proposed criterion, meanwhile strange values excepted during calculations and finally average values for λ, ζ and β for each rock type has been considered. Table 4 presents the parameters for igneous, sedimentary, and metamorphic rocks and coals. Summarily average rounded values vary between 14.50 - 10 for λ, between 4.50 - 2.50 for ζ and between 1 - 0.80 for β.

Application of constant values easily can be done by selecting the proper values for λ, ζ and β from Table 4 and substituting them for the parameters in the proposed criterion. The average rounded values, which are presented for each rock group, can be used for rock types that are not presented in this research. Also from Table 4, σ_c for igneous and metamorphic (eq.11), sedimentary (eq.12) rocks and coals (eq.13) are as below.

$$\sigma_c = C \tag{11}$$

$$\sigma_c = 0.90 \times C \tag{12}$$

$$\sigma_c = 0.80 \times C \tag{13}$$

Table 4: Parameters for the proposed criterion applicable to intact geo-materials

Geo-materials	Averages		
	λ	ζ	β
Andesite	12.49	8.03	1.03
Basalt	8.98	7.95	1.50
Diabase	13.49	4.16	1.17
Diorite	7.05	3.38	1.06
Dunite	7.93	0.51	0.57
Gabbro	29.22	10.40	0.82
Granite	12.19	2.72	0.89
Granodiorite	10.02	1.84	1.10
Lamprophyre	9.21	4.70	1.25
Peridotite	4.79	0.45	0.79
Quartzdiorite	10.24	2.79	1.03
Rhyolite	8.40	3.71	1.41
Tuff	6.51	0.55	0.95
Average Rounded Values (Igneous)	**11**	**4**	**1**
Dolomite	12.84	4.01	1.04
Limestone	6.95	2.24	0.84
Sandstone	9.46	2.11	0.87
Shale	6.55	4.07	0.95
Siltstone	14.97	4.96	0.83
Average Rounded Values (Sedimentary)	**10**	**3.50**	**0.90**
Eclogite	7.01	0.34	0.55
Gneiss	16.91	6.58	0.92
Marble	4.91	0.87	0.95
Quartzite	11.82	3.22	1.17
Schist	13.47	2.43	1.11
Slate	29.02	14.40	1.34
Average Rounded Values (Metamorphic)	**14**	**4.50**	**1**
Coal (Loading, ⊥bp ǁ mc)	17.75	2.93	0.90
Coal (Loading, ǁ bp ǁ mc)	12.58	1.74	0.68
Coal (Loading, ǁ bp⊥mc)	14.91	2.28	0.76
Coal*	13.31	2.42	0.80
Average Rounded Values (Coals)	**14.50**	**2.50**	**0.80**

* No bp and mc considered (bp is bedding plane and mc is main cleat).

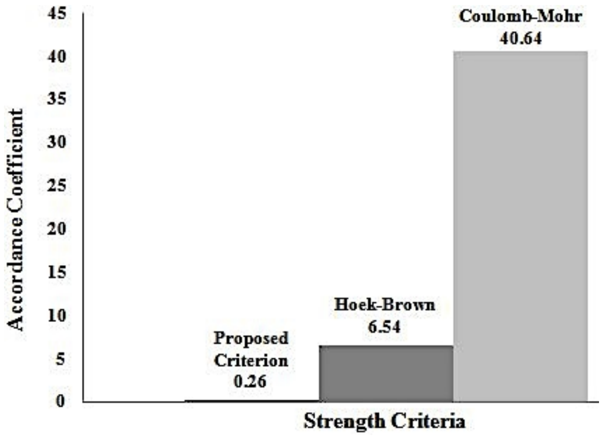

Figure 3. Accordance coefficient for strength criteria with pairs of tensile data

6 CONCLUSION

A comprehensive strength criterion should cover both linearity and nonlinearity and should be applicable for both tension and compression. The proposed criterion is based on this idea and satisfies the features that a comprehensive criterion should enjoy. It contains three parameters; λ, ζ and β, that λ and ζ depend on inherent characteristics of geo-materials and β is an adjustment factor to C. Hence σ_c is a regulated value of C. In this research the average value of σ_t by the proposed criterion is about $1.20\times\mathrm{UTS}$, however this factor for Hoek-Brown and Coulomb-Mohr criteria are 2.20 and 5.20 respectively. Thus the proposed criterion predicts the tensile strength close to UTS and more accurate than others. The coefficient of determination for the proposed strength criterion is adjacent to 1 that shows a good correlation between the criterion and tri-axial data. The accordance coefficient for the proposed criterion shows an excellent accordance between tri-axial data and the criterion. Thus the estimation of tri-axial strength by the proposed criterion in comparison to the selected criteria is more credible. Authors used nonlinear regression to evaluate the proposed criterion in view of correlation and accordance with tri-axial data and calculating the parameters. The proposed constant values can be used in practical purposes with an acceptable accuracy.

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