Consolidation of embankments on soils with anisotropic permeability

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ABSTRACT

Semi-analytic solutions are presented for the consolidation of layered soils that have anisotropic permeability. The method involves applying a Fourier transform and a Laplace transform to the governing equations, solving the equations, and then inverting the transforms to obtain the solution in real time. This means that the solution can be found at any time directly without having to 'march' the solution forward using values obtained at previous times. The results are compared with finite element solutions, and it is shown that the numerical integration schemes used to 'march' the FE solutions forward can greatly affect the results, and recommendations for the integration schemes that give the best FE solutions are given. Design charts are then provided for embankment shaped loadings on consolidating soil layers where the permeability of the soil is anisotropic.

Keywords: Settlement, anisotropy, layered soil, embankments.

1 INTRODUCTION

The problem of the time-dependent settlement of embankments and other structures built on low permeability soils such as clays has been of interest to engineers since Terzaghi (1923) first presented his theory for consolidation of uniform soil layers under one-dimensional conditions. However soil layers are often not uniform and homogeneous, and may consist of several layers of different soil types and each soil type may have anisotropic permeability. Anisotropy of permeability is frequently observed in sedimentary soils where the process of deposition means that the soil is more permeable along the layer than vertically upward (i.e. perpendicular to the layer). The ratio of the horizontal to vertical permeabilities can be very large, and it is therefore necessary to be able to take this into account when making predictions of the rates of consolidation.

Solutions to the consolidation of soils with anisotropic permeability have been obtained in the past through the use of finite difference methods (Poskitt (1970), Davis and Poulos (1972)). Finite difference techniques however are not particularly easy to use, particularly when the soil is layered and so finite element techniques have become more popular for analysis of consolidation problems. Solutions to the problem of anisotropic permeability were presented by Desai and Saxena (1977) for example.

Although finite element methods are very flexible, and any geometry and material properties can be used, there is still a reasonable amount of effort required to set up the mesh and obtain the solution. Numerical errors may be introduced by the proximity of boundaries and by the numerical schemes used to 'march' the solution forward.

In this paper therefore, a semi-analytical solution to the problem of the consolidation of layered soils with anisotropic permeability is presented. As the solutions are semi-analytic, they can be used to calibrate finite element solutions and provide an independent check on the effects of numerical error such as element type, mesh size and boundary position and conditions.

The semi-analytic solution has been incorporated into the computer program CONTAL, and this code has been used to obtain solutions for the consolidation of embankment shaped loadings constructed on clay layers that have anisotropic permeabilities.

2 THEORY

The theory of consolidation that is used here is based on the theory of Biot (1941) which assumes that the soil is saturated, and so any water that flows from an element of the soil is accompanied by a change in volume of the element that is equal to the volume of water that is squeezed out.

The equations governing consolidation can be simplified by applying a Fourier transform (or a Hankel transform if the problem is axi-symmetric) and then a Laplace transform to these equations. Once this is done, the equations are more tractable and can be solved in transform space. These solutions are exact, in that they do not involve any numerical procedures. However to apply the inverse transforms to the solutions is difficult, and so numerical integration is used to invert the Fourier (or Hankel) transforms and a numerical inversion technique (due to Talbot (1979)) used to invert the Laplace transforms. The method is therefore a semi-analytic procedure. It may be noted that the solution can be obtained at any time directly, without having to obtain solutions from previous solutions (i.e. using a ‘marching’ process) as is common with finite difference and finite element solution methods.

3 COMPARISON WITH FINITE ELEMENT SOLUTIONS

As was mentioned in the introduction, finite element solutions may be prone to numerical errors, and so an examination is presented here of one type of error: that of the time integration scheme.

The well known set of finite element equations that is solved is presented in equation (1). In this equation, **K** is the stiffness matrix, **L** is a coupling matrix and **Φ** is the flow matrix that governs the flow of pore water through the soil. The equations (1) may be set up for a non-linear constitutive law, and if the soil has an associated flow rule and isotropic permeability, the equations will be symmetric and only half of the consolidation matrix need be stored. If the soil has anisotropic permeability, or the soil skeleton has a non-associated flow rule, then either the flow matrix or the stiffness matrix is non-symmetric, and the full set of finite element equations must be set up and solved.

$$\begin{bmatrix} K & -L^T \\ -L & -(1-\alpha)\Delta t\Phi \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta q \end{bmatrix} = \begin{bmatrix} \Delta f \\ \Delta t\Phi q_t \end{bmatrix} \qquad (1)$$

In equation 1, the increments of deflection $\Delta\delta$ and of the excess pore water pressure $\mathbf{\Delta q}$ are found from the solutions at a previous time t, $\boldsymbol{\delta_t}$ and $\mathbf{q_t}$ and so the current solutions at time t+Δt may be found where Δt is the time step. This process means that the solution can be ‘marched’ forward in time with each solution being found from a previous solution i.e. $\mathbf{q_{t+\Delta t} = q_t + \Delta q}$. The parameter α in equation 1 arises from the numerical integration scheme that is used to integrate the field quantities with respect to time. It can be shown that for the ‘marching’ process to be unconditionally stable (Booker and Small (1975)) the value of α must lie between 0 and 0.5.

In order to test which value of α gives the best finite element result, the problem of a circular uniform loading q applied to the region $0 \le r \le a$ on the surface of a soil layer of depth h is examined. The entire upper surface of the soil is assumed permeable and the base is assumed impermeable. The parameters used for the solution are presented in Table 1.

Table 1: Properties used in finite element analysis

Quantity	Value
Drained modulus of elasticity	10,000kPa
Drained Poisson’s ratio	0.35
Radius of load *a*	8m
Depth of layer *h*	16m
Horizontal permeability k_h	0.0001m/day
Uniform load *q*	80kPa

The settlement at the central point of the loading versus time is presented in Figure 2 where the semi-analytic solution is presented for two cases. The first case is for the vertical permeability k_v being larger than the horizontal permeability k_h ($k_v = 10k_h$), and the second case where the vertical permeability is less than the horizontal permeability ($k_v = 0.1k_h$). These solutions are compared to finite element solutions where different values of α are used.

It may be seen from the plot that when α=0, the semi-analytic and the finite element solutions are almost identical. However when the value of α=0.5, the finite element solution is shifted in time and is not very accurate. Hence the α=0 scheme is best for general use in such finite element formulations.

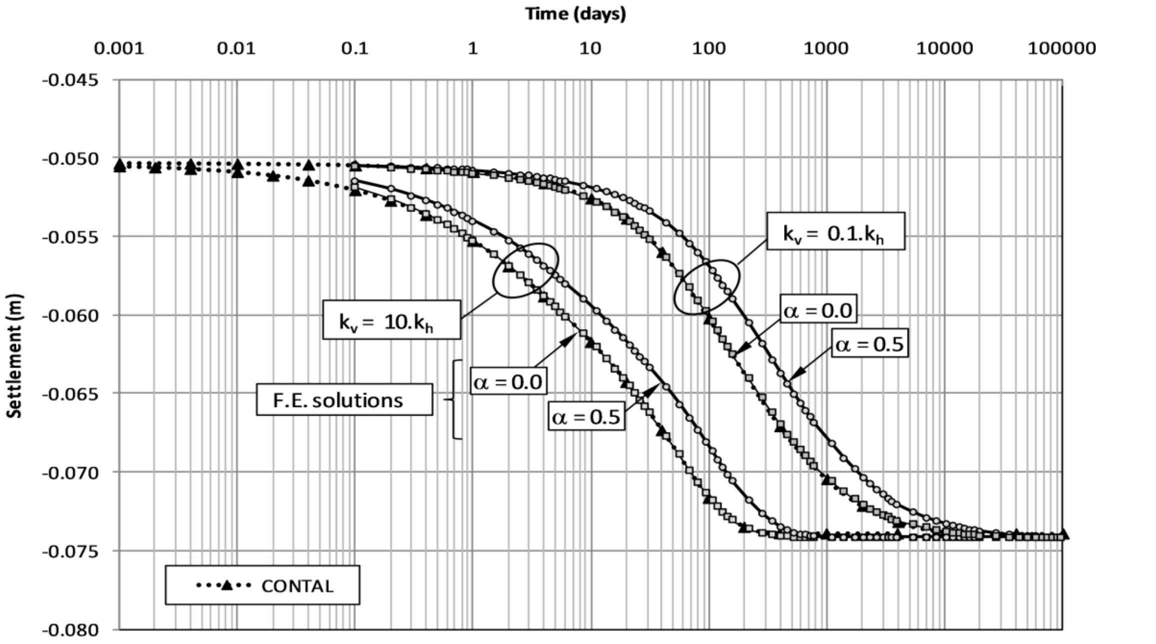


Figure 2. Settlement versus time for central point of circular loading on a soil layer of finite depth.

4 EMBANKMENTS

If the shape of the loading is of an embankment shape, then the Fourier transform needs to be taken of the loading function. For the embankment shown in Figure 3, the transform of the loading q(ρ) is given by

$$q(\rho) = \int_0^\infty q(x) cos\rho x \, dx \tag{2}$$

$$q(\rho) = \frac{\gamma H(t)}{(D-B)\rho^2}\{(1 - cos\rho D) - (1 - cos\rho B)\} \tag{3}$$

where B is the crest half-width, and D is the base half-width of the embankment, γ is the unit weight of the embankment, and H(t) is the height of the embankment at time t.

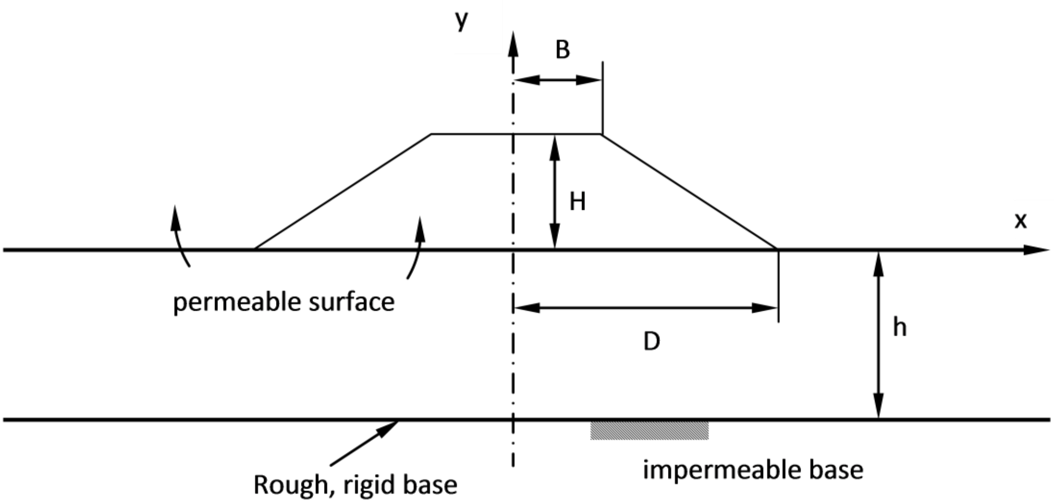


Figure 3. Embankment shaped loading.

The Laplace transform of the loading function in time may also be taken. If it is constant, then the transform of the loading q is simply q/s (where s is the Laplace transform parameter). However

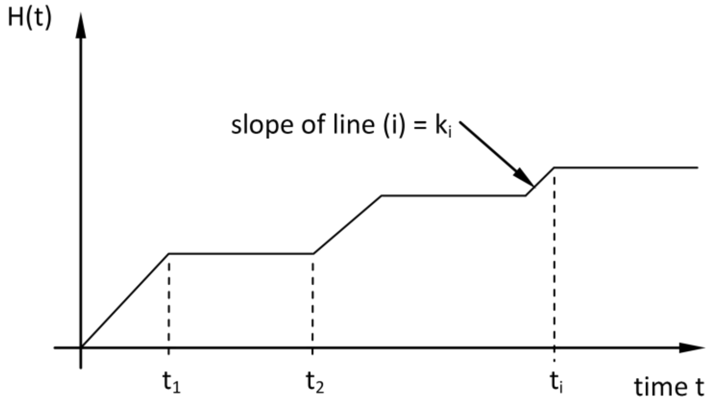


Figure 4. Embankment height as a function of time.

embankments are generally constructed over time, and so the embankment may rise and then stay at a constant height before rising again. Therefore it is useful to obtain the transform of a piecewise linear function for the load as shown in Figure 4. The Laplace transform is given by

$$\mathcal{H}(s) = \int_{-\infty}^{\infty} H(t)e^{-st}\, dt \tag{4}$$

and so for the piecewise linear function

$$\mathcal{H}(s) = e^{-t_1 s}.\frac{k_1}{s^2} + \sum_{i=2}^{n} e^{-t_i s}\frac{(k_i - k_{i-1})}{s^2} \tag{5}$$

Where k_i is the slope of the *i*th segment and t_i are the times at which the loading rate changes as shown in Figure 4 .
If a Laplace transform is applied to equation 3, the value of $\mathcal{H}(s)$ given in equation 5 can be substituted to give the expression for the embankment loading in transform space. The solutions of the consolidation equations for this loading function can then be inverted to obtain the embankment loading solution at any time *t*.

5 EXAMPLES

An example of an embankment that is built up to a height H at time t_1, and then held at a constant height is now given. The embankment geometry is the same as shown in Figure 3 where the side slope is at 2H:1V. This means that D = B+2H. The soil layer (of depth h) is uniform with respect to stiffness properties E and ν, but has a higher lateral permeability k_h than the vertical permeability k_v. The unit weight of material in the embankment is γ and the upper surface is permeable while the base of the soil layer is considered impermeable.

The embankment is built up at a constant rate until a time factor of τ_1 = 0.7 where

$$\tau = \frac{c_v t}{B^2} \tag{6}$$

and

$$c_v = \frac{k_v E(1-\nu)}{\gamma_w(1+\nu)(1-2\nu)}$$

where E is the elastic modulus and ν is the Poisson's ratio of the soil.

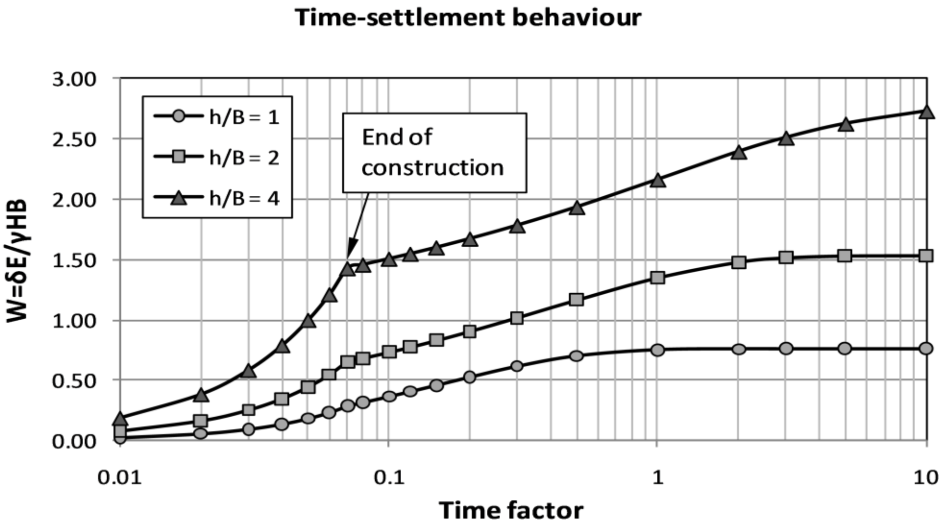


Figure 5. Time-settlement behaviour of an embankment on soil layers of depth h with anisotropic permeability $k_h = 5k_v$.

Solutions are presented for the non-dimensional settlement *w* of the central point of the base of the embankment versus the time factor τ for a permeability ratio of $k_h = 5k_v$.

A second example is one of a tall building that was to be constructed on a raft foundation. The increase in settlement with time of the surrounding ground surface was of interest, and so the structure was modelled as a uniform loading applied over a rectangular region of half-widths 25.7m and 34.6m, that increased with time from zero up to 422kPa at 2 years and then remained constant. One quarter of the plan of the structure is shown in Figures 6a,b as the shaded area.

The soil was layered and the description of the soil layers and their properties are shown in Table 2. The upper surface of the soil was assumed to be permeable, while the base of the layer was assumed to be impermeable.

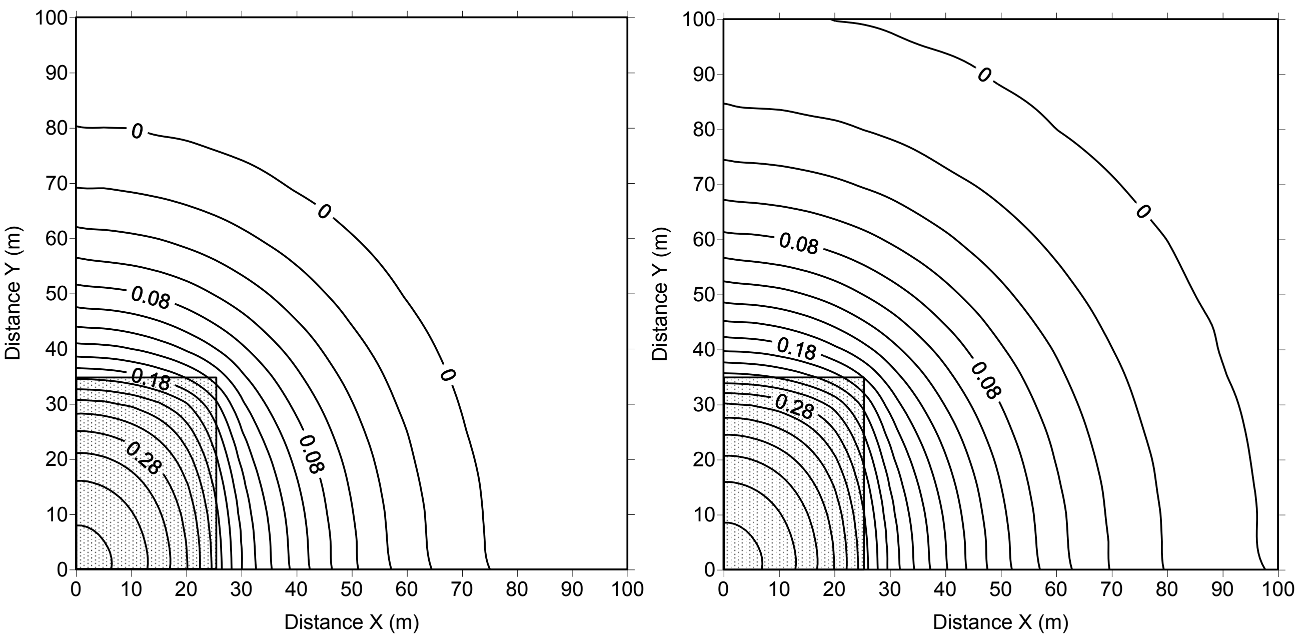
The contours of vertical settlement at 2 years and at large time are shown in Figures 6a and 6b respectively. The increase in settlement with time may be seen from these two figures, as can the spread of the region that has settled.

Table 2 Properties used for layered soil foundation

Layer	Thickness (m)	Modulus (MPa)	Permeability (m/yr)
SAND, Silty SAND, dark grey, very dense, moderately to strongly cemented.	27.8	100	0.3
CLAY, brownish grey to dark grey, high plasticity, stiff to very stiff, with some interbedded layers of silty sand, or organic clay.	46	20	0.000004
Sandy SILT, dark grey, moderately to well cemented, medium plasticity	20	100	0.3

6 CONCLUSIONS

A semi-analytic method has been presented that enables solutions to be obtained to problems involving horizontally layered soils with anisotropic permeability subjected to time-dependent loadings of various shapes. Some solutions have been evaluated to demonstrate the use of the method on some realistic problems.



(a) (b)

Figure 6. Settlement contours (in m) around a rectangular loaded area (a) at 2 years (end of construction) (b) at large time.

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