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# Reasonable Variation Between Slope Stability Analysis Methods

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## ABSTRACT

Slope stability analyses have been routinely performed using various software packages since the 1980s. There have been a variety of software packages that have been developed at the academic and the commercial level to accommodate the analysis of slopes. Numerous software packages are used by geotechnical consultants little understanding of the potential differences that can occur amongst various methods of analysis that have been implemented. Also, some of the earlier software packages were not verified against significant numbers of benchmark examples. Recent research efforts by the authors have attempted to develop libraries of benchmark problems to which slope stability software packages can be compared. This paper summarizes an extensive comparison study of benchmarks examples and other software packages performed during the development of the SVSLOPE software package. Various potential sources of variation are analysed. Comparisons between classic limit equilibrium method of slices are summarized and reasonable variations are presented.

Keywords: slope stability, limit equilibrium, landslides, stability analysis

## 1 INTRODUCTION

There are many sources of potential variation when performing a slope stability analysis. Some of these sources of uncertainty are quantified as parameter uncertainty, model uncertainty, and human uncertainty (El-Ramly et al., 2002) The potential variation in slope stability analysis was recently examined in detail by SoilVision Systems Ltd. during the development of the SVSLOPE® software package. A database of over 150 benchmarks was compiled in order to i) benchmark the software and, ii) gain an increased awareness of potential variation in slope stability analysis. Most historical analysis of the potential variation in the Factor of Safety (FOS) focus on the potential impact of differing material properties. This paper seeks to examine the potential sources of variation within a typical limit equilibrium software package. Most examinations of sources of uncertainty have ignored the potential model uncertainty. Model uncertainty can specifically be divided into three sources, namely; i) differences between analysis methods, ii) difference between critical slip surface searching methods and ii) differences between implementations in various software packages.

Differences between analytical methods are to be expected since different methods of analysis are based on slightly different theories. This is readily illustrated by comparing the Ordinary method of slices and the Bishop method of slices. The differences are related to the assumptions associated with the inter-slice forces designations.

It is important to note the separation between a method of analysis and a critical slip surface searching method. The processes are largely independent and different searching methods can be applied to a wide variety of analysis methods. Searching methods can sometimes be responsible for a larger variation in the results than the variations in the theory associated with the method of analysis.

Also noteworthy is other differences that may exist between the implementation of analysis methods in various software packages.

Most notably this paper attempts to provide answers to the following questions:

- What is the variation expected between different methods of slope stability analysis (i.e. Bishop, GLE, Spencer, etc.)?
- What is the reasonable variation between computed factors of safety amongst software packages?

- What are some causes of observed variations?
- Is the variation the same between the various searching methods?

In summary this paper attempts to provide insight into potential model variation as a source for uncertainty.

## 2 BENCHMARKING HISTORY

Historical attempts to verify the accuracy of slope stability software have focused on developing a library of example models. Notable efforts to develop benchmarks have included the following:

- ACADS study performed by the Monash University Melbourne, Australia (Giam & Donald, 1989)
- Chou:
- Government of HK:

Such efforts have noted the inherent problem of determining a single "correct" answer. Determining such an answer is generally only possible where the example problem is simple and the calculations may be simplified such that they can be performed by hand-calculations. There is significant difficulty with this approach since only simple benchmark problems can be verified using this procedure. This problem was noted in the ACADS study and thus it was decided to adopt the approach where a "correct" answer was established by combining the averages of all software packages as well as using the application of professional judgment. This approach then attempted to allow the benchmarking of more complex problems amongst a variety of software packages. It should be noted that this approach, while acceptable, does not result in a true and absolute "correct" answer. This procedure relies on the law of averages amongst a group of proposed solutions.

Another difficulty encountered in benchmarking is that the complexity of analysis possible by sophisticated slope stability analysis software packages typically exceeds the complexity of most benchmark problems. This was illustrated in the early attempts to verify the Slope-II software package (precursor of SLOPE/W) in 1984. Colorado State University (CSU) attempted to compile a list of benchmark problems as slope stability software was increasingly being used in routine geotechnical analysis. Comprehensive benchmarking processes had not yet been established and therefore there was need for a method to verify calculations. The effort was eventually abandoned at the time for the following reasons: i) the benchmark examples had to be extremely simple to allow establishing of a "correct" answer by hand calculations, and ii) these simple benchmark examples only represented a small fraction of the potential scenarios which could be set up and solved by the Slope-II software package. Comprehensive benchmarking was therefore abandoned at this time.

The approach to compare computed results to a "right" solution is a worthy effort but the ACADS study and the majority of other benchmarking efforts have not quantified the expected variation between software packages or the sources of such variation. This paper attempts to begin to quantify such variation.

## 3 SVSLOPE BENCHMARKING

The difficulties experienced with the true benchmarking of a slope stability package were experienced during the development of the SVSLOPE® slope stability analysis software produced by SoilVision Systems Ltd. A considerable amount of effort was placed on the benchmarking and verification of the software. During this process it was noted that differences were encountered between the results of SVSLOPE and other software packages. Each of these differences was examined to determine the cause of the difference. In all, over one hundred specific example models were selected from research literature and the models were set up and solved in SVSLOPE. The results were then compared to both published results and the results produced by other software packages. The findings are presented in the following sections.

It is also worth noting that each of the over one hundred verification models was compared to benchmark results in more than one method of analysis (i.e., Bishop's M-P, GLE, etc). When this is considered the total number of single comparisons exceeds 300-400 direct comparisons.

## 4 DIFFERENCES BETWEEN ANALYSIS METHODS

There are well-documented differences between the various limit equilibrium slices methods. This is particularly true when comparing the assumptions regarding the inter-slice force function. An extensive review of inter-slice force functions has already been performed (Wilson, 1990) and will not be covered in this paper.

Firstly will be a comparison of different methods for a single example model. The following example is the Cubzac-les-Ponts embankment (1974 in France), which was built and a failure induced for testing and research purposes (Feng, et al, 2012). The model is shown in Figure 1, and the material properties and the results are given in Table 1 and Table 2 respectively. Program defaults for interslice functions (half sine in all cases) were adopted for the comparison of methods using interslice force functions. A circular slip surface and the grid and tangent method was utilized to search for the critical slip surface in this example.

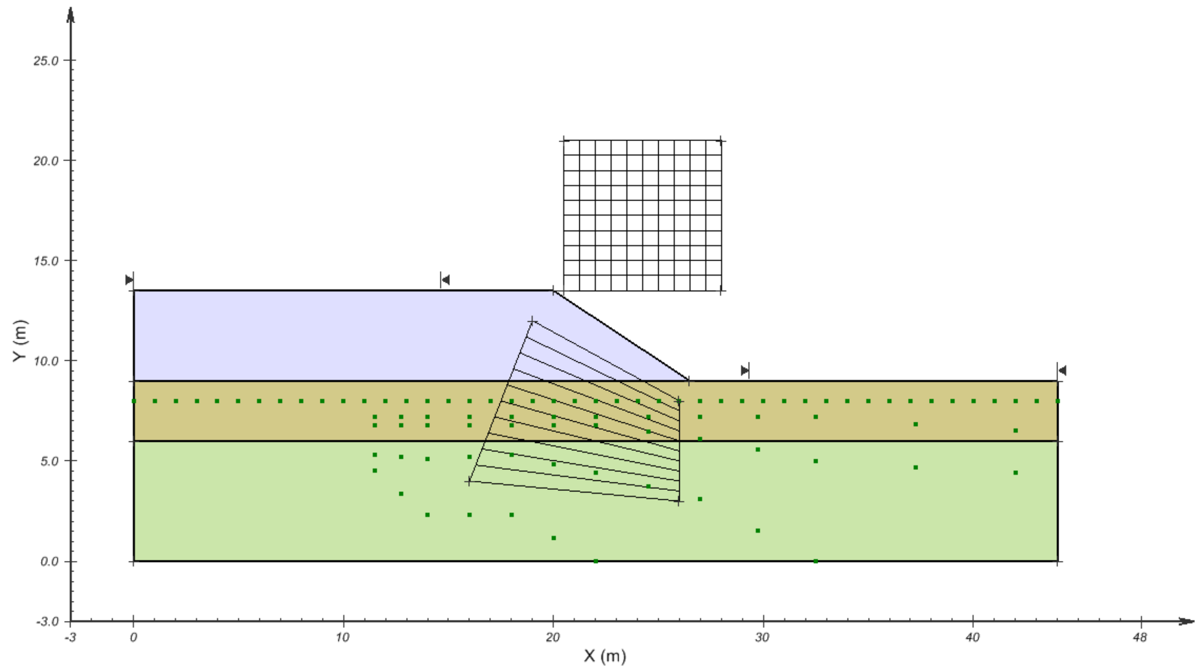


*Figure 1 Geometry of Cubzac-les-Ponts embankment*

*Table 1 Material Properties*

| Material | c (kN/m$^2$) | $\phi$ (degrees) | $\gamma$ (kN/m$^3$) |
|---|---|---|---|
| Embankment | 0.0 | 35.0 | 21.2 |
| Upper Clay | 10.0 | 24.0 | 15.5 |
| Lower Clay | 10.0 | 28.4 | 15.5 |

The discrete pore-water pressures at 30 designated points were used. The pore-water pressures at the case of the slice were interpolation from the given data using a bio interpolation method. The location of the critical slip surface and the corresponding factor of safety are to be determined.

The differences between various methods are presented in Table 2. The variation is presented only in terms of the expected variation between answers in different methods and it is not assumed that the central tendency of all answers represents the “correct” solution.

*Table 2 Factors of safety for the Cubzac-les-Ponts example*

| Methods | SLIDE | SVSLOPE |
|---|---|---|
| Ordinary | 0.662 | 0.656 |

| | | |
|---|---|---|
| Bishop Simplified | 1.314 | 1.317 |
| Janbu Simplified | 1.199 | 1.206 |
| Spencer | 1.334 | 1.334 |
| M-P | 1.336 | 1.338 |
| GLE | 1.336 | 1.336 |
| Min FOS | 0.662 | 0.656 |
| Max FOS | 1.336 | 1.338 |
| Difference(%) | 50.4 | 51.0 |

Group 1 3.2 in Feng, T.Q. et al, 2012

This example shows that sometimes there are significant differences between methods. In other words, a comprehensive understanding of these methods is essential and the factor of safety of sole method could mislead the engineering decisions.

As an alternative approach the variation between all methods can be summarized based on a group of models compiled during the benchmarking process. What will be quantified is the expected difference between various limit equilibrium analysis methods when compared statistically on a large group of benchmark examples. Table 3 is a summary of differences of the FOS among over 140 models between GLE method and other methods in the verification categories of SVSLOPE. It can be seen that the average difference between methods is 0.2-11.8% and the maximum difference between methods is 5-519%. It is worth mentioning that the significant differences usually exist between the methods (e.g., Corps#1, Corps#2,Ordinary, Janbu simplified, etc,), which satisfy either force or moment equilibrium equations but not both. The variation between comprehensive methods (e.g., Spencer, M-P and GLE) is typically much less. The variability in results using only force or only moment equilibrium suggest that only methodologies which solve for both force and moment should be used to improve reliability.

*Table 3 Summary of Factor of safety differences (140 Models)*

| | Bishop | Corps#1 | Corps#2 | Janbu Simplified | L-K | M-P | Ordinary | Spencer |
|---|---|---|---|---|---|---|---|---|
| Min Diff. between GLE and methods (%) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Max Diff. between GLE and Methods(%) | 82 | 519 | 420 | 55 | 113 | 5 | 71 | 7.4 |
| Aver. Diff. between GLE and Methods (%) | 1.9 | 8.9 | 11.8 | 7.9 | 6.3 | 0.2 | 8.6 | 0.6 |

## 5 DIFFERENCES BETWEEN SOFTWARE IMPLEMENTATIONS

Implementing each analysis method in various software packages introduces a host of potential differences depending on i) the development platform, ii) the specific lines of code in the implementation (often thousands) and iii) the qualifications of the programmer of the methodology. Even when two software packages are run side-by-side on the same model there may be slight variation brought on by i) differences in the number of slices, ii) convergence issues, iii) searching methodology to find the critical slip surface as well as a host of other potential areas for the solutions to diverge.

Given the number of software packages available on the market today it is useful for the practicing engineer to be able to effectively quantify the expected differences between software packages. This will allow them to make a judgement call on whether a difference is statistically significant of a problem or within reasonable limits.

In this comparison the software packages SVSLOPE, SLIDE, SLOPE/W, XSTABL, UTEXAS were compared using different limit equilibrium method of slices methods analyzing published examples. Examples from a wide variety of deterministic benchmark examples were used which compared both un-reinforced as well as reinforced slopes.

A typical example may be seen in Figure 2, which was presented by Pockoski and Duncan (2000). The model is distributed as VS_55 in the SVSlope software. The material properties and the results of the analysis may be seen in Table 4 and Table 5, respectively. In this analysis the grid and tangent searching method was used in each software package with the same searching guidelines and restricted to only search for circular slip surfaces.

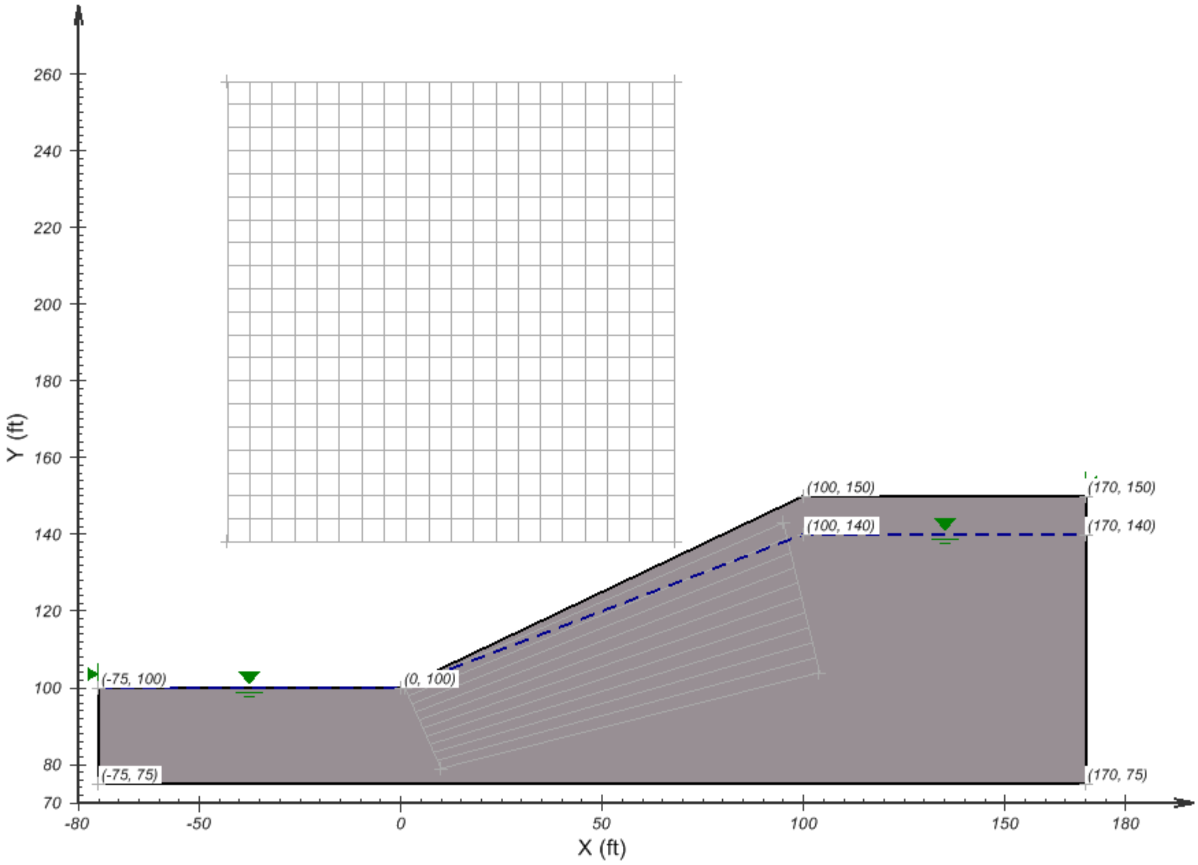


*Figure 2  Geometry of the Pockoski and Duncan Slope model*

*Table 4  Material properties for the Pockoski and Duncan (2000) model*

| Material | c (psf) | $\phi$ (degrees) | $\gamma$ (pcf) |
|---|---|---|---|
| Sandy Clay | 300 | 30 | 120 |

*Table 5  Factors of safety between software packages*

| Methods | Ordinary | Bishop Simplified | Janbu Simplified | Lowe-Karafiath | Spencer |
|---|---|---|---|---|---|
| UTEXAS4 | - | 1.29 | 1.15 | 1.32 | 1.3 |
| SLOPE/W | 1.04 | 1.29 | 1.15 | - | 1.3 |
| WINSTABL | - | 1.29 | 1.2 | - | 1.34 |
| XSTABL | - | 1.29 | 1.24 | - | - |
| RSS | - | 1.29 | 1.15 | - | - |
| SLIDE | 1.052 | 1.293 | 1.151 | 1.318 | 1.3 |
| SVSLOPE | 1.058 | 1.292 | 1.151 | 1.307 | 1.298 |
| Diff (%) SVSLOPE-SLIDE | 0.57 | 0.08 | 0.00 | -0.83 | 0.15 |
| Diff.(%) Max-Min | 1.70 | 0.23 | 7.26 | 0.98 | 3.13 |

Group 1 3.37 in Feng, T.Q. et al, 2012

From this analysis it may be seen that the expected variation in calculations between SVSLOPE and SLIDE should typically not exceed 1%. Variations of less than 0.5% are ideal. However, the variations between minimum and maximum FOS among difference software exceed 1%, there is likely a good reason.

Table 7 presents a summary of the comparison between SVSLOPE and SLIDE or SLOPE/W using the verification models in Feng, T.Q. et al (2012). In this comparison grid and tangent searching methods along with circular slip surfaces were used in the analysis. The deviations of the methods between different software packages are within +/- 3% except for L-K method in which the minimum and maximum differences exceed 10%. L-K method needs to be further investigated for the larger differences.

*Table 6 Summary of software comparison*

| Methods | Number of Models | Min Diff (%) | Max Diff (%) |
|---|---|---|---|
| Ordinary | 41 | -1.86 | 1.93 |
| Bishop Simplified | 93 | -1.72 | 1.93 |
| Janbu Simplified | 59 | -2.11 | 2.80 |
| Corps#1 | 17 | -3.02 | 0.47 |
| Corps#2 | 17 | -2.82 | 0.505 |
| L-K | 21 | -12.47 | 11.32 |
| Spencer | 70 | -2.5 | -2.14 |
| M-P | 30 | -1.76 | 0.48 |
| GLE | 46 | -1.942 | 2.19 |

Difference =(SVSLOPE-SLIDE)/SVSLOPE (% ) or
Difference =(SVSLOPE-SLOPE/W)/SVSLOPE (%)

## 6 DIFFERENCES BETWEEN SEARCH METHODS

One of the significant areas of potential differences between slope stability analysis is between searching methods. The most practical approach in practice is to use one search method and use one or two other search methods as a check. However, in some cases, different search methods might provide significant differences in the computed FOS. It is the experience of the authors that the difference of non-circular slip surface searching methods can potentially be higher than searching methods which only involve circular slip surfaces. Table 8 is the summary of search method comparison using the verification model (VS_13) in Feng, T.Q. et al (2012). It can be seen that significant differences (15%) can be noticed in this single example. Differences up to 30% were noticed by SoilVision Systems Ltd. Developers in other benchmark problems when comparing searching methodologies.

*Table 8 Summary of search method comparison*

| Methods | Grid and Tangent | Entry Exit | Slope Search | Auto Refine | Path Search | Max Diff (%) |
|---|---|---|---|---|---|---|
| Ordinary | 1.062 | 1.031 | 1.035 | 1.010 | 0.896 | 15.63 |
| Bishop Simplified | 1.180 | 1.176 | 1.180 | 1.162 | 1.178 | 1.525 |
| Janbu Simplified | 1.012 | 0.974 | 1.012 | 1.002 | 0.993 | 3.75 |
| Corps#1 | 1.156 | 1.126 | 1.154 | 1.175 | 1.137 | 4.17 |
| Corps#2 | 1.244 | 1.265 | 1.274 | 1.264 | 1.227 | 3.68 |
| L-K | 1.198 | 1.198 | 1.246 | 1.210 | 1.211 | 3.85 |
| Spencer | 1.178 | 1.176 | 1.173 | 1.182 | 1.169 | 1.10 |
| M-P | 1.180 | 1.179 | 1.176 | 1.184 | 1.178 | 0.68 |
| GLE | 1.180 | 1.179 | 1.176 | 1.184 | 1.178 | 0.68 |

Max Diff =(Max-Min)/Max (% )

## 7 CONCLUSIONS

The research presented in this paper provides a summary of the reasonable and possible variations in a slope stability analysis carried out with differing software packages. These variational analysis were conducted during the benchmarking process for the SVSLOPE limit equilibrium software package. As a result of this analysis it is possible to draw the following conclusions:

- It can be seen that the average difference between analysis methods is 2.0-11.8% and the maximum difference between methods is 519% when a library of 140 models is considered. The variation between comprehensive analysis methods (GLE, M-P, Spencer) is less.
- The variation between software packages for the same exact model should be less than 0.5% or 1% in extreme cases. If the searching method is allowed to vary the difference could jump to between 10-20% or higher.

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