Modelling the Behaviour of Sand with Fines Using Equivalent Granular Void Ratio

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ABSTRACT

Void ratio has been used as a state variable in modelling soil behaviour under Critical State Soil Mechanics (CSSM). However, it may not be a good state variable for sandy soils because a little variation of fines (particle diameter<0.075mm) content in sand significantly affects the position of the steady state line (SSL). Thus, one cannot have a single CSSM framework as variation of fines content is very common in the natural condition, even at a single site. Equivalent granular void ratio, e^* was proposed to achieve a single SSL for such soils. However, back analysis was used to obtain e^*. The author has developed a semi-empirical model to predict e^* from simple soil grading properties. This not only brings a single trend of steady state data points but also leads to the concept of equivalent granular state parameter that allows prediction of sandy soil behaviour under monotonic and cyclic loading for unknown fines content.

Keywords: sands, fines, equivalent granular void ratio, steady state

1 INTRODUCTION

The void ratio has been used as a state variable in modelling clay behaviour under critical state soil mechanics (CSSM) frameworks for many years (Roscoe*, et al.*, 1958, Schofield and Worth, 1968). Following this line, the void ratio of clean sand is also considered as a state variable and is being used in modelling clean sand behaviour (Been and Jefferies, 1985, Li, 2002). However, clean sand usually mixed with some fines (particle diameter<0.075mm) contents, and the fines content even at a single site may varies. Research in the last 20 years has revealed that void ratio may not be a good state variable for such soils. This is because at the same void ratio a little variation of fines content significantly alters their behaviours such as steady state (SS), quasi-steady state (QSS), cyclic resistance to liquefaction, cyclic/static instability, small strain shear modulus etc (Vaid, 1994, Zlatovic and Ishihara, 1995, Lade and Yamamuro, 1997, Salgado*, et al.*, 2000, Rahman*, et al.*, 2012). Most notably, the steady state line (SSL) in $e\text{-}log(p')$ space, which is the anchor concept of CSSM, shifted downwards with fines content i.e. sandy soil at a single site may have many SSLs; where e is void ratio and p' is mean effective stress (Thevanayagam*, et al.*, 2002). Thus, these soils' behaviour cannot be modelled under a single CSSM framework unless CSSM framework for each individual fines content is considered separately. This is a difficult task and impractical in geotechnical practice. To overcome this issue, Kuerbis *et al.* (1988) proposed an 'equivalent' void ratio, referred to as skeleton void ratio, and showed that undrained monotonic triaxial tests for sand with fines at a constant skeleton void ratio exhibited approximately similar behaviour. Along this line, Thevanayagam *et al.* (2002) modified the concept of skeleton void ratio to a more generalised form referred to as equivalent granular void ratio, e^* and showed that a single trend of steady state (SS) data points for sand with fines in $e^*\text{-}log(p')$ space can be obtained. This trend line is called equivalent granular steady state line (EG-SSL). Many researchers confirmed this observation (Ni*, et al.*, 2004, Yang, 2004, Chiu and Fu, 2008). The EG-SSL could be a starting point for modelling the behaviour of sand with fines under the same CSSM framework. However, this has not been verified yet. Moreover, converting e^* from e was essentially a back analysis process that requires making certain fundamental assumptions and known SS data points for a range of fines content. This significantly reduced its effectiveness as a prediction tool. Thus, the objective of this paper is to present a 'prediction' approach for obtaining e^* from e and to validate the applicability of the EG-SSL under single CSSM framework. Then, the state parameter, as defined by Been and Jefferies (1985), was modified and generalized to equivalent granular state parameter to model monotonic undrained behaviour of sand with fines, irrespective of fines contents. This was validated with a large number of triaxial data from experimentation and published datasets. Further, cyclic instability type behaviour under cyclic loading was predicted from the correlation developed from monotonic loading.

2 LITERATURE REVIEW

2.1 Equivalent granular void ratio, *e**

Kuerbis *et al.* (1988) suggested that fines simply occupied the voids in the sand skeleton and therefore, the behaviour was controlled by the sand skeleton only. Thus, the sand skeleton void ratio (by neglecting the fines) should be an appropriate state variable. Thevanayagam *et al.* (2002) suggested a more generalized concept that when an amount of fines is added to sand, a part of it will be placed in the sand skeleton void but the rest of it will be placed in between sand grains and therefore will have an active contribution to the soil behaviour. Such a complicated microscopic particle arrangement can be presented by a simple schematic diagram as shown in Figure 1 and the equivalent granular void ratio, *e** can be defined as in Eq. (1).

$$e^* = \frac{e + (1-b) f_c}{1-(1-b) f_c} \tag{1}$$

where f_c is fines content and b is the active fraction of fines in transferring forces between sand grains. An intrinsic assumption behind Eq. (1) is a 'fines-in-sand' soil matrix, i.e. f_c is less than the threshold fines content, f_{thre} at which any increment of fines content in the soil matrix becomes 'sand-in-fines'.

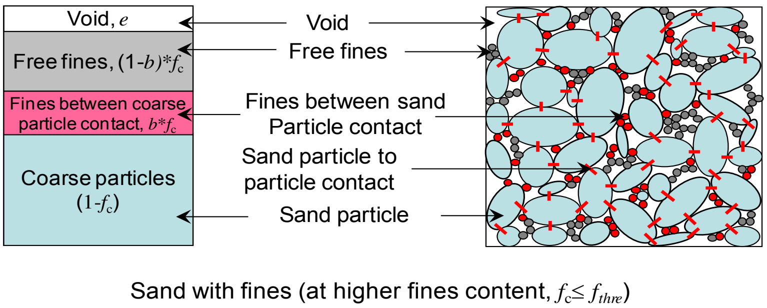

*Figure 1: Schemtic diagram of particle arrangement of sand-fines mix and formulation of e**

2.2 Prediction of *b*

When f_c is small relative to f_{thre}, $b \approx 0$ is a good approximation. But, at higher f_c values, $b \neq 0$ and the determination of b can be a challenge. Thevanayagam *et al.* (2002) selected a suitable value of b so that a family of SSLs in the *e-log(p')* space was coalesced into a single line/curve in the *e-log(p')* space. This was essentially a back analysis process where SSLs were input. This required a large number of tests as a prerequisite. Furthermore, a single SSL in the *e-log(p')* space was in fact an assumption that by the very nature of back-analysis cannot be verified. Despite of the practical and theoretical deficiencies of this approach, it had been followed by many others (Ni, *et al.*, 2004, Yang, 2004). Thus, the main challenge is the determination of b without extensive test results and back-analysis. Rahman *et al.* (2008, 2009), based on the physical meaning of b and binary packing studies of McGeary (1961), argued that b is a function of f_c and the size of fines relative to the host sand; and therefore proposed the following semi-empirical equation from simple input parameters.

$$b = \left[1 - \exp\left(-0.3 \frac{(f_c / f_{thre})}{k}\right)\right]\left(r \frac{f_c}{f_{thre}}\right)^r \tag{2}$$

where $k = (1 - r^{0.25})$, $r = d_{50}/D_{10} = 1/\chi$, d_{50} is the median size of fines and D_{10} is the 10% fractile of host sand. Thus, k and r can be obtained from the grading curves. The other input parameter is f_{thre}, which may be taken as 0.30 as a first approximation (Rahman, *et al.*, 2008). Alternatively, it can be estimated by the following empirical equation developed in Rahman and Lo (2008) based on nine published databases with $\chi \geq 1.83$.

$$f_{thre} = 0.40\left(\frac{1}{1+\exp(0.5-0.13\chi)} + \frac{1}{\chi}\right) \tag{3}$$

where $1/\chi = d_{50}/D_{10}$ and no additional input parameters are needed in using Eq. (3). The predicted *b*, with Eqs. (2) and (3), is able to give *e** as an alternative state variable and the EG-SSL for many datasets (Rahman, *et al.*, 2011, Rahman and Lo, 2012).

3 EXPERIMENTAL PROGRAM AND RESULTS

An experimental study was carried out, to verify the proposed modelling concept, by covering a wide range of testing conditions: i) p'_0 ranging from 100kPa to 1300kPa, ii) e_0 ranging from 0.455 to 0.892 and iii) 5 different fines contents in the range of 0 to 30%. A total of 33 triaxial tests were conducted. The host sand is a uniform size quartz sand (median size, d_{50}=0.27mm and uniformity coefficient, Uc=1.26) whereas the silt is a well graded silt with a d_{50}= 0.0055mm and Uc=21.56. A modified moist tamping method was used for specimen preparation. The techniques to ensure uniformity of the prepared specimen and subsequent deformation during shearing are detailed in Rahman *et al.* (2011).

3.1 Experimental Results

The SSLs are curved and moved downwards with f_c as shown in Figure 2.a. However, the EG-SSL, obtained from the prediction approach using Eqs. (2) and (3), was also curved and can be represented by a power function as in Figure 2.b. The corresponding scatter in terms of root-mean-square-deviation (RMSD) is 0.052, which is acceptably small. The very small scatter of the data points implies that the EG-SSL can be obtained from testing on clean sand (or sand with a given f_c).

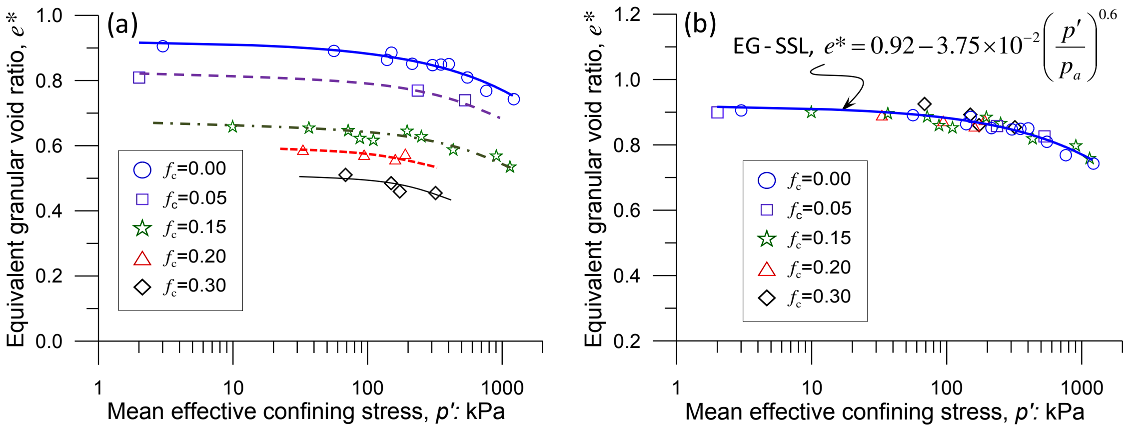

Figure 2: (a) SSLs for sand with fines; (b) EG-SSL for sand with fines

The experimental results showed that the deviatoric stress-strain and effective stress path (ESP) responses during undrained shearing is strongly influence by f_c, where *e** is a better state index than *e* as illustrated in Figure 3. The specimen with lowest *e** showed strain hardening and with highest *e** showed strain softening. Depending on strain hardening/softening, the undrained behaviours can be divided in four types: non-flow, limited flow, transition and flow. According to CSSM, these behaviours are highly related to state parameters. Been and Jefferies (1985) defined a state parameter, ψ as the difference between current void ratio and void ratio at the same mean effective stress, *p′* on the SSL. Having the EG-SSL, the definition of ψ is generalised so that *e** can be incorporated. Thus, the generalized state parameter can be represented by the equivalent granular state parameter, ψ^*, defined as:

$$\psi^* = e^* - e^*_{SS} \tag{4}$$

where e^*_{SS} is the corresponding *e** value at the same *p′* on the EG-SSL. Thus, the ψ^* might be able to capture the types of behaviours of sand with fines, irrespective of f_c. The four types behaviours are presented in initial mean effective stress, p'_0 – initial equivalent granular state parameter, $\psi^*(0)$ space as shown in Figure 4.a. Clearly, $\psi^*(0)$ is a suitable parameter to predict these types of behaviours. The $\psi^*(0)$=0 line separated non-flow from the other behaviours. Flow, limited flow and transition manifested deviatoric strain softening after the deviator stress attained a peak value, q_{IN} (as in Figure 3). The stress ratio at q_{IN} on ESP is called instability stress ratio, η_{IN} which is a key observation in modelling undrained behaviour (Imam, *et al.*, 2002). The η_{IN} were presented in η_{IN}-ψ*(0) space as shown in Figure 4.b. A good correlation was observed with ψ*(0). In the case of transition and limited flow, the deviatoric stress-strain response returns to gradual strain hardening after deviatoric strain softening to a transient minimum value referred to as quasi-steady state (QSS). The q_{QSS} at QSS is also key observation in undrained shearing and presented in q_{QSS}/p'_0-ψ*(0) space as shown in Figure 4.c. The normalised rate of strain hardening, H_n after QSS is presented in H_n-ψ*(0) space as shown in Figure 4.d. A good correlation is observed; this implies that the relation obtained from clean sand or sand with a known fines content can be used for unknown fines content as well. Moreover, ψ*(0) could be a state index irrespective of fines as ψ is used for clean sand.

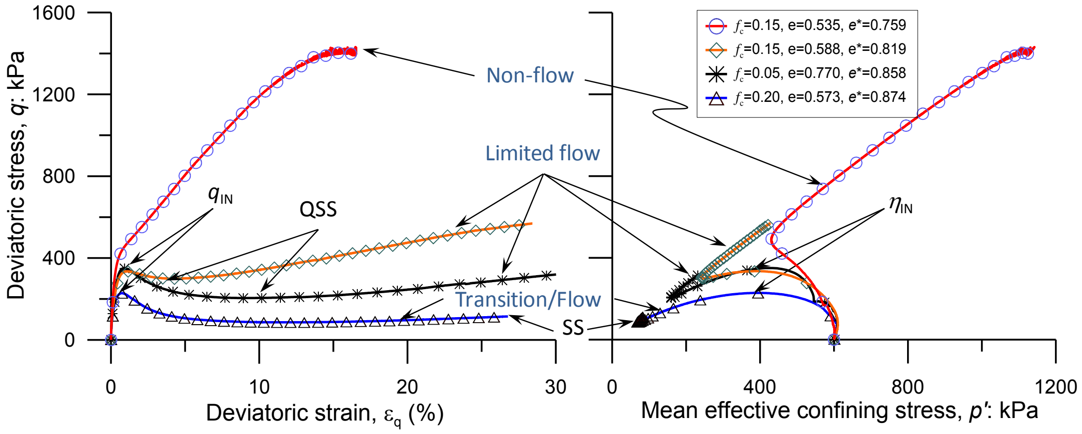

Figure 3: Type of undrained monotonic responses observed for sand with fines.

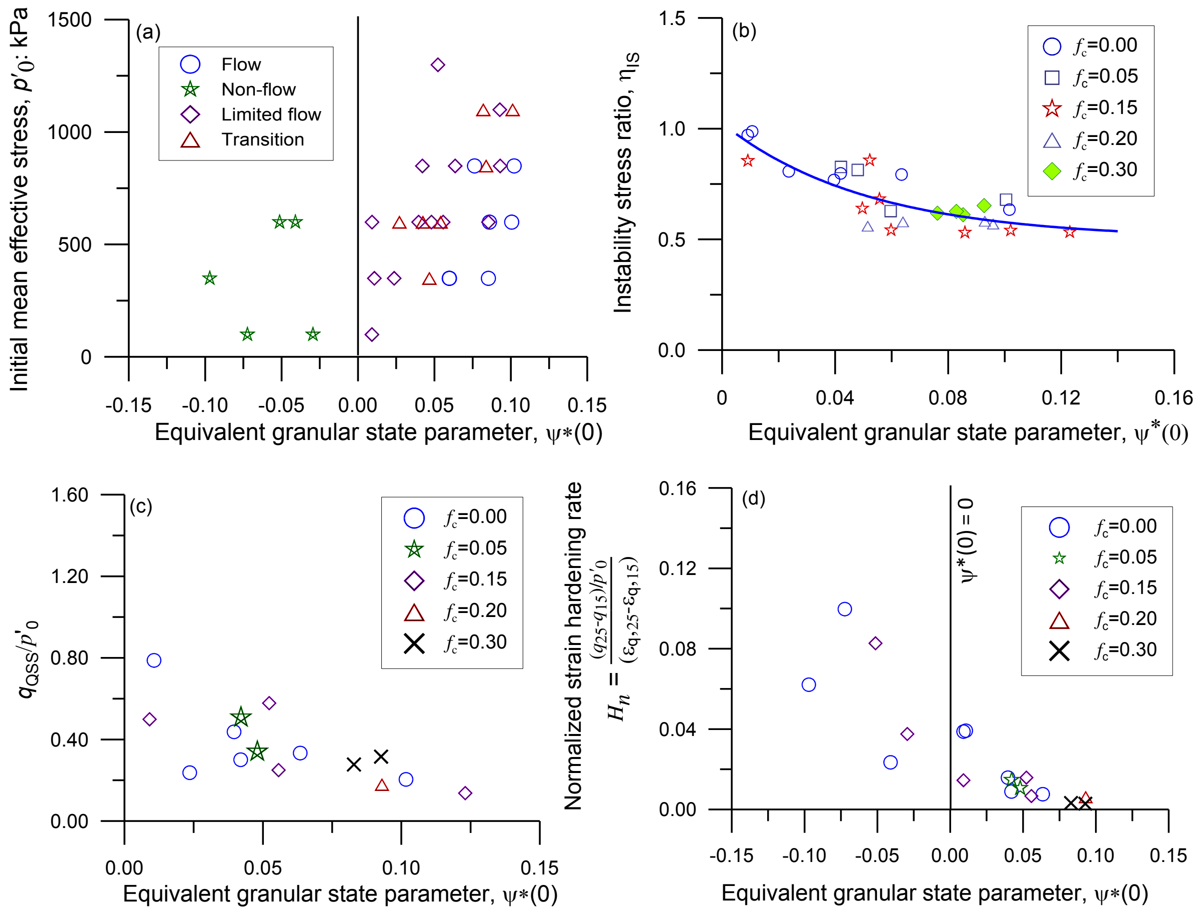

Figure 4: Relation between undrained behaviour and equivalent granular state parameter; (a) types of behaviour, (b) in η_{IN}-ψ(0) space, (c) q_{QSS}/p'_0-ψ*(0) space, (d) in H_n-ψ*(0) space*

The correlations developed in the above can be used to predict cyclic instability behaviour of sand with fines, irrespective of f_c. For example, a cyclic test f_c=0.30 was conducted at p_0'=350 kPa and e=0.488 i.e. ψ*(0)=+0.052. The applied peak, q_{peak} and trough, q_{min} were 112 kPa and -39 kPa respectively. The initiation of strain softening was predicted for ψ*(0)=+0.052 from Figure 4.b and shown by η_{IN} (black) line in Figure 5. The scatter of data points around the trend line in Figure 4.b is presented by dotted lines in Figure 5. In the first 5 load cycles, ESP moved left side with loading cycles and the leftward movement per cycle was getting smaller. Then, the ESP touched with the η_{IN} line at point A on 6th cycle and the leftward movement of the ESP started to increase with load cycles and developing large axial strain (Figure 5). The prescribed q_{peak} value cannot be developed rather plummet downwards. Furthermore, the attained q_{peak} value was reducing with load cycles. Thus, cyclic instability occurred on η_{IN} line at point A, because a maintained q value would result in run-away deformation. The above finding is applicable to tests conducted with different combinations of q_{peak} and q_{min}, irrespective of f_c as long as ψ*(0)>0 and cyclic instability occurred in the compression side of stress space.

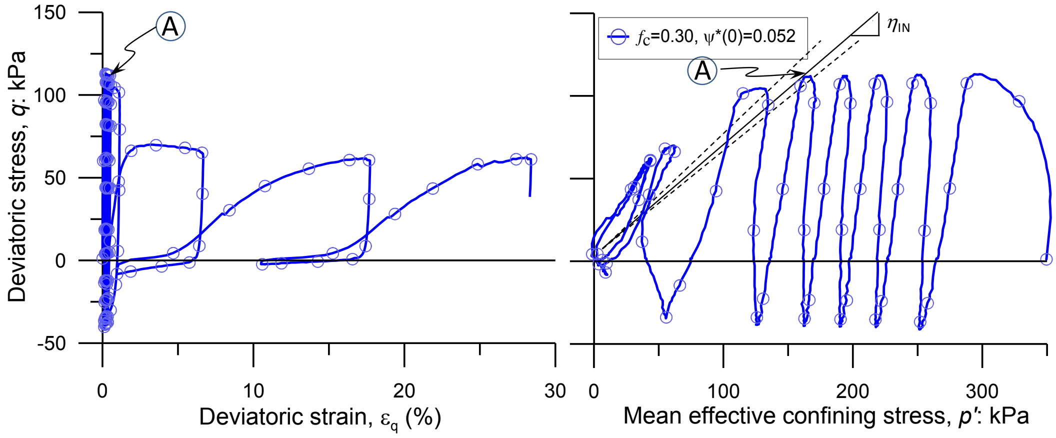

Figure 5: Prediction of cyclic instability behaviour for ψ>0*

3.2 Published datasets

The effective stress and stress-strain paths for sand with fines in published literature are very rare. The key observations for published datasets are presented in this section. The η_{IN} from two datasets for Marine sand with Kaolin fines (Chu and Leong, 2002) and Hokksund sand with silt (Yang, 2004) were presented in η_{IN}-ψ*(0) space as shown in Figure 6.a. A good correlation is observed. These correlations can be used to predict cyclic instability behaviour for those soils under cyclic loading irrespective of f_c. Only five data points for Hokksund sand with upto 30% silt showed clear strain hardening behaviour and they are presented in H_n-ψ*(0) space as shown in Figure 6.b. It shows a good correlation with ψ*(0).

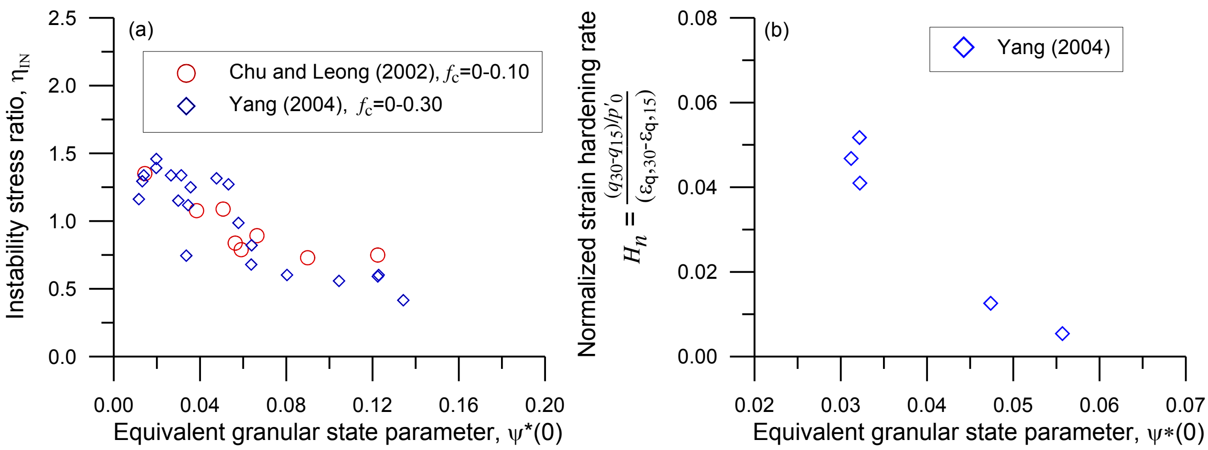

Figure 6: Published datasets and their relation with ψ(0); (a) in η_{IN}-ψ*(0) space, (b) in H_n-ψ*(0) space*

4 CONCLUSIONS

The development of the concept of equivalent granular void ratio, e^* and equivalent granular state parameter ψ^* for sand with fines are discussed. Then, the effectiveness of e^* and ψ^*, in prediction of undrained behaviour of sand with fines, is evaluated with extensive experimental and published data. The major outcomes of this study are-

- The e^* can be predicted from Eqs. (2) & (3), where fines content, f_c and soil grading properties are inputs. e^* and ψ^* are suitable state variables that correlate with key observation of undrained behaviour for sand with fines presented in this article.
- The triggering of cyclic instability can be predicted from the η_{IS}-$\psi^*(0)$ relationship pre-established from monotonic undrained behaviour.
- The significance of having the equivalent framework is that it can be established for clean sand or sand with a known f_c and then can be used for other f_c.

It is noted that this approach relies on a single EG-SSL which in turn requires the SSLs for different fines content to be parallel. While this is observed in many sands with fines, it should be noted that some soils may not exhibit parallelism. These soils are not the context of this article.

5 ACKNOWLEDGEMENTS

The author would like to thank A/Prof S. R. Lo (UNSW@ADFA, Australia) and Prof. Julie Mills (UniSA, Australia) for their careful reading and suggestions to improve the quality of the manuscript.

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