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1D Compression Calculation for Composite Geomaterial

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ABSTRACT

A composite geomaterial is formed by blending earth materials with a special material, e.g., expanded polystyrene beads or rubber tire chips, in designated proportions. The composite geomaterial takes the advantages of low unit weight, improved shear strength and integrity, over conventional earth fills, when the materials are used for various geo-infrastructures, e.g., embankments, utility trenches and retaining walls. To study the deformation behaviour of the composite material helps understand the response of the material if subjected to field vertical or lateral loads. In this study, a one-dimensional (1D) compression calculation was developed to depict the compressibility of the material. The calculation was able to account for the effect of mixture proportion, and can be summarised in a unique mathematical form. Case study was illustrated to demonstrate the calculation of the compressibility.

Keywords: void ratio, compressibility, composite geomaterial, mixture proportion, regression

1 INTRODUCTION

A composite geomaterial is a specially structured mixture by blending earth mass, in designated proportions, with other materials, e.g., expanded polystyrene (EPS) beads (Deng and Xiao 2010), and rubber tire chips or beads (Wartman *et al*. 2007). So formed composite geomaterials can be regarded as choices of construction materials, which exhibit benefits of low unit weight, improved shear strength and integrity over conventional earth fills, when the materials are used for various geo-infrastructures, e.g., embankments, utility trenches and retaining structures (Humphrey 1992).

The mixture of a composite material leads to complex hybrid characteristic, which makes the behavioural response of the geomaterial, e.g., the compression deformation of the geomaterial when placed in the field and subjected to mechanical loads, different from that of conventional pure earth fills (e.g. sand). More than often, the hybrid characteristic was relevant to the mixture ratio used to make the composite geomaterial, which were observed and discussed by Youwai and Bergado 2003, Zornberg *et al*. 2004, Wartman *et al*. 2007, Deng and Feng 2009, Deng and Xiao 2010, etc. Of these investigations, mixture ratio was unanimously acknowledged as one of critical variables affecting the mechanical response of geomaterials. Specifically, the strength and deformation of the composite geomaterial was associated with the proportion of the composite mixture. That is, pure-soil based field design methodology will be challenged, if composite geomaterials are to be selected to construct geo-infrastructure systems.

A method was established in this study to depict the 1D deformation of a composite geomaterial, attempting to setup a simplified but generalised methodology addressing the proportion-dependent compressibility of a composite geomaterial. The method was able to account for the variable of mixture proportion, while predict the compressibility of the composite geomaterial in terms of the variation of void ratio of geomaterial. The method enables decision-makers to seek a performance-based design for a backfilling infrastructure system.

2 APPROACH DEVELOPMENT

2.1 *e*-log*p* curve

An *e*-log*p* curve, or consolidation curve, is the plot of void ratio of a geomaterial against a series of 1D effective consolidation loads applied to the geomaterial, and is often employed to calculate the compressibility of the geomaterial. For a normally consolidated soil, the laboratory and field *e*-log*p* curves are often illustrated as shown in Figure 1. For composite geomaterials of varying mixture proportions, the *e*-log*p* curves can reasonably be approximated by using the curves shown in Figure 2. In Figure 2, *e*-log*p* curves for mixtures of three proportions (i.e., η_1, η_2 and η_3) are presented. The initial void ratios of the three mixtures differ in terms of individual proportions. Along with the increase of consolidation loads, the curves converge eventually due to the consolidation process, basically in a manner comparable to that indicated in Figure 1.

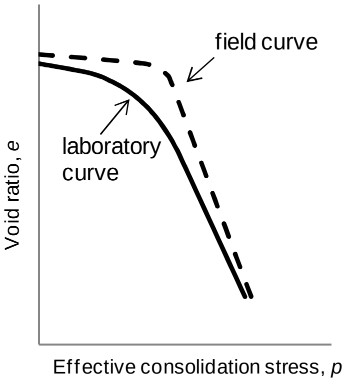

Figure 1. Void ratio vs log effective consolidation stress of normally consolidated soil

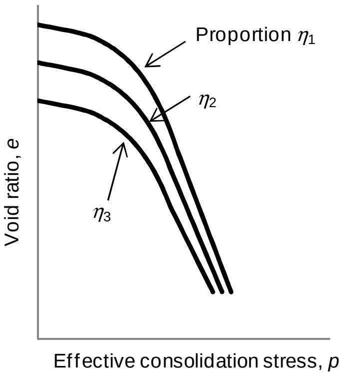

Figure 2. Void ratio vs log effective consolidation stress of three mixtures

Model generalisation was performed for curves in Figure 2. A piece-wise linear geometry, as indicated in Figure 1, was used to approximate the trend of the curves, which set ground for compressibility calculation in the next section, while maintaining the accuracy of the calculation. Furthermore, mixture proportion η was treated as an independent variable, which determined the difference between curves. On top of the model generalisation, void ratio *e* was mathematically solvable in terms of the piece-wise geometry and the variable of mixture proportion. The model generalisation can be extended to any *e*-log*p* curves only if the proportion is the sole variable of a mixture.

2.2 Derivation of Void Ratio

Figure 3 draws the piece-wise linear geometry of the curves indicated in Figure 2. It is clear that to seek a generalised mathematical equation of the geometry leads to the broad depiction of compressibility. The equation should be figured out in terms of variable η, the proportion of the mixture. That is, the equation is a function of variable η.

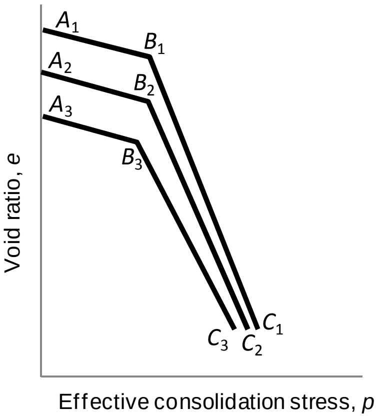

Figure 3. Geometry of consolidation curves

Eqs. (1) and (2) were used to depict the upper-left linear approximations (i.e., lines A_1B_1, A_2B_2 and A_3B_3) and lower-right linear approximations (i.e., lines B_1C_1, B_2C_2 and B_3C_3), respectively.

$$e = e_k - k\lg P \tag{1}$$

$$e = e_\lambda - \lambda\lg P \tag{2}$$

where, variables e_k and e_λ denote the intercepts; variables k and λ denote the line slopes. These variables are functions of variable η, and are determined by regression of test observations in terms of η.

It should be noted that the geometry may not necessarily be approximated, either linearly or as limited segments as possible, although two linear segments were ideal in the case of Figure 2. In reality, the geometry is merely a close depiction of a testing curve, and could be any shape it would fit, which would be illustrated in the next section.

3 CASE STUDY

One case study was conducted to illustrate the calculation of void ratio in light of the above derivation. The case involved the mixtures of sand and EPS bead, which lead to the production of composite backfills of reduced unit weight and enhanced strength and integrity, relative to common earth fills.

Three series of mixtures were produced by blending sand and EPS bead in accordance with the proportions listed in Table 1, in which proportion η represents the dry mass ratio of EPS bead over sand. As EPS beads are formed by a pre-puff process, voids of small sizes exist in the matrix of a bead. That is, the internal void volume contributes to the compression and should be taken into account in the estimation of void ratio. Call general void ratio e to account for the entire void volume, and the general void ratio can be calculated by using Eq. 3.

$$e_0 = \left(V - \frac{m_s}{G_s \times \rho_w} - \frac{m_{EPS}}{\rho_{E3}} \right) \Big/ \left(\frac{m_s}{G_s \times \rho_w} + \frac{m_{EPS}}{\rho_{E3}} \right) \tag{3}$$

where, V denotes the initial volume of a sample, m_s and m_{EPS} denote the mass of sand and EPS bead, respectively, G_s denotes the specific gravity of sand, ρ_{E3} denotes the density of the bead prior to pre-puffing.

Table 1: Mix proportion and regression variables

η (%)	e_0	k	e_k	λ	e_λ	ζ	e_ζ
0.5	1.26	0.187	1.450	0.461	1.911	0.402	1.761
1.5	2.43	0.487	2.887	1.265	4.197	0.622	2.682
2.5	3.43	0.742	4.057	2.095	6.315	0.767	3.222

The mixtures were subjected to a conventional 1D consolidation test, and the test results are presented in Figure 4. It is indicated that individual compression curves are apparently divided into three segments in terms of the consolidation loads. Each segment is reasonably fit by using a semi-log linear approximation, which is elaborated in Figure 5. It is also shown that loads of 50 kPa and 200 kPa are break points, which divide the approximation into the three segments.

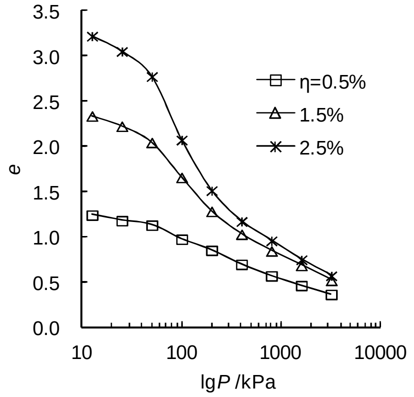

Figure 4. Void ratio vs log effective consolidation stress of sand-EPS bead mixtures

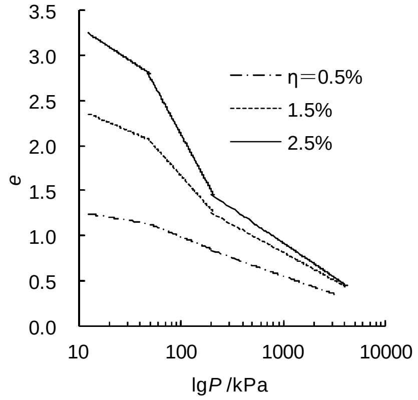

Figure 5. Approximation of void ratio vs log effective consolidation stress

On top of Eqs. (1) and (2), Eq. (4) was introduced to approximate the third segment of individual piece-wise lines in Figure 5.

$$e = e_{\zeta} - \zeta \lg P \tag{4}$$

where, variables e_{ζ} and ζ denote the intercepts and slope of the third segment, respectively.

In a spreadsheet, intercepts (i.e., e_k, e_{λ} and e_{ζ}) and slopes (i.e., k, λ and ζ) were calculated and presented in Table 1. A regression of these variables in terms of proportion η was conducted to derive the mathematical representation of each variable. The regressions are shown in Figure 6 for intercepts and in Figure 7 for slopes, respectively.

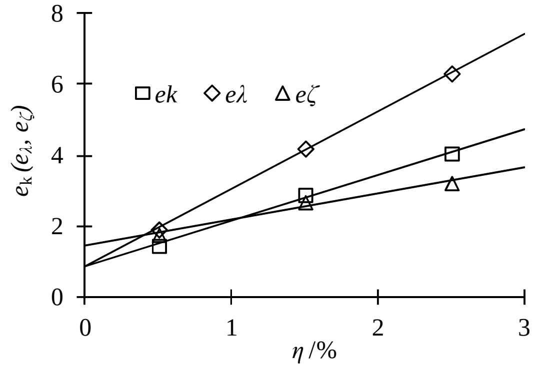

Figure 6. Regression of intercepts on mix proportion

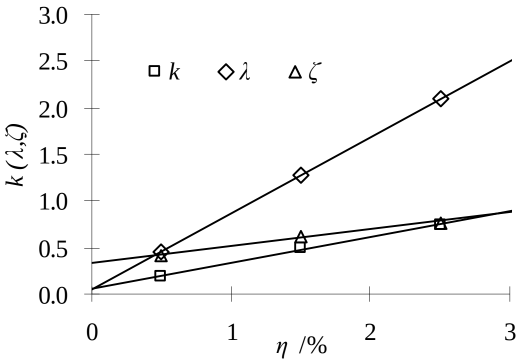

Figure 7. Regression of slopes on mix proportion

A linear regression is implicated in both Figures 6 and 7, which means that intercepts and slopes of the *e*-log*p* segments vary linearly along with the proportion of the mixture. That is, the two sets of variables (i.e., e_k, e_{λ} and e_{ζ}, and k, λ and ζ) can be mathematically presented using Eqs. (5) and (6).

$$e_k\ (e_{\lambda} \text{ or } e_{\zeta}) = a_1 \times \eta + b_1 \tag{5}$$

$$k\ (\lambda \text{ or } \zeta) = a_2 \times \eta + b_2 \tag{6}$$

where, parameters a_1, b_1, a_2 and b_2 are inferred from a spreadsheet regression, as shown in Table 2.

Table 2: Regression parameters for intercepts and slopes

Regression Parameter	k	λ	ζ	e_k	e_{λ}	e_{ζ}
a_1	–	–	–	1.304	2.202	0.730
b_1	–	–	–	0.843	0.839	1.460
a_2	0.277	0.817	0.182	–	–	–
b_2	0.056	0.048	0.324	–	–	–
R^2	0.99	0.99	0.99	0.99	0.99	0.98

The ultimate solution to void ratio of the mixture against consolidation load and mixture proportion is presented in Eq. (7).

$$e = (a_1\eta + b_1) - (a_2\eta + b_2)\lg P \tag{7}$$

in which, parameters a_1, b_1, a_2 and b_2 are selected (from Table 2) in terms of the level of load *P*.

In this study, there were two break points within the load range of 3200 kPa, i.e., 50 kPa and 200 kPa, which divided the *e*-log*P* geometry into three segments. In the first segment (0<*P*≤50 kPa), the set of parameters for k and e_k were used, i.e., a_1=0.130, b_1=0.843, a_2=0.277, and b_2=0.056. Likewise, the parameters for the other two segments can also be determined. The mathematical equations of the void ratio can be presented in Eqs. (8)–(10).

$$e = (1.304\eta + 0.843) - (0.277\eta + 0.056)\lg P \text{, if } 0<P\leq 50 \text{ kPa} \tag{8}$$

$$e = (2.202\eta + 0.839) - (0.817\eta + 0.048)\lg P \text{, if } 50 < P \leq 200 \text{ kPa} \tag{9}$$

$$e = (0.730\eta + 1.460) - (0.182\eta + 0.324)\lg P \text{, if } 200 < P \leq 3200 \text{ kPa} \tag{10}$$

In light of the equations above, the void ratio of the sand-EPS bead mixture can be plotted against consolidation load and mixture proportion, as shown in Figure 8, by which, the compressibility of any mixture of the investigated mixture components can be estimated in terms of the proportion of the composite geomaterial and the load placed on the geomaterial.

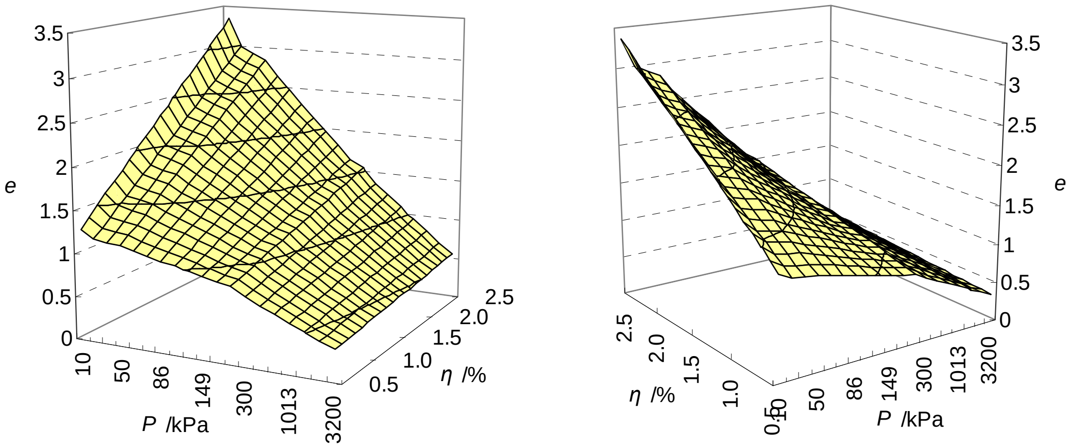

Figure 8. Void ratio against consolidation loading and mixture proportion

The approach presented in this study also contributes to the development of constitutive modelling of a composite geomaterial. Void ratio is one of important variables in geomechanic modelling in terms of critical state theory. To have a unique mathematical representation of void ratio is a premise from which a constitutive relationship can be drawn. The concept introduced in this study makes a point on this by showcasing the derivation of void ratio of any composite geomaterial when subjected to 1D or isotropic consolidation.

4 CONCLUSION

A method for estimating the 1D compressibility of a composite geomaterial was presented, which lead to the derivation of the void ratio of the geomaterial against consolidation load and mixture proportion. The derivation was undertaken by a piece-wise linear approximation of individual compression curves, and a regression analysis of the approximations. Critical regression parameters were collected and used to describe the void ratio in a unique mathematical form. The calculation was illustrated by showcasing the consolidation test and void ratio derivation of sand-EPS bead mixtures. The concept presented in this study is also meaningful to model composite geomaterials in terms of critical state theory.

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