Effect of Diameter-to-Length Ratio on the Strength of Cylindrical Specimens in Triaxial Tests

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Summary A homogeneous rock like material was selected to investigate the effect of diameter-to-length ratio on the strength of specimens in triaxial tests. Cylindrical specimens having different diameter-to-length ratios from 0.384 to 3.0 were tested under triaxial compressive stresses. The results show that the effect of diameter-to-length ratio on the axial compressive strength increases as the confining pressure increases. A higher value of axial stress is required to fail specimens under confining pressure and confinement due to frictional stresses. The ductile behaviour and the scatter of axial compressive strength of specimens increase with an increase in both the confining pressure and the diameter-to-length ratio. The coefficient of triaxial strength in Bieniawski criterion and Hoek and Brown criterion increases with an increase in the diameter-to-length ratio. The effect of diameter-to-length ratio on the strength of specimens under triaxial compressive stresses is similar to the effect of diameter-to-length ratio on the strength of bolted pillars.

1. INTRODUCTION

When a rock specimen is compressed axially between the platens of a testing machine, the frictional stresses are induced in its ends. The specimen tends to expand radially as it shortens because of Poisson's effect but frictional constraint between the specimen and platens tends to prevent expansion. A standard specimen (having length-to-diameter ratio, L/D of 2.5) is under triaxial stresses at its ends and fails with double cones and shear failure. But the specimen is under uniaxial stresses at the centre (mid-height) and fails with vertical micro-cracks. Brady and Blake (1968) used finite element method of stress analysis to obtain the state of stresses produced in cylindrical specimens having diameter-to-length ratios of 0.5 to 2 under uniaxial load and they showed that distribution of the triaxial stresses through the specimen changes with diameter-to-length ratio, and at $D/L \leq 1.0$ a larger portion of the specimen comes under the influence the uniaxial state of stress.

The values of frictional stresses (horizontal stresses) inside the specimen increase when its ends become closer together or its length decreases keeping the diameter constant. This is not a complete definition of the phenomenon of shape effect. Another important factor affecting the values of the frictional stresses is the diameter (cross sectional area) of the specimen. The values of frictional stresses on the ends of specimen increase with an increase in diameter keeping the length constant. But the distribution of stresses inside the specimens is similar for similar shape (the same diameter-to-length ratio) specimens having different sizes. The effect of shape on the compressive strength of a specimen is related to the effect of dimensions of diameter and length and these two effects combined can be interpreted as the effect of diameter-to-length ratio.

The values of horizontal stresses increase from the periphery to the centre of a specimen. Specimen having a diameter-to-length ratio greater than 1 fails at periphery under uniaxial load and the failure extends to the centre of specimen with an increase in the axial load. The periphery of a specimen tested under triaxial compressive stresses comes under confinement due to confining pressure. Induced horizontal stresses inside such a specimen are a function of diameter-to-length ratio (shape) and confining pressure.

Very little attention has been paid to the effect of diameter-to-length ratio on the strength of rock under triaxial compressive stresses. Strength-shape relationship under triaxial compressive stresses was investigated for specimens having diameter-to-length ratio from 0.25 to 1 (length-to-diameter ratio from 1.0 to 4.0) by Mogi (1966). From the results, he concluded that the effect of shape on the compressive strength of specimens becomes less critical in a triaxial test. A larger portion of the specimen comes under the influence of the uniaxial state of stress for this range of diameter-to-length ratios in a uniaxial test and differences in the strengths are also negligible. Therefore, the effect of diameter-to-length ratio on the strength of rock specimens under triaxial compressive stresses needs further investigation.

A homogenous rock like material consisting plaster (patternstone U), crushed sand, water and borax has been used to investigate the effect of diameter-to-length ratio on the strength of cylindrical specimens under triaxial compressive stresses. After determining the optimum components and curing conditions of model material, cylindrical specimens having different diameter-to-length ratios from 0.384 to 3.0 were prepared to test under triaxial compressive stresses.

2. PREPARATION OF SPECIMENS

A crushed sand having grain sizes between 0.24 mm and 1.2 mm was selected. After comparison of different percentages of sand, plaster, borax and water, the weight ratio of plaster to sand equal to 30%, the weight ratio of water to total weights of sand and plaster equal to 9% and the weight ratio of borax to water equal to 0.15% were chosen.

First, plaster and sand were mixed by the mixer machine. Then, water with borax were poured into the container of mixer. The mixer was run for 1.5 minutes. At this stage a soft mixture of model material was produced to cast specimens.

During hardening, due to the chemical reaction of plaster with water and crystallisation, the temperature of the model material rose and 1 hour after casting, the specimens were removed from the moulds. Specimens were cured at room temperature of about 19-22 °C and humidity of about 40-50% for a period of two weeks. The weights of 25 specimens were measured during 17 days after casting. After about two weeks, the specimens attained a constant weight (Figure 1).

The ends of specimens were ground in order to make them parallel according to standards (ISRM, 1978; ASTM, 1994) by griding machine. The compressive strength is independent of the size of specimens made from model material consisting plaster and filler materials (Hunt, 1973 and Moomivand, 1993). All internal and external factors affecting the compressive strength of specimens were kept constant. At the end, four groups of cylindrical specimens of 44.5 mm diameter with diameter-to-length ratios of 0.384, 1.0, 2.0 and 3.0 were prepared.

3. TESTING PROCEDURE

Four groups of cylindrical specimens having diameter-to-length ratios of 0.384, 1.0, 2.0 and 3.0

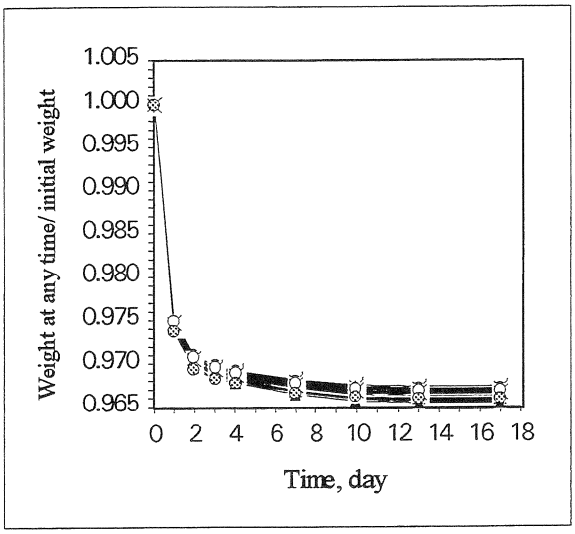

Figure 1. The drying curve of 25 specimens at the room temperature (19 - 22°C).

were tested in the triaxial cell under confining pressures of 0 MPa to 12 MPa. Two test procedures were followed, namely, individual test and multiple failure state test (ISRM, 1978, 1983). In the individual test, after initial axial load (0.11 kN) and deformation were observed, the lateral pressure was applied slowly. Lateral pressure was increased to a value equal to axial stress (hydrostatic pressure) until the predetermined value of confining pressure. The axial load was then increased continuously at a constant rate of displacement (200 μm/second) until the specimen failed under triaxial compressive stresses. A series of specimens from any group was tested using the individual test under different confining pressures.

In the multiple failure state test, the applied axial load and confining pressure at the first stage of failure is the same as in an individual test. The axial load was increased keeping confining pressure constant until the corresponding peak value was observed in axial load-displacement curve. The second stage started by increasing confining pressure manually to the second predetermined level and axial loading was continued until peak value was observed. The third stage of testing was the same as the second.

4. ANALYSIS OF RESULTS

Axial compressive strength increases with increase in both the confining pressure and diameter-to-length ratio for all groups. The effect of diameter-to-length ratio on the axial compressive strength increases as the confining pressure increases. The following criteria were selected for analysis of the results using a computer program (DataFit, 1992):

Bieniawski criterion (1974):

$$\sigma_1 = \sigma_c + B\sigma_3^{\alpha} \quad (1)$$

Normalised form of Bieniawski criterion:

$$\frac{\sigma_1}{\sigma_c} = 1 + B'\left(\frac{\sigma_3}{\sigma_c}\right)^{\alpha} \quad (2)$$

Hoek and Brown criterion (1980):

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \sqrt{m\frac{\sigma_3}{\sigma_c} + s} \quad (3)$$

where σ_1 is axial compressive strength in MPa;
σ_c is unconfined compressive strength in MPa;
σ_3 is confining pressure in MPa;
α is a constant in Bieniawski criterion;
B is coefficient of triaxial strength in Bieniawski criterion;
B' is coefficient of triaxial strength in normalised form of Bieniawski criterion;
s is a constant in Hoek and Brown criterion (s=1 for intact rock); and
m is coefficient of triaxial strength in Hoek and Brown criterion.

Bieniawski criterion fits better to the results than Hoek and Brown criterion. The relationship between axial compressive strength and confining pressure for specimens having different diameter-to-length ratios are given in Figure 2. The ductile behaviour and the scatter of the axial compressive strength values increase with an increase in both the confining pressure and the diameter-to-length ratio.

The values of B, B' and α in Bieniawski criterion with correlation coefficient of the best fit and standard deviation for the four different groups of results having diameter-to-length ratios of 0.384 to 3.0 with different average unconfined compressive strengths are given in Table 1. Table 2 gives the details of results for Hoek and Brown criterion.

Tables 1 and 2 show the higher values of correlation coefficient and lower values of standard deviation for Bieniawski criterion in comparison to Hoek and Brown criterion. The relationships between B, B' and diameter-to-length ratio are given in Figure 3.

The values of B and B' increase with an increase of diameter-to-length ratio. The best functions describe the relationships are as follows:

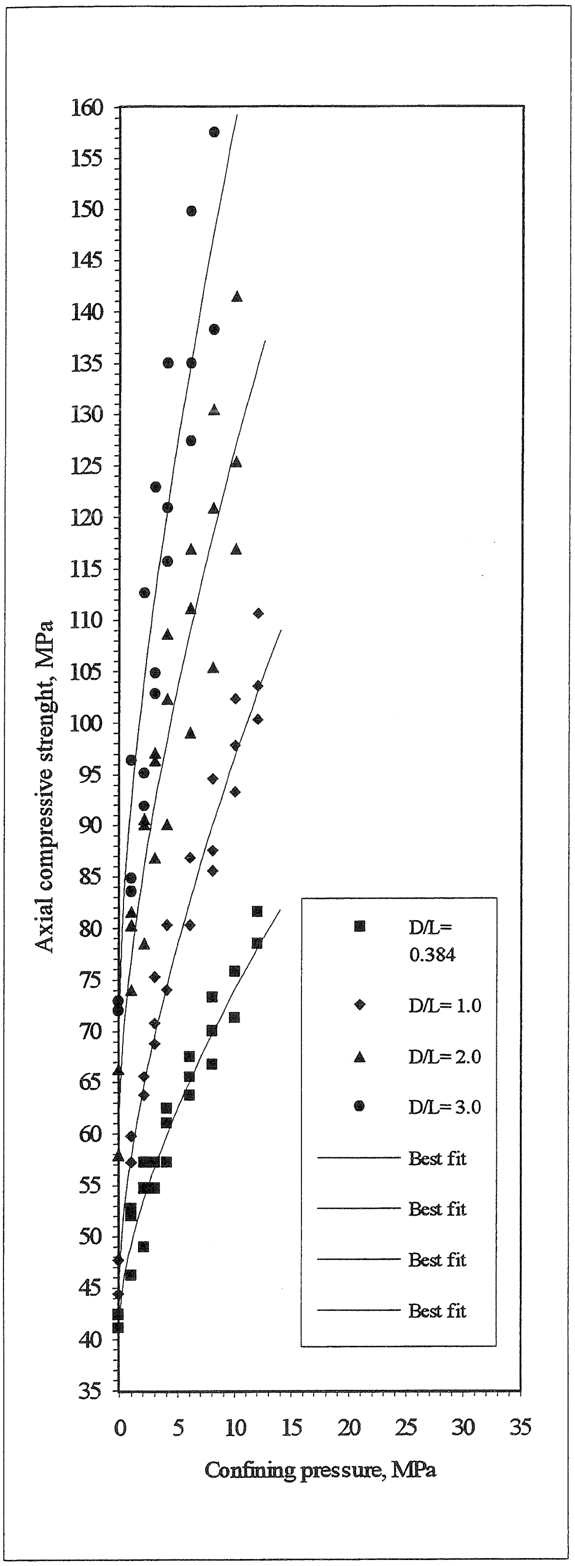

Figure 2. Relationship between axial compressive strength and confining pressure for specimens having different diameter-to-length ratios.

$$B = 7.395 + 5.563\left(\frac{D}{L} - 0.384\right)^{0.815} \quad (4)$$

Correlation coefficient (r)=0.995
Standard deviation = 0.5308

$$B' = 2.002 + 1.262\left(\frac{D}{L} - 0.384\right)^{0.642} \quad (5)$$

Correlation coefficient (r)=0.989
Standard deviation = 0.148

Table 1. Values of B, B' and α in Bieniawski criterion, correlation coefficient of the best fit (r) and standard deviation (stdv) for four different groups of results having diameter-to-length ratios of 0.384 to 3.0 with different average unconfined compressive strengths (σ_{ca}).

Group	$\frac{D}{L}$ ratio	σ_{ca} MPa	B	B'	α	r	stdv MPa
A	0.384	41.80	7.395	2.002	0.65	0.98	2.48
B	1.0	45.98	11.652	3.051	0.65	0.98	3.25
C	2.0	62.06	14.923	3.520	0.65	0.94	7.48
D	3.0	72.52	19.891	4.441	0.65	0.95	8.71

Table 2. Values of *m* in Hoek and Brown criterion, correlation coefficient of the best fit (r) and standard deviation (stdv) for four different groups of results having diameter-to-length ratios of 0.384 to 3.0 with different average unconfined compressive strengths.

Group	$\frac{D}{L}$ ratio	σ_{ca} MPa	*m*	r	stdv MPa
A	0.384	41.80	5.97	0.968	3.717
B	1.0	45.98	13.13	0.977	5.202
C	2.0	62.06	18.33	0.927	8.891
D	3.0	72.52	27.99	0.948	8.584

Dividing Equation (5) by $B'_{0.384}$ ($B'_{0.384}$=2.002 for specimens having diameter-to-length ratio of 0.384) the relationship between ratio of $\frac{B'}{B'_{0.384}}$ and diameter-to-length ratio is obtained as follows:

$$B' = 1 + 0.630\left(\frac{D}{L} - 0.384\right)^{0.642} \quad (6)$$

Relationships between axial compressive strength and diameter-to-length ratio under different confining pressures were analysed. The following equation gives the best fit to the results:

$$\sigma_1 = \sigma_{c0.384}\left(a + b\left(\frac{D}{L}\right)^c\right) \quad (7)$$

where $\sigma_{c0.384}$ is unconfined compressive strength of specimen having diameter-to-length ratio of 0.384;
a is a constant for any confining pressure;
b is a constant for any confining pressure; and
c is constant power for all confining pressures.

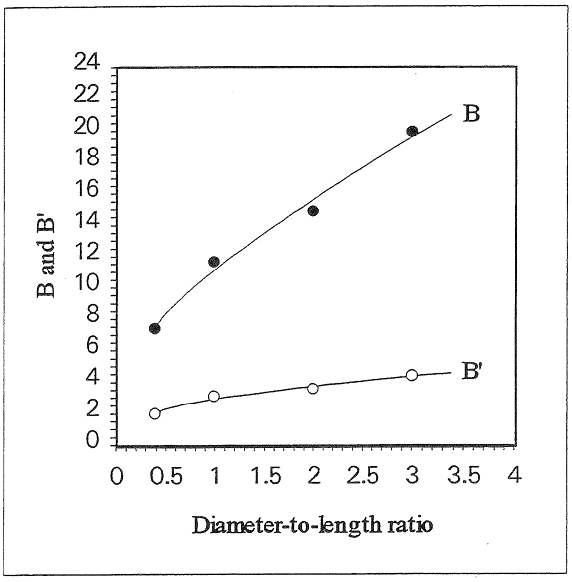

Figure 3. Relationships between coefficients of triaxial strength B and B' (in the normalised form) of Bieniawski criterion and diameter-to-length ratio.

The values of a, b and c for different confining pressures with correlation coefficient and standard deviation are given in Table 3 and the relationships are shown in Figure 4. Coefficients a and b increase with an increase of confining pressure.

Table 3. Values of a, b, c, correlation coefficient (r) and standard deviation of the best fit to the results for different confining pressures (σ_3).

σ_3 MPa	a	b	c	r	Standard deviation MPa
0	0.794	0.371	0.84	0.970	3.273
1	0.994	0.458	0.84	0.964	4.303
2	1.033	0.549	0.84	0.952	6.029
3	1.081	0.627	0.84	0.963	5.863
4	1.097	0.737	0.84	0.971	6.217
6	1.167	0.830	0.84	0.974	6.660
8	1.223	0.935	0.84	0.972	7.776
10	1.354	0.960	0.84	0.954	7.210
12	1.436	1.072	0.84	0.961	3.891

The values of frictional stresses at the ends of specimen having diameter-to-length ratio of 0.384 are higher than at the centre. The frictional stresses are developed at the centre with a decrease in the length of specimen (increasing diameter-to-length ratio with the same diameter). The values of horizontal compressive stresses increase from the periphery to centre of specimen and a higher axial compressive stress is required to extend failure from periphery to the centre of a specimen under uniaxial

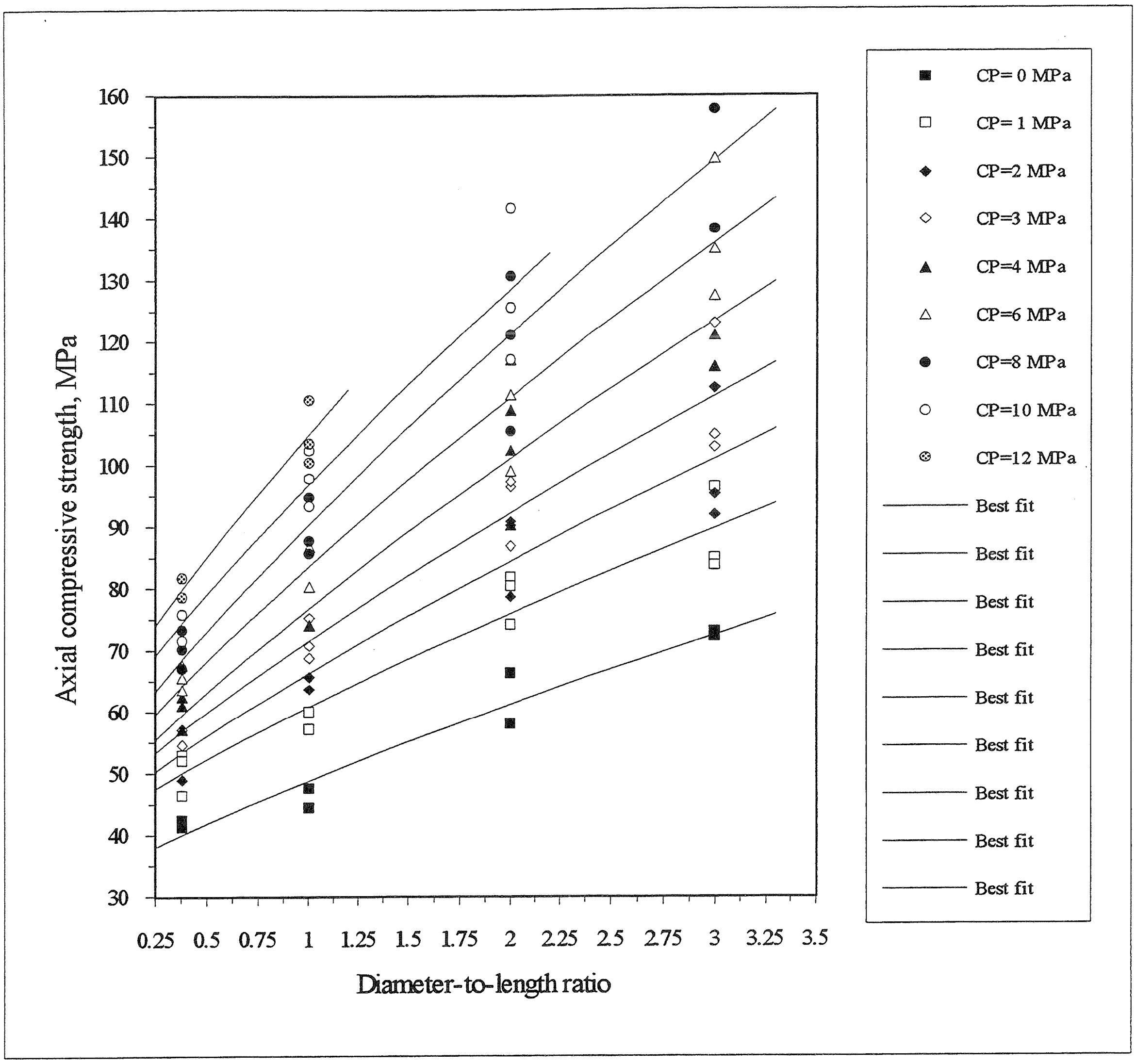

Figure 4. Relationship between axial compressive strength and diameter-to-length ratio of specimens under different confining pressures (CP).

load. Such a specimen that is supported by frictional stresses (horizontal stresses), goes under prefect confinement by adding confining pressure. Therefore, a higher value of axial stress is required to fail specimens under coffining pressure and confinement of frictional stresses. Diameter and length of specimens as external factors have a critical effect on the stress distribution and confinement of specimens under confining pressure.

The bolts in a pillar make confinement as well as confining pressure. Effect of diameter-to-length ratio on compressive strength of bolted pillars is similar to the effect of diameter-to-length ratio on the axial compressive strength of specimens under confining pressure.

5. CONCLUSIONS

The axial compressive strength increases with an increase in the diameter-to-length ratio and the confining pressure for all groups of test results. The effect of diameter-to-length ratio on the axial compressive strength increases as the confining pressure increases. The ductile behaviour and scatter of axial compressive strength of specimens increase with an increase in both the confining pressure and the diameter-to-length ratio. The horizontal stresses (confinement) of specimens are related to the confining pressure and the frictional stresses (the diameter-to-length ratio).

Bieniawski criterion and Hoek and Brown criterion were used for analysis of the results. Bieniawski criterion fits better to the results than Hoek and Brown criterion. Coefficients B and B' in Bieniawski criterion and *m* in Hoek and Brown criterion increase with an increase in diameter-to-length ratio. The increase in coefficients of triaxial strength criteria is related to the value of distribution of frictional stresses.

Relationships between axial compressive strength and diameter-to-length ratio under different confining pressures were analysed. One function (Equation 7) having different coefficients fits the 9 groups of test results under different confining pressures. The coefficients of Equation (7) increase with an increase in the confining pressure.

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