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Estimation of Horizontal Stresses in Pillars

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Summary The strength-shape relationship of a pillar has been analysed by a new physical model keeping all internal and external factors affecting the compressive strength of the model material constant. The model material has also been tested under triaxial compressive stresses. The effect of width-to-height ratio on the compressive strength has been compared with the effect of confining pressure on the axial compressive strength. The induced horizontal stress due to the shape of model pillar specimens under ultimate load has been estimated. The relationship between horizontal stress and width-to-height ratio of a pillar sandwiched between hard roof and floor under any value of vertical stress lower than the maximum vertical stress (compressive strength) has also been estimated. Horizontal stress increases with an increase in width-to-height ratio and vertical stress (depth).

1. INTRODUCTION

The shape of mine pillar is either square or rectangular prism. The shape of a rectangular pillar is a function of the width-to-height ratio and length-to-width ratio, and a square pillar is a rectangular pillar whose length-to-width ratio is one. From laboratory model studies of rectangular shape specimens, Moomivand (1993) and Vutukuri and Moomivand (1993) showed that the length has no effect on compressive strength. The mode of failure of all specimens was double pyramids and it did not change with length-to-width ratio. Also, it can be concluded that the distribution of stresses inside the rectangular shape specimens (pillars) is independent of its length. Therefore, the effect of shape on the compressive strength of a rectangular pillar is related to the dimensions of width and height, and the effect of width and height (width-to-height ratio) on the compressive strength of rectangular and square pillars is the same.

The effect of shape on the compressive strength of pillars is investigated by modelling and the results are compared with the effect of confining pressure on the axial compressive strength of model material in a conventional triaxial test. The horizontal stress in model pillar specimens under ultimate load is estimated from this comparison. The horizontal stress is also estimated by comparison between strength-shape relationship of model pillars and rock mass strength criterion. The induced horizontal stress as a function of width-to-height ratio and vertical stress (depth) of pillars has been analysed.

2. EFFECT OF WIDTH-TO-HEIGHT RATIO ON THE COMPRESSIVE STRENGTH

For determining the effect of width-to-height ratio on the compressive strength, it was tried to keep the size (volume) of specimens constant (1000 cm^3). The effect of size on the compressive strength of 42 cubic models having edge dimensions of 34.3 mm, 50.1 mm, 70.6 mm and 102.0 mm was studied. Statistical analysis of the compressive strength data of four groups using SPSS computer program showed that no two groups were different at the 0.05 level. All internal and external factors affecting the compressive strength of the model pillar specimens having different shapes have been kept constant in the tests.

Square prismatic specimens having different values of width-to-height ratios were tested. The compressive strength of the specimens increases with an increase in the width-to-height ratio similar to increase in the axial compressive strength due to the confining pressure in a conventional triaxial test. The lateral stresses in a model pillar specimen related to its end constraint increase with an increase in the width-to-height ratio. The specimens with width-to-height ratio from 0.25 to 2.0 failed completely after yielding. For specimens having $2<W/H\leq3.5$, differences between compressive strength (yielding strength at periphery of the specimens) and breaking strength inside the specimen increased by an increase in width-to-height ratio. The core of model pillar specimens behaved in a ductile manner when $W/H>3.5$. For specimens with $W/H\geq4$, the axial load increased more than twice after yielding at the periphery and a high percentage of the interior of specimens was intact and had ductile behaviour.

Mode of failure of the specimens having width-to-height ratio of 0.25, typifies double pyramids at the ends and the area between the double pyramids in the middle section of the specimen fails with vertical cracks. It is concluded that the specimen at the centre is under uniaxial stresses and at the ends it is under triaxial stresses. When width-to-height ratio increases, the distance between double pyramids decreases, as at width-to-height ratio of 0.4 and 0.5 most of the specimens failed along their diagonals. For $W/H \geq 0.76$; the mode of failure is double pyramids and the double pyramids cross each other at mid height of the specimens and fracture angle approximately is a constant with increase in the width-to-height ratio. The best function to fit the results is as follows:

$$\sigma_c = \sigma_{c1}\left(A + B\left(\frac{W}{H}\right)^{\alpha}\right) \quad (1)$$

where σ_c is the compressive strength in MPa;

σ_{c1} is the compressive strength of cubic pillar specimen and equal to 40.113 MPa for the model material; and

A, B and α are constants and are equal to 0.588, 0.412 and 0.843 respectively.

When W/H =1; A+B=1 and $\sigma_c = \sigma_{c1}$.

After analysis of the test results by different equations, the following Bieniawski equation (1968) in dimensionless form was found to be more close to the results than other equations:

$$\sigma_c = \sigma_{c1}\left(0.64 + 0.36\,\frac{W}{H}\right) \quad (2)$$

The dimensionless relationships for different materials such as coal and model material under different boundary conditions are very close to relationships derived for pillar specimens sandwiched between hard roof and floor. Crouch and Fairhurst (1973) tested coal specimens under two different platens of steel and sandstone. The compressive strengths under the two different boundary conditions are very similar.

3. EFFECT OF CONFINING PRESSURE ON THE AXIAL STRENGTH

For comparison between the compressive strength of pillar specimens (σ_c) and width-to-height ratio and axial compressive strength (σ_1) and confining pressure (σ_3), 36 cylindrical specimens of 44.75 mm diameter with diameter-to-height ratio of 0.44 were prepared from the same model material used for modelling square prisms. The specimens were tested in a conventional triaxial test under confining pressures from 0 MPa to 35 MPa. The brittle-ductile transition occurred at the confining pressure of 17.5 MPa and the axial compressive strength of 90.17 MPa. The results were analysed using DataFit (1992) program to calculate the parameters in Bieniawski criterion and Hoek and Brown criterion. Bieniawski criterion for intact rock, 1974:-

$$\sigma_1 = \sigma_{ci} + B_1\sigma_3^{\beta} \quad (3)$$

$$\frac{\sigma_1}{\sigma_{ci}} = 1 + B_i\left(\frac{\sigma_3}{\sigma_{ci}}\right)^{\beta} \quad (4)$$

Hoek and Brown criterion for intact rock, 1980:-

$$\frac{\sigma_1}{\sigma_{ci}} = \frac{\sigma_3}{\sigma_{ci}} + \sqrt{1 + m_i\,\frac{\sigma_3}{\sigma_{ci}}} \quad (5)$$

where σ_{ci} is the unconfined compressive strength of cylindrical specimens with diameter-to-height ratio of 0.44 (29.5 MPa);

β is a constant and equal to 0.697 for model material;

B_1 is coefficient of triaxial strength and equal to 8.33 for model material;

B_i is coefficient of triaxial strength in normalised form and equal to 2.943 for model material; and

m_i is coefficient of triaxial strength and equal to 9.40 for model material.

Bieniawski criterion fits better the results than Hoek and Brown criterion.

4. COMPARISON BETWEEN EFFECTS OF W/H RATIO AND CONFINING PRESSURE ON THE AXIAL COMPRESSIVE STRENGTH

The relationship between compressive strength and width-to-height ratio of the square pillar specimens tested in a testing machine when $0.3 \leq W/H \leq 5.0$ is comparable with the relationship between axial compressive stress at failure and confining pressure in a triaxial test. For specimens with W/H>5, failure occurs around the periphery at the same compressive stress. The specimens also undergo a large amount of deformation as well as store large amount of energy. Periphery of a specimen behaves in a brittle fashion for any value of width-to-height ratio and the core of the specimen behaves in a ductile fashion when W/H>3.5. The current failure criteria of rock are applicable up to the brittle range and the mechanism of failure after transition from brittle zone to ductile zone is unknown. Increase of the compressive strength as a function of confinement at core of a specimen with width-to-height ratio greater than its critical value can be risky.

If it is assumed that axial load is parallel to height of a pillar, the distribution of horizontal stresses inside a pillar under pressure have four symmetric planes parallel to axial load and one symmetric plane perpendicular to axial load at mid height of a pillar. Therefore, distribution of stresses from one pint to another point inside 1/16th volume of a pillar having a particular shape and under a particular vertical stress is a variable. In triaxial test, distribution of confining pressure on the surface of the cylindrical specimen is uniform and it is well known that the fracture angle decreases with an increase of confining pressure but the fracture angle is approximately constant with an increase in width-to-height ratio. For this reason, the exponents α and β in Equations (1) and (4) are not exactly equal.

The average unconfined compressive strength of cylindrical specimens is equal to 29.5 MPa. From Equation (1), compressive strength of a square prismatic specimen having W/H=0.3 is equal to 29.5 MPa. The compressive strength of the square pillar specimens is divided by the average unconfined compressive strength of cylindrical specimens (29.5 MPa) and the relationship in dimensionless form between ratio of the compressive strengths and width-to-height ratio is determined as follows:

$$\frac{\sigma_c}{\sigma_{ci}} = A' + B'\left(\frac{W}{H}\right)^{\alpha} \qquad (6)$$

where A' and B' are constants and are equal to 0.799 (i.e. 1.359A) and 0.561 (i.e. 1.362B).

The relationship between axial compressive strength and confining pressure, and compressive strength-shape relationship are given in Figure 1. It shows good correlation between the effects of width-to-height ratio and confining pressure on the axial compressive strength. The brittle-ductile transition occurred at σ_1 of 90.17 MPa. The maximum value for width-to-height ratio in Equation (6) is equal to 5 for the model material. The compressive strength (σ_c) of specimens having width-to-height ratio of 5 is equal to axial compressive strength of cylindrical specimens (σ_1) at brittle-ductile transition.

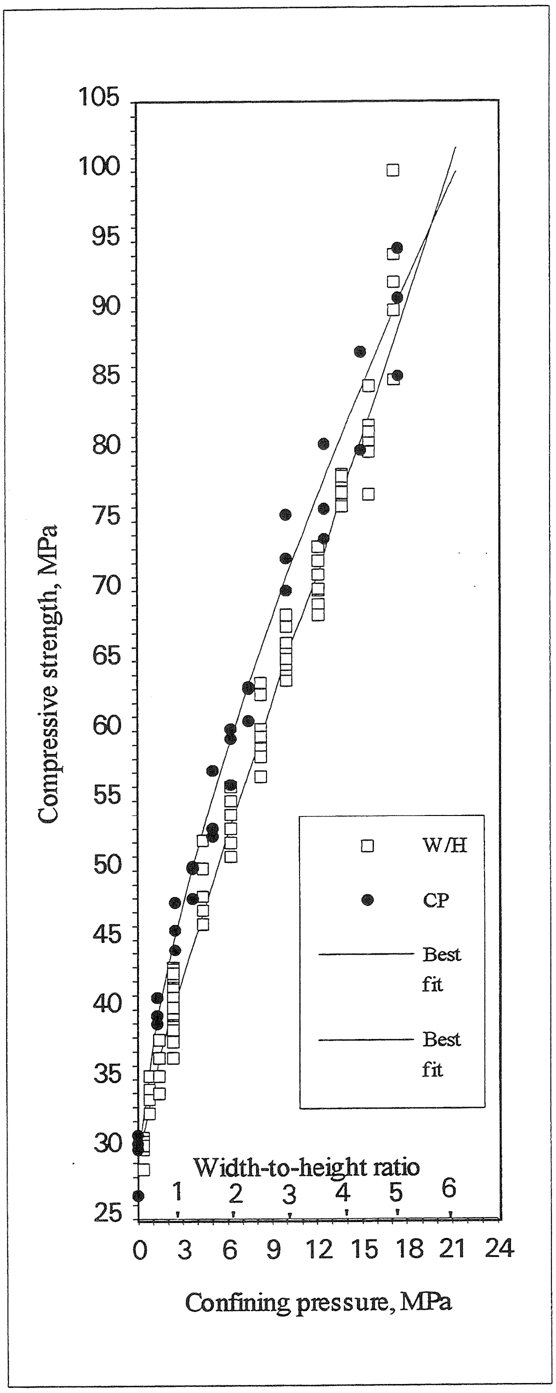

Figure 1. Comparison between effects of width-to-height ratio (W/H) and confining pressure (CP) on the axial compressive strength.

5. ESTIMATION OF THE HORIZONTAL STRESS

Equation (4) is comparable with Equation (6). If σ_C (as a function of the width-to-height ratio) and σ_1 (as a function of σ_3) become equal, two ratios of σ_c/σ_{ci} and σ_1/σ_{ci} will be equal.

Therefore

$$A' + B'\left(\frac{W}{H}\right)^{\alpha} = 1 + B_i\left(\frac{\sigma_3}{\sigma_{ci}}\right)^{\beta} \qquad (7)$$

The horizontal stress (σ_3) of a specimen under ultimate load is estimated as follows:

$$\sigma_3 = \sigma_{ci}\left(\frac{A' + B'\left(\frac{W}{H}\right)^{\alpha} - 1}{B_i}\right)^{\frac{1}{\beta}} \qquad (8)$$

$$\sigma_3 = \sigma_{ci}\left(\frac{f(s)-1}{B_i}\right)^{\frac{1}{\beta}} \quad (9)$$

where $f(s)$ is shape function $\left(f(s) = A' + B'\left(\frac{W}{H}\right)^{\alpha}\right)$.

From Equation (6), $\sigma_{ci} = \frac{\sigma_c}{f(s)}$ and the estimated horizontal stress under ultimate load can be calculated from the following equation:

$$\sigma_3 = \frac{\sigma_c}{f(s)}\left(\frac{f(s)-1}{B_i}\right)^{\frac{1}{\beta}} \quad (10)$$

In the same way the strength-shape relationship can be compared with the Hoek and Brown criterion as follows:

$$A' + B'\left(\frac{W}{H}\right)^{\alpha} = \frac{\sigma_3}{\sigma_{ci}} + \sqrt{1 + m_i\frac{\sigma_3}{\sigma_{ci}}} \quad (11)$$

$$f(s) - \frac{\sigma_3}{\sigma_{ci}} = \sqrt{1 + m_i\frac{\sigma_3}{\sigma_{ci}}} \quad (12)$$

$$(f(s))^2 - 2f(s)\frac{\sigma_3}{\sigma_{ci}} + \left(\frac{\sigma_3}{\sigma_{ci}}\right)^2 = 1 + m_i\frac{\sigma_3}{\sigma_{ci}} \quad (13)$$

$$\left(\frac{\sigma_3}{\sigma_{ci}}\right)^2 - \left(m_i + 2f(s)\right)\frac{\sigma_3}{\sigma_{ci}} + (f(s))^2 - 1 = 0 \quad (14)$$

$$\frac{\sigma_3}{\sigma_{ci}} = \frac{m_i}{2} + f(s) \pm \sqrt{\left(\frac{m_i}{2} + f(s)\right)^2 - \left((f(s))^2 - 1\right)} \quad (15)$$

$$\sigma_3 = \sigma_{ci}\left(\frac{m_i}{2} + f(s) \pm \sqrt{\left(\frac{m_i}{2} + f(s)\right)^2 - \left((f(s))^2 - 1\right)}\right) \quad (16)$$

Equation (16) has two answers and the following is the right answer for estimating σ_3:

$$\sigma_3 = \sigma_{ci}\left(\frac{m_i}{2} + f(s) - \sqrt{\left(\frac{m_i}{2} + f(s)\right)^2 - \left((f(s))^2 - 1\right)}\right) \quad (17)$$

$$\sigma_3 = \frac{\sigma_c}{f(s)}\left(\frac{m_i}{2} + f(s) - \sqrt{\left(\frac{m_i}{2} + f(s)\right)^2 - (f(s))^2 + 1}\right) \quad (18)$$

Relationships between estimated σ_3 and width-to-height ratio from Equations (9) and (17) are shown in Figure 2. By putting values of B_i, β and m_i in Equations (10) and (18), the horizontal stress due to the shape of a model pillar specimen under ultimate load (σ_c) is estimated. When the axial stress increases, the frictional stresses will increase and development of frictional stresses to the body of a pillar due to the width-to-height ratio results in a very complicated distribution of horizontal stresses. From this analysis, only the effective horizontal stress under ultimate load is estimated.

The strength-shape relationship of a pillar is comparable with the rock mass compressive strength criterion. Among different failure criteria for rock mass, the modified Bieniawski criterion by Vutukuri and Hossaini (1992) is as follows:

$$\frac{\sigma_1}{\sigma_{cm}} = 1 + B_m\left(\frac{\sigma_3}{\sigma_{cm}}\right)^{\beta} \quad (19)$$

where σ_{cm} is unconfined compressive strength of rock mass; and
B_m is a coefficient of triaxial strength of rock mass.

In the same way, using B_m instead of B_i in Equation (9), the horizontal stress in a pillar under ultimate load is estimated as follows:

$$\sigma_3 = \sigma_{cm}\left(\frac{f(s)-1}{B_m}\right)^{\frac{1}{\beta}} \quad (20)$$

$$\sigma_3 = \frac{\sigma_c}{f(s)}\left(\frac{f(s)-1}{B_m}\right)^{\frac{1}{\beta}} \quad (21)$$

The horizontal stress under any value of vertical stress (σ_v) lower than maximum vertical stress of a pillar can be estimated using σ_v instead of σ_c as follows:

$$\sigma_3 = \frac{\sigma_v}{f(s)}\left(\frac{f(s)-1}{B_m}\right)^{\frac{1}{\beta}} \quad (22)$$

where σ_v is the average vertical stress and equal to axial load divided by cross-sectional area of a pillar.

σ_3 in Equations (22) is a function of *width-to-height ratio* ($f(s)$) and σ_v for a particular type of rock. As an example, putting β=0.6 and B_m= 4 for coal pillars in Equation (22), the relationships between estimated horizontal stress and width-to-height ratio under different vertical stresses from 0 MPa to 20 MPa are given in Figure 3. Figure 3 shows that horizontal stress increases with an increase in both the width-to-height ratio and the vertical stress (depth). The width-to-height ratio and the vertical stress play main role on the development of horizontal stress.

6. CONCLUSIONS

Increase of the compressive strength of a pillar with an increase in the width-to-height ratio is related to the development of horizontal stresses, as axial strength increases with an increase in the confining pressure of specimens in triaxial tests. The horizontal stress of a pillar is estimated by comparison between the strength-shape relationship and the failure criterion of rock.

The effective horizontal stress due to the width-to-height ratio of a pillar sandwiched between hard roof and floor under ultimate load has been estimated. The horizontal stress of a pillar under any value of vertical stress lower than maximum vertical stress at yielding is also estimated. The two factors of *shape* and *vertical stress* (depth) play a main role on the estimated horizontal stress for a particular type of rock.

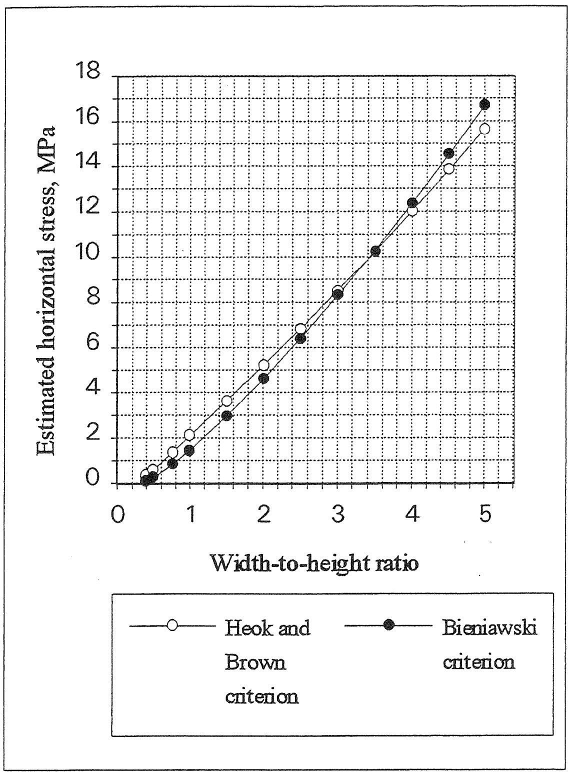

Figure 2. Relationships between estimated horizontal stress of model pillar specimens under yielding and width-to-height using Bieniawski criterion and Hoek and Brown criterion.

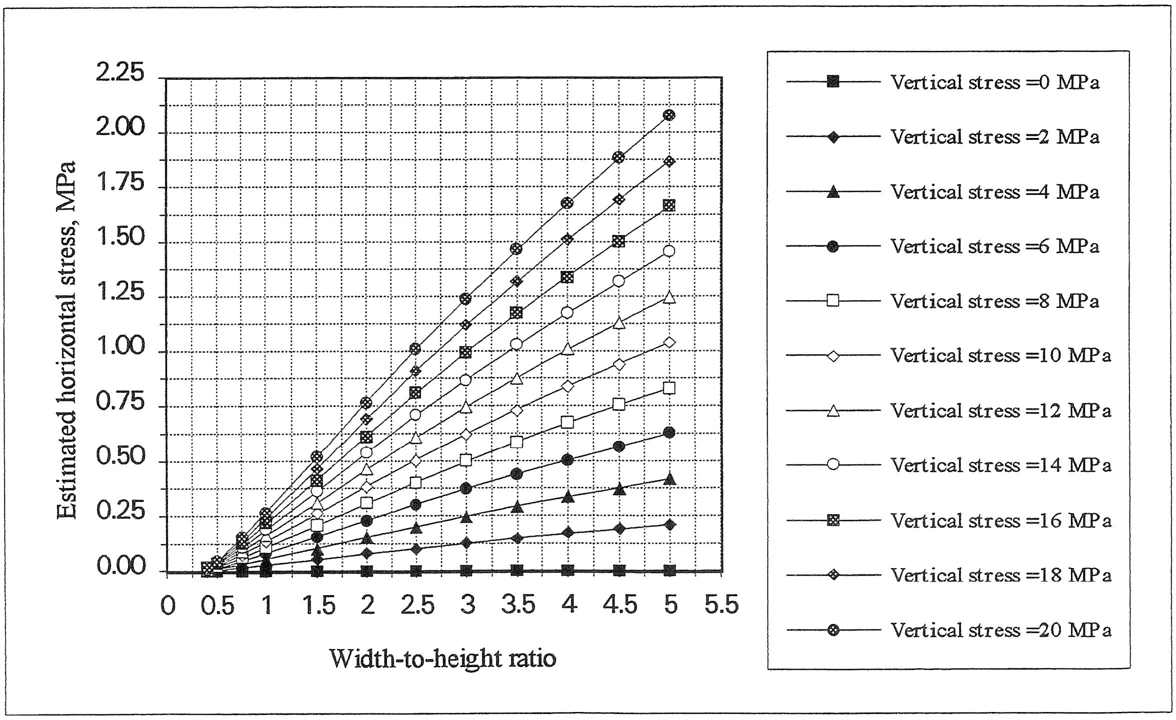

Figure 3. Relationships between estimated horizontal stress and the width-to-height ratio of coal pillars having B_m=4 under average vertical stress from 0 MPa to 20 MPa.

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