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Comparison of Deterministic and Probabilistic Analysis of Rock Slope Sliding Failure Mechanisms

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Summary Current desk top computing power enables rapid, deterministic analysis, and will hopefully lead to the development of equally comprehensive probabilistic tools. In the paper, the power of probabilistic analysis is demonstrated through comparison with deterministic analysis of rock slope stability by means of simple examples of tetrahedral wedge sliding. A spreadsheet with a probabilistic simulation add-in is used for the analysis. Amongst other results, it is shown that for a given Factor of Safety (FOS), the probability of sliding (P(s)) increases and the FOS distribution becomes skewed as structure variability increases. Also demonstrated is the use of probabilistic analysis for reinforcement design, and consequent reduction of P(s) and sliding mass. Wide acceptance of the method will, however, require the development of new stand-alone packages.

1. INTRODUCTION

The mining industry views geomechanical engineering as a mature science demanding rapid accurate analysis. Improvements in desk top computing technology have allowed the development of efficient, comprehensive, though largely deterministic software to address these particular demands.

The validity of the analytical result from such software depends primarily on the quantity and quality of the input data and, traditionally, an appropriate design factor of safety (FOS) has been used to implicitly account for uncertainty and variability of data.

For many years, however, the limitations of such deterministic analysis has been recognised, and attempts made to adopt probabilistic methods able to explicitly model the variability of input data. Much research of probabilistic methods has been undertaken by various workers and some probabilistic software has been marketed commercially (e.g. through CANMET in Canada). Despite this, the widespread use of such methods, particularly in rock mechanics, has been limited by the previous lack of computing power and software, the limited understanding of statistical procedures and the consequent difficulty in deriving and interpreting the results of probabilistic analysis.

However, current computing power and a better understanding of the statistics of geomechanical data should enable development of useful tools for probabilistic analysis. Such tools need to be sophisticated, comprehensive, highly interactive and reliable. Past development has generally been precluded by cost, the limited market and the consequent high unit price of the software.

However, existing computer analysis tools are available for rapid development of relatively robust, specialised, interactive packages. In particular, the spreadsheet has become a standard analysis tool in a wide range of engineering disciplines, and has proved useful for validating computer algorithms and code prior to development of specialised stand-alone packages. Spreadsheet capabilities have been extended through probabilistic simulation add-in packages using Monte Carlo and Latin Hypercube sampling methods.

It is timely, therefore, to demonstrate the capabilities of probabilistic analysis in rock mechanics design, considering the desk top computer power and analytical tools now available. In this respect, the paper compares the results of deterministic and probabilistic analysis in rock slopes by means of simple examples of tetrahedral wedge sliding.

Failure by this mode is controlled by several geomechanical parameters including, amongst others, shear strength, continuity and orientation of rock defects and groundwater condition. For the purpose of this demonstration, however, only structure orientation variability is modelled in order to illustrate how this, in particular, influences FOS distribution, probability of sliding, sliding mass and reinforcement requirements. For the purpose of the exercise, a spreadsheet with a probabilistic simulation add-in has been used.

2. ANALYTICAL PROCEDURE

2.1 Analytical Method

The rigorous method of tetrahedral wedge (wedge) analysis, as detailed in Hoek & Bray (1977), is used. The method is easily implemented in a spreadsheet, with the algorithm being analysed incrementally and the results of individual formulae being viewed directly as cells in the spreadsheet. This facilitates development, verification and debugging of the code. The method is extended through additional analytical procedures, as follows:

- The wedge crest width and the distance of the intersection of the sliding planes behind the slope crest are determined and used to calculate the slope face area of the wedge and the required reinforcement length.
- Individual reinforcement loads are specified at selected horizontal and vertical element spacings and the gross force is then calculated from these using the wedge face area.
- Kinematically invalid geometries (where the sliding surface or line do not intersect the slope) and over-reinforced slopes (negative calculated factor of safety) are assigned a very large FOS to facilitate subsequent probability analysis.

Probability distributions are assigned and probabilistic analysis were undertaken using the spreadsheet's simulation capability. The results are displayed as output parameter distributions and associated statistics. For a wedge analysis including 9 input parameters described by probability distributions, it has been found that a Monte Carlo simulation of 1000 iterations requires approximately one minute to complete using a 55MHz, 486 PC.

2.2 Input Parameters

Input parameters described by probability distributions include:

- dip and dip direction of sliding surfaces;
- friction and cohesion of sliding surfaces; and
- unit weight of rock.

Input parameters which are not varied for a particular analysis include:

- slope dip and dip direction;
- tension crack dip and dip direction;
- upper surface dip and dip direction; and
- groundwater condition.

2.3 Output Parameters

Output parameters provided for each demonstration case include:

- results of a deterministic analysis calculated using the mean values of input parameters (***deterministic results***); and
- results of a probabilistic simulation involving 1000 iterations (***simulation results***).

The particular simulation results provided in this demonstration include distributions and statistical data for::

- factor of safety;
- reinforcement length;
- sliding mass; and
- sliding direction

3. SCOPE OF EXAMPLE ANALYSES

During the course of his work, the author has used probabilistic methods for analysis of complex sliding geometry and shear strength combinations. In several real slope situations, probabilistic analysis has resulted in slope redesign where deterministic analysis has indicated adequate stability. These real situations have given us new insights into the effect of data variability on slope stability. Nevertheless, results of real analysis require careful interpretation and can mask the basic effects of data variability. Comparison of probalistic and deterministic methods is demonstrated, therefore, by means of a very simple geomechanical model in which only structure orientation has been varied.

Figures 1 and 2 show lower hemisphere, equal area projections of pole and density plots for a domain of four sets each containing 300 structures. These were constructed using the simulation package and have normally distributed, uncorrelated dip and dip directions as defined by the parameters shown in Table 1. The envelopes shown on the projections define an area equal to three standard deviations about the mean dip and dip direction of each set. The apparent skewness of the sets is due to the non-linear nature of the projection axes. Figures 1 and 2 also show the slopes orientations used in the demonstration analysis.

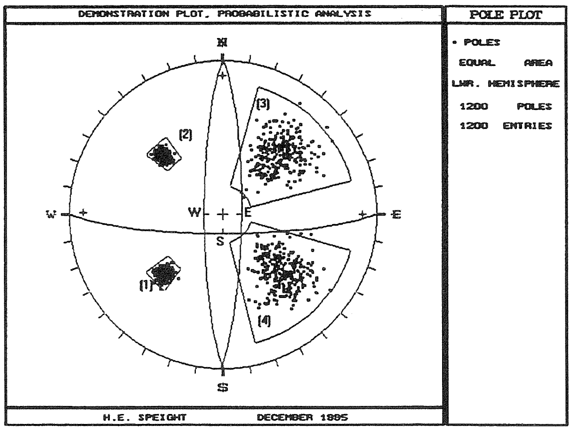

Figure 1. Pole plot of structural domain

Tables 2, 3 and 4 show trial slope design details, geomechanical data and reinforcement specifications.

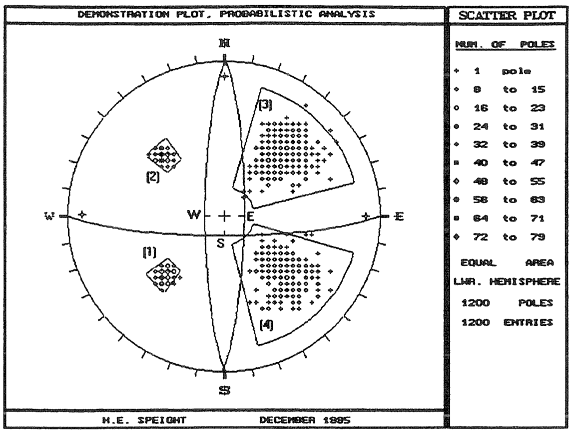

Figure 2. Pole density plot of structural domain

Table 1. Structure orientation parameters

SET NO.	DIP (deg)		DIP DIRECTION (deg)	
	MEAN	ST. DEV.	MEAN	ST. DEV.
1	45	2.5	45	2.5
2	45	2.5	135	2.5
3	45	10	225	10
4	45	10	315	10

Table 2. Slope design parameters

SLOPE	HEIGHT (m)	DIP (deg)	DIP DIRECTION (deg)
E	20	80	90
W	20	80	270
S	20	80	180

Table 3. Other geomechanical parameters

PARAMETER	VALUE
unit weight	25 kN/m^3
structure friction	35 deg
structure cohesion	0 kN
water	none

Table 4. Reinforcement parameters

PARAMETER	VALUE
single element load	500 kN
element dip	3 deg. below horizontal
horizontal spacing	5 m
vertical spacing	5 m

4. RESULTS OF EXAMPLE ANALYSES

Three trial analyses of unreinforced and reinforced tetrahedral wedge sliding have been undertaken, including:

- sliding on Sets (1) and (2) out of Slope (E) for the unreinforced condition only;
- sliding on Sets (3) and (4) out of Slope (W) for the unreinforced and reinforced conditions; and
- sliding on Sets (2) and (3) out of Slope (S) for the unreinforced and reinforced conditions.

4.1 Analysis of Slope (E)

Figure 3 presents probability distributions of FOS against sliding on Sets (1) and (2) out of Slope (E).

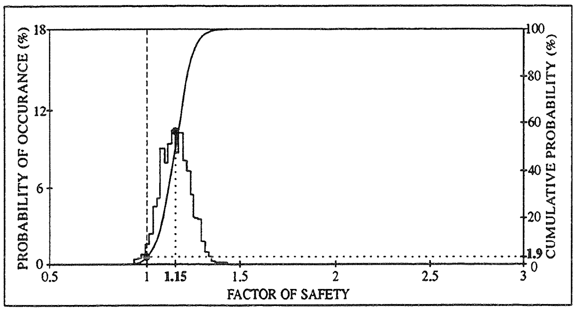

Figure 3. FOS distribution, Slope (E)

Figures 4 and 5 show distributions of the trend and plunge respectively of the line of intersection between structures in Sets (1) and (2).

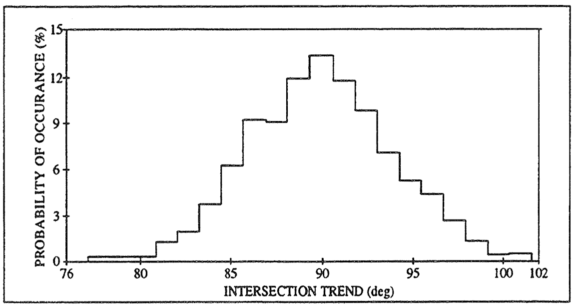

Figure 4. Trend of intersection, Sets (1) - (2)

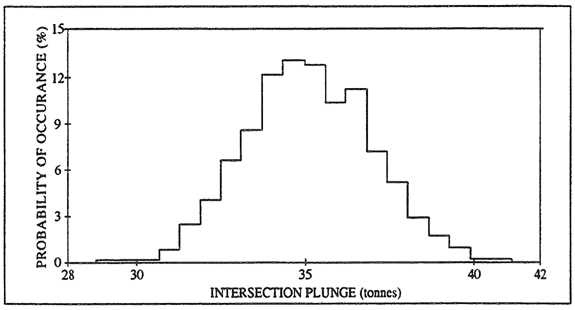

Figure 5. Plunge of intersection, Sets (1) - (2)

4.2 Analysis of Slope (W)

Figure 6 presents probability distributions of FOS against sliding on Sets (3) and (4) out of Slope (W).

Figures 7 and 8 show distributions of the trend and plunge respectively of the line of intersection between structures in Sets (3) and (4).

Figure 9 shows distributions of the sliding mass defined by Sets (3) and (4) and reinforcement length required.

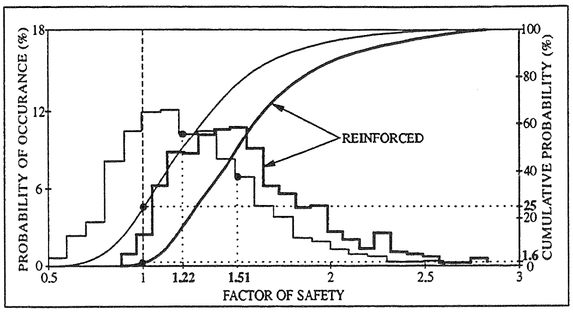

Figure 6. FOS distribution, Slope (W)

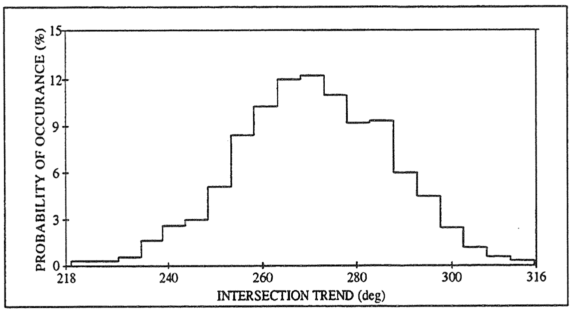

Figure 7. Trend of intersection, Sets (3) - (4)

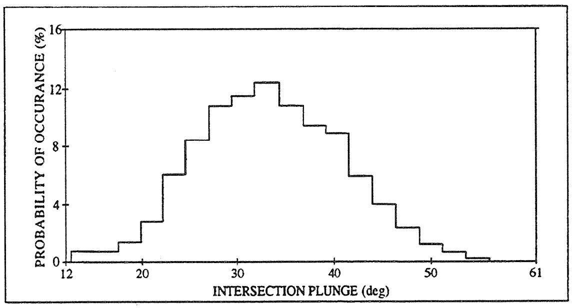

Figure 8. Plunge of intersection, Sets (3) - (4)

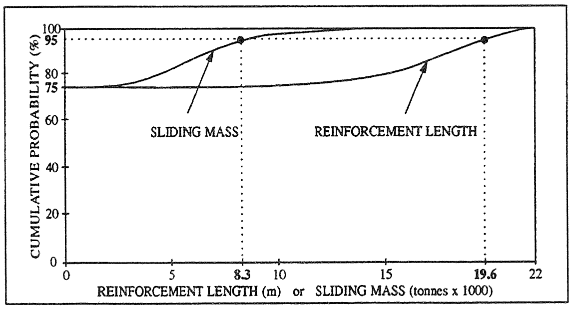

Figure 9. Mass and reinforcement, Sets (3) - (4)

4.3 Analysis of Slope (S)

Figure 10 presents probability distributions of FOS against sliding on Sets (2) and (3) out of Slope (S). Figures 11 and 12 show distributions of the trend and plunge respectively of the line of intersection between structures in Sets (2) and (3). Figure 13 shows distributions of the sliding mass defined by Sets (2) and (3) and reinforcement length required.

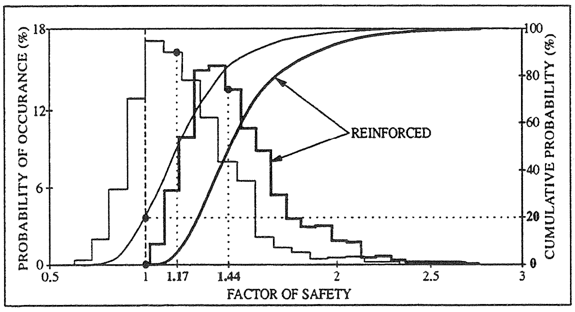

Figure 10. FOS distribution, Slope (S)

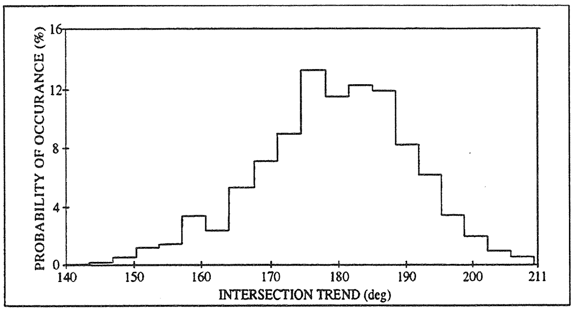

Figure 11. Trend of intersection, Sets (2) - (3)

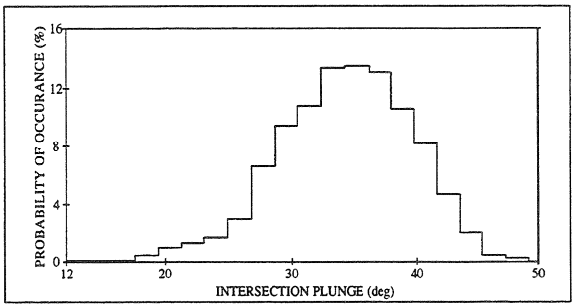

Figure 12. Plunge of intersection, Sets (2) - (3)

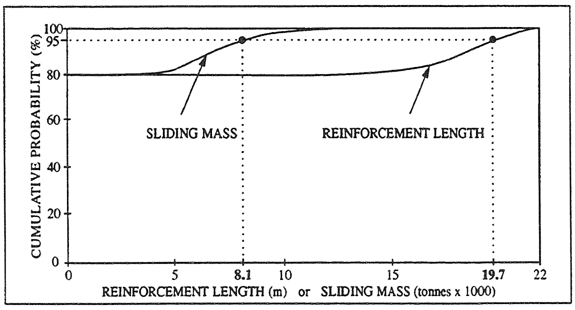

Figure 13. Mass and reinforcement, Sets (2) - (3)

Table 5 presents results of the analysis in the form of statistical data for the various distributions.

5. DISCUSSION OF RESULTS

These simple analyses allow comparison of the results of deterministic and probabilistic analysis and highlight the strength of the probabilistic method.

Table 5. Results of analysis

SLOPE	ANALYSIS	PARAMETER	STABILITY & FOS		LINE OF INTERSECTION	
			UNREINF.	REINF.	TREND (deg)	PLUNGE (deg)
E	deterministic	mean	1.15	-	90.0	35.3
	simulation	mean	1.15	-	90.0	35.2
		stnd. dev.	0.08	-	3.96	1.85
		coeff. of var.	0.07	-	-	-
		median	1.15	-	-	-
		P(s)	1.9%	-	-	-
		skewness	0.13	-	0.00	0.00
W	deterministic	mean	1.15	1.42	270.0	35.3
	simulation	mean	1.28	1.58	269.8	33.6
		stnd. dev.	0.39	0.41	15.90	7.75
		coeff. of var.	0.30	0.26	-	-
		median	1.22	1.51	-	-
		P(s)	25.0%	1.6%	-	-
		skewness	1.26	1.40	0.00	0.16
S	deterministic	mean	1.15	1.42	180.0	35.3
	simulation	mean	1.21	1.51	180.0	34.3
		stnd. dev.	0.28	0.34	11.70	5.45
		coeff. of var.	0.23	0.23	-	-
		median	1.17	1.44	-	-
		P(s)	20.0%	negligible	-	-
		skewness	1.99	6.50	-0.48	-0.48

This is demonstrated in the following sections through discussion of:

- results of stability analysis;
- orientation of lines of intersection; and
- sliding mass and reinforcement.

5.1 Results of Stability Analysis

Except for Slope (E), Both unreinforced and reinforced slopes have been studied (Refer to Section 5.1.2 for explanation).

5.1.1 Unreinforced slopes

Figures 3, 6 and 10 show FOS distributions for unreinforced Slopes (E), (W) and (S) respectively. The deterministic FOS for all unreinforced trial slopes is 1.15. However the probability of sliding P(s) in Slope (E) is only 1.9% compared with 25% and 20% for Slopes (W) and (S) respectively. This may be attributed to the relatively small orientation variability of Sets (1) and (2).

In reality, the variability of Sets (3) and (4) is not uncommon. It may be argued, however, that the calculated P(s) for Slope (W) is unacceptable for the corresponding deterministic FOS. This illustrates the inadequacy of using FOS alone for assessing slope stability and for undertaking slope design when input parameters are highly variable.

It must also be noted that P(s) for Slope (S) is still high despite the low variability of Set (2). This suggests that high P(s) may be expected even when only one of the contributing sets has high variability.

The FOS distribution for Slope (E) is approximately normal, having a low skewness value. The mean and median simulation FOS equal the deterministic FOS.

The corresponding distributions for Slopes (W) and (S) are, however, skewed to the left (being greatest for Slope (S)), despite the original structural data being normally distributed. This may be attributed to the fact that FOS is related to the ratio of the tangents of the angles of friction and sliding. FOS cannot, therefore, be less than zero and becomes large at low sliding angles. The approximately normal FOS distribution for Slope (E) is probably due to the small variability in sliding direction (see Section 5.2).

It is also significant that for each of Slopes (W) and (S), **the deterministic and simulation means are not the same**. The means of the simulation FOS for these slopes (1.28 and 1.21 respectively), and the corresponding medians (1.22 and 1.17) are greater than the deterministic FOS (1.15). This observation may be academic since, arguably, the use of mean FOS is redundant in probabilistic analysis. Nevertheless, in the Author's experience, mine operators prefer that both FOS and P(s) are provided. If an FOS must be quoted, the Author recommends use of the median simulation FOS since (1) 50% of simulation FOS values lie below the median and (2) kinematically invalid cases are included explicitly when determining the median.

5.1.2 Reinforced slopes

Figures 6 and 10 show FOS distributions for reinforced Slopes (W) and (S) respectively. The probability of sliding of the unreinforced Slope (E) is very low hence a reinforced case has not been studied for this slope.

Application of the reinforcement parameters detailed in Table 4 results in a deterministic FOS of 1.42 for both slopes, however median simulation FOS values for Slopes (W) and (S) are 1.51 and 1.44 respectively. The resulting P(s) are respectively 1.6% and "negligible", and arguably these slopes are now over-reinforced. For open pit batter slopes, a P(s) of say 5% or greater may be more appropriate.

For both slopes, there has been little change in the respective coefficients of variation for the unreinforced and reinforced cases, despite the increase in simulation FOS following reinforcement. This is reflected in a corresponding increase in standard deviation (s.d), and implies that the application of reinforcement has increased the absolute range of FOS values calculated during simulation.

5.2 Plunge and Trend of Line of Intersection

Figures 4, 7 and 11 show distributions of the trend of lines of intersection of sliding masses in Slopes (E), (W) and (S) respectively. Figures 5, 8 and 12 show the corresponding plunge distributions.

As may be expected, the deterministic mean and simulation mean trends are essentially equal to the dip directions of their respective slopes. Trends appear to be normally distributed.

The deterministic mean plunge of the line of intersection is 35.3° for all trial slopes. The simulation mean for Slope (E) is similar whilst those for Slopes (W) and (S) are slightly shallower. Plunge distributions for all slopes are approximately normal. For Slope (E) the low plunge variability (s.d. of 1.85) probably accounts for the approximately normal distribution of the corresponding FOS.

5.3 Sliding Mass and Reinforcement Length

Figures 9 and 13 show cumulative probability distributions of sliding mass and reinforcement length for Slopes (W) and (S) respectively. As may be expected, for each slope the percentage of simulation cases which are stable, and therefore have no sliding mass or reinforcement requirement, equal 100% - P(s) for the slope.

The effect of reinforcement on FOS and P(s) for each of these slopes has been discussed in Section 5.1.2. Probabilistic methods also allow estimates of the potential sliding mass and the required reinforcement length to be made to an appropriate level of reliability. This minimises the risk of (1) under-reinforcement if wedge geometry is determined based on mean structure orientations and (2) excessive reinforcement length if the geometry of the sliding mass is based on worst-case (shallowest) structure orientation.

For Slope (W) it may be seen that 95% of simulation wedge geometries have a sliding mass less than 8,300 tonnes and may be captured by reinforcement length of 19.6m. (excluding anchor length). The corresponding values for Slope (S) are 8,100 tonnes and 19.7m respectively.

Again it should be noted that potential sliding mass and reinforcement length are similar for the two slopes despite the low variability of Set (2) in Slope (S).

6. CONCLUSIONS

The following general conclusions may be drawn:

- In the absence of commercial software, spreadsheets and probabilistic simulation add-in packages are powerful tools for probabilistic geomechanical analysis.
- As well as estimating probability of sliding, probabilistic analysis can be useful for assessing the impact of excavation engineering issues such as potential sliding mass and reinforcement requirements.
- Even simple probabilistic analyses can demonstrate the effect of significant variability in input parameters and the essential differences between deterministic and simulation FOS.
- Increased use of such tools for probabilistic analysis of actual engineering problems will lead to a greater understanding of (1) the effect of data variability and (2) the contribution of particular geomechanical parameters to excavation stability.

The spreadsheet used for these example analyses is reasonably robust, however its use requires familiarity with spreadsheets and their idiosyncrasies.

It is likely, therefore, that universal adoption of probabilistic methods in rock mechanics will require the development of sophisticated, comprehensive, highly interactive, stand-alone software. Ideally these would be operated within, and make use of the graphics capabilities afforded by the Windows environment.

7. REFERENCES

Hoek, E. and Bray, J. (1977). *Rock Slope Engineering*, 2nd ed., The Institution of Mining and Metallurgy, London, 402p.