Determination of Critical Failure Surface in Embankments Based on Modified Displacement Vector

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Summary Analysis of embankment stability based on modified displacement vectors makes it possible to determine the most critical failure surface and corresponding factor of safety. By definition, of a family of consistent curves based on modified deformation vectors will introduce a unique system which shows the probable failure surfaces. Local factors of safety for each one of the family of curves can be defined from finite element analysis. Finally, total factor of safety can be determine based on mean local factors of safety. The factors of safety for the family of probable failure surfaces have almost regular variation. Therefore, with determination of a few surfaces, the minimum safety factor and its corresponding failure surface can be obtained. In this research, linear elastic and elastoplastic Bounding Surface Models have been used for soil behaviour in finite element method. Comparison of the results shows that the elastoplastic model gave more satisfactory results than elastic model. In order to verify the purposed method, the final results were compared with the classical modified Bishop and Janbu method, and these two methods show good agreement.

1. INTRODUCTION

Analysis of stability for embankments can be divided into two groups: limit plastic equilibrium method, and limit analysis method. Limit plastic equilibrium, usually referred to as the classical method, has many disadvantages such as: reducing the number of unknowns to change the problem from undetermined to determined, calculations considered only at failure, solution is only based on trail and error procedure and there is no convergence, properties such as void ratio, preconsolidation ratio, Poisson's ratio can not be considered in computation, stress distribution within embankment can not be considered and others. Due to the above disadvantages the limit analysis method was used in this research, based on Bounding Surface and linear elastic models. The Bounding Surface model was first introduced by Dafalias in 1975, then modified by others such as Dafalias and Hermann (1987) for isotopic cohesive soils and Anandarajah and Dafalias (1987) for anisotropic behaviour. In this research plane strain, with various embankment heights and slopes was assumed. Finite element analysis was used with the only loading being the weight of the soil. Modification of nodal displacement were performed based on previous researches such as Nimitchai (1979) and Naylor (1987). To determine the displacement matrices for each node and for each loading step, the virtual and removable part of it must be eliminated. This will be described later in this paper. The scheme used to determine the safety factor was based on existing driving and resisting shear stresses at each section.

2. DEFORMATION ANALYSIS

Natural soil which may be subjected to embankment load, has already deformed under its own weight until reaches equilibrium. Under embankment construction additional stresses due to embankment layers will add to initial natural soil weight which will introduce new deformation in the natural soil and embankment layers. The complication of deformation analysis develop when the weight suddenly releases in finite element analysis; each point (node) in comparison with undeformed condition (before subjecting to load) would introduce deformations which are very large, incorrect, unreliable and part of which are virtual.

For a loading of n layers embankment, which has n+1 levels, virtual deformation of ith level is equal to a sudden weight release deformation due to construction layers up to that layer (second level). The amount of vertical and horizontal deformations for each level of embankment at each step number of loading can be shown by the matrices given in Equations (1) and (2).

$$V = \begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1(n-1)} & v_{1n} \\ v_{21} & v_{22} & v_{23} & \cdots & v_{2(n-1)} & v_{2n} \\ & v_{32} & v_{33} & \cdots & v_{3(n-1)} & v_{3n} \\ & & v_{43} & \cdots & v_{4(n-1)} & v_{4n} \\ & & \vdots & & \vdots & \vdots \\ & \text{zero} & & & v_{n(n-1)} & v_{nn} \\ & & & & \vdots & \vdots \\ & & & & & v_{(n+1)} \end{bmatrix} \quad (1)$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1(n-1)} & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2(n-1)} & u_{2n} \\ & u_{32} & u_{33} & \cdots & u_{3(n-1)} & u_{3n} \\ & & u_{43} & \cdots & u_{4(n-1)} & u_{4n} \\ & & \vdots & & \vdots & \vdots \\ & \text{zero} & & & u_{n(n-1)} & u_{nn} \\ & & & & \vdots & \vdots \\ & & & & & u_{(n+1)} \end{bmatrix} \quad (2)$$

In the above matrices the first index indicates level number and the second one shows the step number of loading. Therefore the amount of modified deformation for each node can be expressed by given equations.

$$u^*_{in} = u_{in} - u_{i(i-1)} \quad (3)$$

$$v^*_{in} = v_{in} - v_{i(i-1)} \quad (4)$$

In these equations u^*_{in} and v^*_{in} are modified horizontal and vertical components of deformations respectively in *i*th level and *n*th step number of loading.

It can be seen from these equations for analysis of an embankment with *n* constructed layers, *n* times analysis should be performed, where for each analysis one layer should be added to previous ones. With an increasing number of layers, the accuracy of results would increase. In this research only four layers were chosen for finite element analysis of embankments.

3. PARAMETERS FOR FINITE ELEMENT MODEL OF EMBANKMENTS

In this research the finite element method was used for analysis of slopes with various material properties, slope angles and boundary conditions. However, in this paper only one condition is presented as an example. In this example a slope of 1 to 2 with a height of 8 metres was chosen. The section were discretised to 360 elements and 413 nodes, geometrical dimensions of the model is shown in Figure 1. In the analysis the weight of natural soil was introduced as initial stress in semi-infinite space into the computer program. Material properties and model calibration parameters were chosen similar for embankment and natural soil from Dafalias (1986).

By using the relationship between parameters of critical state soil mechanics and classical parameters c and ϕ under triaxial consolidated drained conditions, a friction angle of 26.5° and a cohesion of zero were used as the basis of computation.

4. FAMILY OF CURVES CONSISTENT WITH DEFORMATION VECTOR

In Figure 2 modified deformation vectors for each node based on finite element analysis for normally consolidated clay are shown. It is possible to draw a family of curves, in which each point on each curve is tangent to deformation vector. In other words, from mathematical point of view for each starting point (usually at top layer of embankment) only one curve can be drawn which has the above described condition. With access only to limited number of nodes in finite element model the main difficulty here is that, there are only limited numbers of deformation vector directions available to obtain each curve. In other words for a set of failure points in x - y plane only tangential slope of one curve can be obtained. Therefore, it is not possible to define a rule based on mathematical principle to obtain curves equation (tangent slope of curves). In the following treatment a proposed method will be discussed to show how to define a desired family of

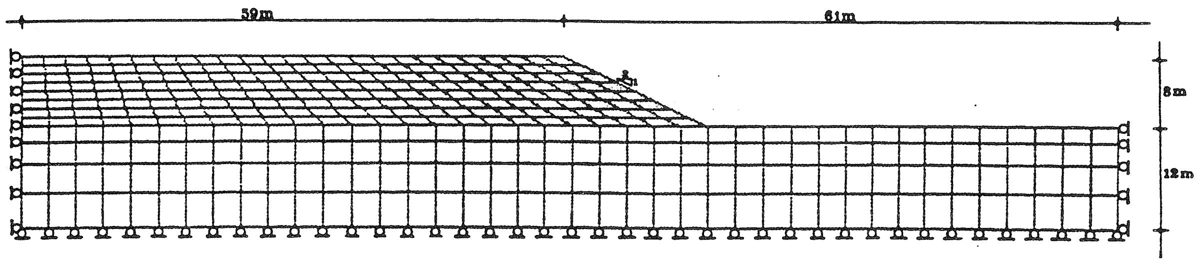

Figure 1. Finite element model used in the analysis of the proposed embankment slope.

curves. As shown in Figure 2 starting point $A_1(x_1, x_2)$ at top of embankment can be chosen, with introducing a small growth in the direction of modified deformation vector from point A_1, the new point $A_2(x_2, y_2)$ can be obtained. At this point by using interpolation direction of deformation vector can be calculated and therefore it makes it possible to introduce new small growth in the direction of modified deformation vector and obtain the third point $A_3(x_3, y_3)$. With continuation of this procedure a set of points A_1, A_2, A_3,... A_n, can be obtained based on numerical methods. It is possible to obtain the equation of these points such as curve fitting procedure which pass through all these points. It should be noted that for choosing larger growth the accuracy of the results would not be affected very much because of the slopes of the introduced curves slowly vary along the curves. Figure 3 shows curves based on growths: ΔL=0.5, 1.0, 1.5, 2.0 metres. It can be seen that all these curves are almost coincident. It is possible to choose a set of points at the top layer of the embankment as starting points and, based on the discussed procedure, obtain a family of curves. These family of curves for an embankment slope with given boundary condition and soil properties are unique and can introduce shape of probable surfaces. Figure 4 shows six possible failure surfaces which were obtained based on above procedure. The accuracy of this procedure will be discussed later in this paper.

5. LOCAL FACTOR OF SAFETY

At each node on embankment section model, local factor of safety can be defined by ratio of resistant shear stress over existing shear stress along deformation vectors. With assumption of Mohr-Coulomb failure criterion, factor of safety can be written as:

$$FS_{local} = \frac{c_i + \sigma_{ni} \tan \phi_i}{\tau_{ni}} \qquad (5)$$

Where ϕ_i and c_i are the friction angle and cohesion respectively, σ_{ni} and τ_{ni} are the normal shear and shear stress along the deformation vectors, which can be obtained from the equations below:

$$\sigma_{ni} = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\theta_i + \tau_{xy} \sin 2\theta_i \qquad (6)$$

$$\tau_{ni} = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta_i + \tau_{xy} \cos 2\theta_i \qquad (7)$$

where σ_x and σ_y are normal and shear stresses which can be obtained from finite element analysis based on assumption of plain strain condition, elastic or elastoplastic behaviour of material, θ is the slope of the deformation vector with horizontal axis and can be obtained from below equation:

$$\theta = \tan^{-1}\left(\frac{v_{in}^*}{u_{in}^*}\right) \qquad (8)$$

Based on the above definition, the total factor of safety base on local ones in the direction of family of curves can be defined as (where ΔL_i is length in the direction of the deformation vector):

$$FS_{local} = \frac{\sum (c_i + \sigma_{ni} \tan \phi_i)\Delta L_i}{\sum \tau_{ni} \Delta L_i} \qquad (9)$$

6. DETERMINATION OF THE MOST PROBABLE FAILURE SURFACE

The most probable failure surface location and its corresponding total factor of safety can be determined without any pre-assumption and, with some skill, the repetition of calculation can be reduced. By studying various behaviour such as elastic or elastoplastic, it can be seen that variation of safety factor in family of curves, only has one minimum value. It should be noted that the local factor of safety at any point along the failure surface may have a factor of safety less than one, but at another location along the surface with a higher factor of safety, may not allow slippage to take place, with the resultant of total factor of safety greater than one. Usually it is possible only to choose a limited number of family of curves and obtain satisfactory results. As an example, consider the purposed embankment in Figure 4 with six failure surfaces (family of curve) as explained in the previous section. The computed factor of safety and their corresponding failure surfaces, for elastic and elastoplastic conditions, are shown in Table 1.

Table 1. Factors of safety for proposed embankment.

Curve No.	1	2	3	4	5	6
Elastoplastic Behaviour	*	14.3	9.11	2.07	0.96	0.97
Linear Elastic Behaviour	*	*	*	*	6.58	4.27

In this table (*) indicates that unacceptable values of factor of safety were obtained due to the boundary conditions assumed in the model. In order to verify the purposed method, for the sake of accuracy, the results are compared with classical methods such as modified Bishop and Janbu methods and shown in Figure 5. It can be seen that the purposed method has enough accuracy when compared to the classical solution with less computation. It should be noted that the main goal here is not to confirm classical solution, but only to justify our results.

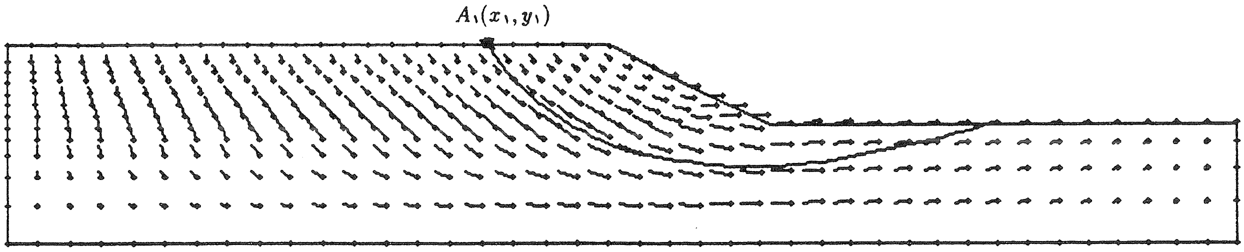

Figure 2. Modified deformation vectors and a sample curve from the family of curves.

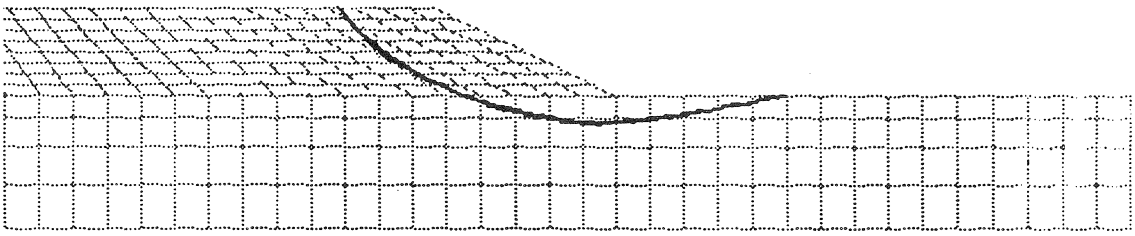

Figure 3. A curve from the family of curves due to ΔL= 0.5, 1.0, 1.5, 2.0.

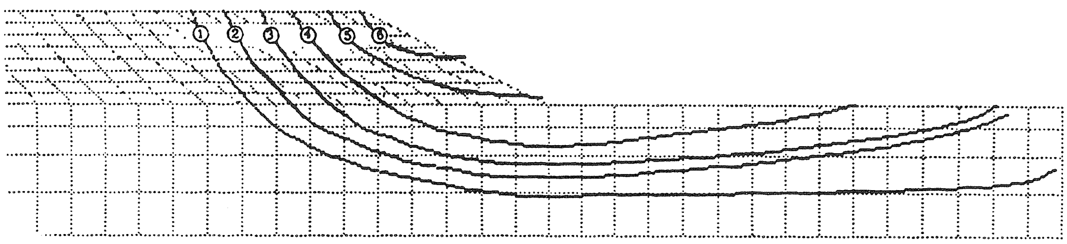

Figure 4. Shape of possible failure surface for elastoplastic behaviour.

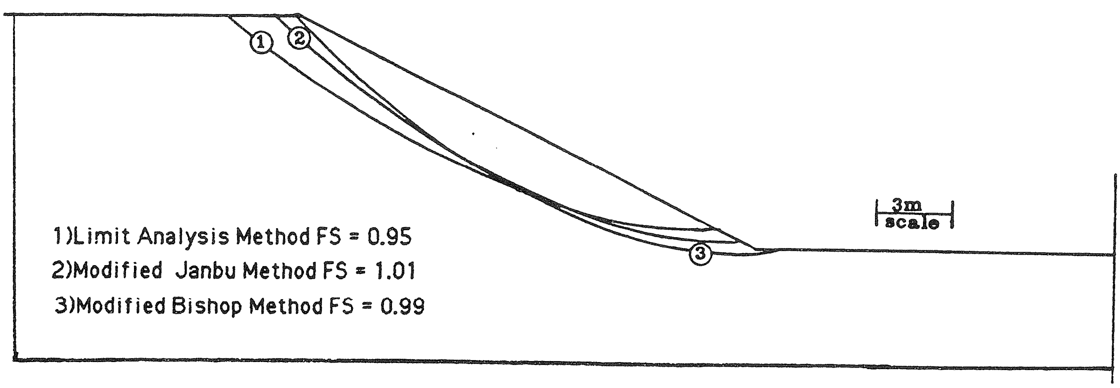

Figure 5. Comparison between the most critical failure surface due to classical methods and limit analysis method.

7. RESULTS AND CONCLUSIONS

It is possible to obtain good information regarding the mechanism of failure of an embankment slope by using modified deformation vectors from a finite element model incorporating elastic and elastoplastic behaviour. Modification of deformation vectors can be performed by the concept of initial stresses in natural soil and the behaviour of embankment layers and cancelling virtual parts of horizontal and vertical deformation components at each level.

At any section of embankment a family of curve can be defined so that each one of these curves are tangent to modified deformation vector, and shape of these curves are good patterns of probable failure surface. With definition of local factor of safety based on existing and resisting shear stresses on family of curves at each point, the overall (average) factor of safety can be computed along assumed failure surface. By comparison of factor of safety for limited number of assumed surfaces, the minimum factor of safety and its corresponding failure surface can be obtained. The purposed method would converge to final results with very limited number of trials in a set of family curves and has good agreement with classical methods.

8. REFERENCES

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