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The paper was published in the proceedings of the 7th Australia New Zealand Conference on Geomechanics and was edited by M.B. Jaksa, W.S. Kaggwa and D.A. Cameron. The conference was held in Adelaide, Australia, 1-5 July 1996.

Analysis of Laterally Loaded Pile Groups with Unequal Dimensions

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Summary Pile group analysis has generally been limited to the situation where each pile in the group has the same length, diameter and stiffness. In this paper the analysis of pile group response to lateral loading is extended to consider the case in which the piles are not restricted to being identical. The existence of a critical length of pile beyond which additional pile length is ineffective is evident in the results of the analysis. The elementary case of two unequal piles is presented first and then an example of a nine pile group is investigated to examine the possibility of varying the types of pile used at the centre, corners and sides to achieve a design aim.

1. INTRODUCTION

Elastic continuum based approaches to the analysis of groups of piles allow for the interaction between piles to be modelled. Although the effects of non-linear soil behaviour and the shielding of piles within a group are expected to alter the response, the elastic analysis remains a sound basis from which to study these phenomena. Poulos (1971a) presented a pile analysis based upon elastic interaction factors for the influence of the loading of one pile upon the deformation of an identical pile. Very little attention has been paid to the problem of lateral loading of a group of dissimilar piles.

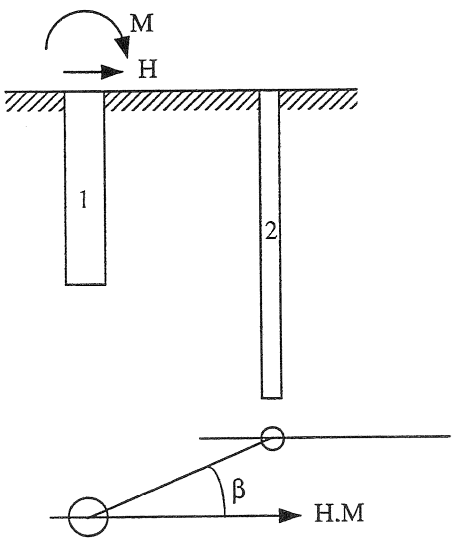

Figure 1. Definition of the two pile problem.

The elevation and plan of the general two pile problem is shown in Figure 1. In general, piles 1 and 2 may have different values of bending stiffness, EI_1 and EI_2, length, L_1 and L_2, and diameter, d_1 and d_2. In common with most pile group analyses, the shear and moment loading of the pile head is here restricted to being in one plane, and the interaction is calculated for the deformations in planes parallel to the plane of loading.

2. ANALYSIS

The analysis (PALLAS) follows the modified boundary element method, in which the pile is modelled by a finite difference approximation of the equations of beam bending and the soil is modelled as an elastic half-space, Poulos (1971a) and Hull (1987). To model the soil, use is made of the closed form solution for uniform horizontal loading of a vertically oriented rectangular plate in an elastic half-space, presented by Douglas and Davis (1964). This solution is used for calculating the interaction matrix for pile self-influence in the soil, while the numerical integration of the equation of Mindlin (1936) is carried out to determine the influence of an element of one pile in the soil upon the elements of all other piles in the soil. The two calculation techniques were found to give the same answers for pile self-influence factors, but both were employed, since the use of the closed form solution is more efficient than the numerical integration method for assessing the matrix of pile self-influence factors.

A significant difference between this and most pile group analyses, is the inclusion of the effect of all piles in the soil-structure interaction matrix. The bending of every pile and the interaction of every pile and soil interface element with every other pile-soil interface element is considered. This leads to a large set of equations to be solved in order to model the behaviour and interaction of all piles in a group.

Previous analyses have been restricted to groups of identical piles, usually employing interaction factors derived from the procedure of analysing just two piles to predict the response of a pile group. This approach tends to ignore the stiffening effect of the presence of a large number of piles in the soil mass.

The results of the analysis for the behaviour of single piles have been compared with the boundary element based solutions of Poulos (1971b) and Banerjee and Davies (1978), and the finite element based solutions of Randolph (1981), Kuhlemeyer (1979) and the results from the author's own finite element program, Hull (1987). By employing the average displacement across the face of the soil element, the results from the current boundary element analysis agree much more closely with the results from the finite element method than do previous boundary element approaches, which employ the displacement at the centre of the soil element. The group analysis is also capable of incorporating a non-linear pile-soil interface response, but this paper restricts attention to the interaction in the elastic problem.

It has been shown that the results for the response of a single pile can be presented more elegantly if use is made of the existence of a critical pile length beyond which any additional length of pile does not alter the response of the pile-head to head loading. Randolph (1981) has presented an expression for the critical length ℓ_c of a pile in an elastic continuum. Based upon the results of his finite element analyses of single piles under lateral loading, and from the consideration of dimensional analysis, his ℓ_c can be approximately represented by (1).

$$\frac{\ell_c}{d} = 2.93\left[\frac{EI}{E_c d^4}\right]^{\frac{2}{7}} \tag{1}$$

This equation was originally cast in a form using the soil shear modulus and Poisson's ratio. Since the effect of Poisson's ratio is less than 3%, it has been rearranged in terms of E_c, the Young's modulus of the soil at a depth equal to the critical length.

Hull (1987) has also derived an equation for the pile critical length from the closed form results from the simpler Winkler (Subgrade Reaction) model of soil response, Hetenyi (1948), the elastic continuum based boundary element analysis used here and an elastic finite element analysis.

$$\frac{L_c}{d} = \pi\sqrt{2}\left[\frac{EI}{E_c d^4}\right]^{\frac{1}{4}} \tag{2}$$

The Winkler model results in a damped periodic solution for the pile deformation pattern which has been found to also apply to piles in an elastic continuum. The wavelength of the Winkler solution has been considered in the choice of (2) for the critical length L_c. Both equations are similar, but since (2) has been developed from the analysis used here, and only a set of equations for pile response for use with (1) were developed by Randolph for similar piles, (2) will be employed here.

The critical length L_c provides a suitable dimension with which to non-dimensionalise the presentation of results. Although the critical length equation applies to any soil profile with a Young's modulus that varies linearly with depth, attention here will be limited to the case of a uniform soil profile with Young's modulus E_s and Poisson's ratio 0.5.

3. TWO PILE INTERACTION

The interaction effect between two piles has been presented by Poulos (1971a) as α interaction factors for the deflection and rotation of the head of an unloaded pile due to shear and moment loading of a second pile. The standard interaction factor is defined to be the ratio of the head response of the pile *i*, due to the loading of another pile *j*, to the response of a single pile to the same load. The response may be the head deflection, u or rotation, θ and the loading may be a head shear force, H or moment M. The standard interaction factors are α_{uH}, $\alpha_{uM}=\alpha_{\theta H}$ (because from reciprocal theory the cross-products of deflection due to moment, and rotation due to shear are equal) and $\alpha_{\theta M}$.

However, when the two piles are unequal there is the complication that, in general, the single pile response is not the same for both of the piles. The reciprocal theorem will still lead to the equality of the cross-products of deformations and loads at two points in an elastic system. However, when the response of pile *i* due to loading on pile *j* is divided by the response of a single pile of type *i* the reciprocal theorem does not lead to equal interaction factors, i.e. $\alpha_{uM}\neq\alpha_{\theta H}$.

The interaction factors of Poulos are presented in terms of the variation of the pile spacing to diameter ratio (s/d), the relative stiffness of the pile to the soil as measured by the value of $K_R=EI/E_sL^4$ and the pile length to diameter ratio (L/d). As well as these, the inclination of a line joining the two pile centres to the plane in which the lateral loading acts, the departure angle β, affects the interaction factors.

From consideration of the symmetry of the problem of a single pile, at any radius from the pile centre the displacement of the soil in the direction of lateral loading, u, will be given by an equation of the form

$$u = u_o + u_\Delta \cos 2\beta \tag{3}$$

where u_o is the mean of, and u_Δ is the difference between the displacements occurring at β values of 90° and zero. The interaction factor variation with departure angle would be expected to follow this pattern too.

For two piles with the same diameter and stiffness at a spacing of three diameters, each with $L_c/d=25$ and various values of L/d, Figure 2 presents α_{uH}, the interaction factor for displacement u due to lateral load H, defined here to be the deflection of the unloaded pile divided by the deflection of the loaded pile. In the figure α_{uH} has been calculated from equation (3) (with $\alpha_o=(\alpha_0+\alpha_{90})/2$ and $\alpha_\Delta=(\alpha_0-\alpha_{90})/2$ calculated from the values of interaction factor at β values of zero, α_0 and 90°, α_{90}) plotted as a line and the actual computed (numerical) values of interaction factor at intermediate β values plotted as symbols.

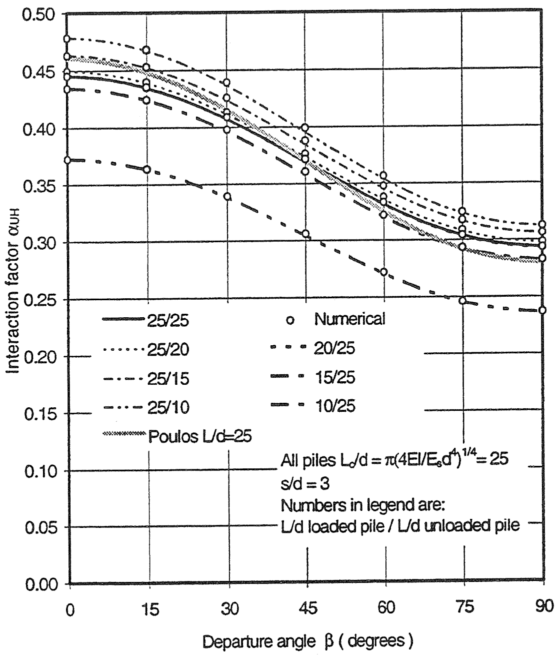

Figure 2. Variation of interaction factor with departure angle β.

The matching of the theoretical curve (based upon the values of the computed interaction at two values of β) to the numerical interaction factors computed directly from the analysis suggests Equation 3 can be used with any value of β. The interaction factor from Poulos (1971a) for s/d=3 is also plotted in Figure 2, again using Equation 3, and the results of both analyses are seen to compare favourably for L/d=25 for the full range of departure angles.

Figure 2 also presents the variation of the interaction factor α_{uH} for a series of cases in which first the L/d of the loaded pile is kept at 25 and the unloaded pile takes on values of L/d 20, 15 and 10, then a series of cases in which the L/d of the unloaded pile is kept at 25 and the loaded pile takes on values of L/d 20, 15 and 10. From the figure it can be seen that the smallest length loaded pile produces a smaller value of interaction factor than the case of equal pile lengths, as might be expected. Whereas the smallest length unloaded pile produces interaction factors slightly above that for the case of equal pile lengths.

It should be emphasised that the two-pile interaction factors, such as those produced here, may still be used to approximately calculate u_i the deflection of pile *i*, due to N loaded piles. However, such a use would be limited to piles having the same diameter and relative pile to soil stiffness and would not fully model the influence of the presence of all the other piles. In regard to this, the standard pile self-influence factor is actually only equal to unity for pile spacings approaching infinity; since the surrounding piles in a group stiffen the response of any pile compared to an isolated single pile.

To account for a reasonable range of pile geometries and placements, that are possible within a group of unequal size piles, the extremely large number of possible variations seems unlikely to be catered for by a sensible set of two pile interaction factors. Also, the fact that the pile self-influence factor is somewhat affected by the number and spacing of piles in the group, leads to the conclusion that each pile group is best assessed using the analysis directly.

4. NINE PILE GROUP STUDY

As an illustration of the results obtainable from the analysis a nine pile group on a grid spacing of 3 pile diameters will be considered. The aim was to obtain the same shear load per pile in the group. A further condition was that each pile has a pinned connection to a rigid raft which results in the piles all deflecting the same amount. The piles are of the same diameter and stiffness. Each analysis required 3 minutes on a 486 /66 Mhz computer. By keeping the same pile stiffness and diameter in a uniform soil, each pile has a critical length of 25 pile diameters.

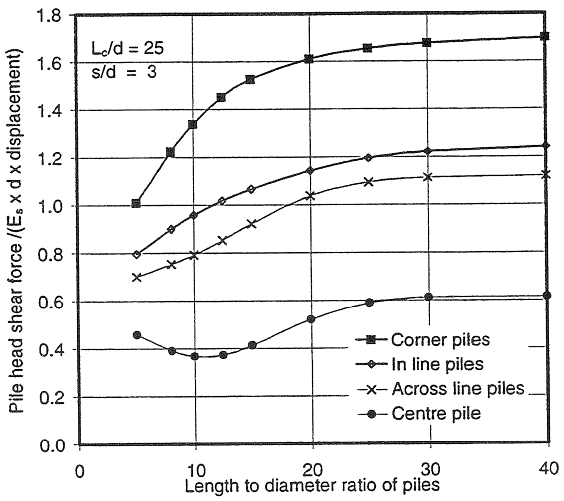

Figure 3. Variation of pile head shear with pile length in a nine pile pinned head group.

In Figure 3 the pile head shears developed as the group undergoes a common displacement are plotted when all pile lengths vary together from 5 to 40 pile diameters. In the figure the "in line" piles refer to the two mid-side piles in line with the applied horizontal group load and "across line" piles are those two mid-side piles disposed at a right angles to the loading. From the figure it is clear that only a minor variation with L/d is evident in the shear generated at the pile heads once the L/d exceeds 25. This result is consistent with the existence of the critical length for lateral loading in pile groups, and suggests that for flexible piles (L/d greater than L_c/d) the length of the pile does not have an important effect upon the group response.

Also evident in Figure 3 is the wide range of shear forces developed in the nine pile group when it is composed of one length of pile. In fact the central pile carries 60 % less shear force than a mid-side pile and the corner pile carries 60 % more than the mid-side piles. Further the critical length concept implies that the increasing of any pile length will not alter the response appreciably. The shorter pile length groups are essentially composed of rigid piles and the shear loads developed are becoming more uniform across the raft and the centre pile becomes more effective.

Four cases of a nine pile group with the corner piles kept at a constant length (L/d = 10, 15, 20 and 25) and the remaining piles all changing together from 5 to 40 pile diameters are shown in Figure 4

It is evident that the closer the L/d of the corner pile becomes to 25, the pile head shear force developed with displacement reaches a limiting value which does not appreciably increase with increasing the length of the corner pile.

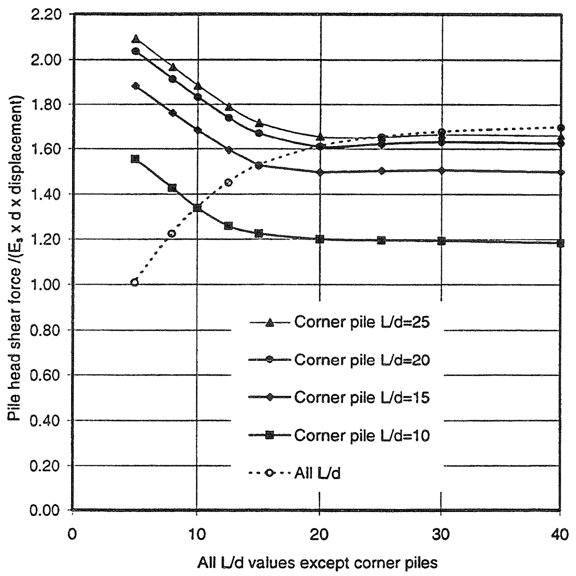

Figure 4. Variation of pile head shear for corner piles of constant length and other pile lengths varying.

The developed shear force response of the corner piles from Figure 3 (where all piles are of an equal but varying length) is also reproduced in Figure 4 for the purpose of comparison. Above a length of 25 pile diameters on the x axis there is only a small variation between the dashed curve, when all piles are of equal length, and the uppermost curve, in which the corner piles are always 25 diameters long. The similarity between these two curves for L/d greater than 25 is again due to the existence of a critical length of pile appearing in the results of the group analysis.

The two sets of curves in Figure 3 and Figure 4 suggest that the length of the corner piles must reduce in order to attract less load. Further, it is apparent that the length of the centre pile will need to increase in order to attract more shear load. However, it would be uneconomical to increase the centre pile beyond the critical length, since there would be no appreciable increase in head shear.

In view of the above it was decided to initially try corner piles with lengths less than 10 pile diameters, the centre pile with a length of 25 pile diameters and both midside piles with lengths of 15 pile diameters. The lengths of piles that resulted in all pile head shear forces of the same magnitude were arrived at after 4 trials.

The corner piles were found to have L/d ratios of 7.9, the "in line" midside piles 11.6, the "cross line" midside piles 11.9 and the centre pile 25. Other solutions would be possible for this problem but the value of group shear load per unit of pile group displacement will be maximised when the longest pile is of the same length as the critical pile length. For this arrangement the average non-dimensionalised pile head shear load per unit of pile group displacement (and obviously the actual pile head loads) was 1.075. For comparison the same average non-dimensional shear load for a group of piles with all lengths equal to 25 pile diameters is 1.30. The exercise of sharing the load equally among the piles has reduced the stiffness of the pile group.

The average length of pile in the solution of the nine pile group problem is 15 pile diameters, and the stiffness of this pile group may be compared with that of a group in which all piles are 15 pile diameters long. The average non-dimensional shear load for a group of piles with all lengths equal to 15 pile diameters is 1.16. The desirability of achieving a uniform pile shear load distribution must be weighed against a drop in group stiffness.

5. CONCLUSIONS

The response of an unloaded pile due to loading of a neighbouring pile can be represented by Equation 3, which is comprised of two factors that may be described as the mean pile influence factor, α_o and the amplitude pile influence factor, α_Δ. Equation (3) defines the variation of interaction with respect to variation of departure angle β.

Reciprocal theory is satisfied by the deformations and loads applied to piles of unequal size, but the standard two-pile influence factors do not reflect the theory. This means the two-pile interaction method becomes unwieldy for groups of unequal sized piles.

The pile self influence factor has been found to vary with the number of piles, and the spacing of the piles in a group. Although two-pile influence factors can model the influence of the loading of other piles in a group, the influence arising because of the presence of the other piles is not modelled. This has normally been found to be of small effect, but can help to explain why large groups of piles are often poorly modelled by two-pile influence factor methods.

Because of the unsuitability of the standard two-pile interaction method for pile groups with piles of unequal size, and the existence of the other piles in the soil mass, a full analysis is deemed preferrable in the design of such pile groups.

Pile group response has been found to display the effects that are consistent with the existence of a critical length of pile. This feature can be used to more efficiently design piles to resist working load deformations by limiting the pile lengths to be a maximum of L_c, provided axial load requirements are met.

A problem in which the lengths of the piles in a nine pile, pinned head group were each varied in order to achieve the same shear force at each pile head has been solved to illustrate a possible use for the group analysis. The design aim was met at the cost of a reduction in group stiffness when compared to a group in which all piles were of the same length.

As new design requirements are raised by increasingly more complex projects the analysis method that is employed must be advanced to a state where every possible advantage can be extracted from the design.

6. ACKNOWLEDGEMENTS

This paper constitutes part of an investigation into the response of pile foundations at the Centre for Geotechnical Research, The University of Sydney under the general direction of Professor H G Poulos. The assistance of Dr. P T Brown in the reviewing of this paper is gratefully acknowledged.

7. REFERENCES

Banerjee, P.K. and Davies, T.G. (1978). The Behaviour of Axially and Laterally Loaded Single Piles Embedded in Non-Homogeneous Soils, *Géotechnique*, Vol. 28 No. 3, pp. 309-326.

Douglas, D.J. and Davis, E.H. (1964). The Movement of Buried Footings Due to Moment and Horizontal Load and the Movement of Anchor Plates, *Géotechnique*, Vol. 14 No. 2, pp. 115-132.

Hetenyi, M. (1948). Beams on Elastic Foundations, *Ann Arbor*, Michigan, University of Michigan Press.

Hull, T.S. (1987). The Static Behaviour of Laterally Loaded Piles, Ph.D. Dissertation, University of Sydney, Australia.

Kuhlemeyer, R.L. (1979). Static and Dynamic Laterally Loaded Floating Piles, *JGED*, *ASCE*, Vol. 105 No. GT2, pp. 289-304.

Mindlin, R.D. (1936). Force at a Point in the Interior of a Semi-Infinite Solid, *Physics*, Vol.7, pp.195-202.

Poulos, H.G. (1971a). Behaviour of Laterally Loaded Piles: II - Pile Groups, *JSMFD*, *ASCE*, Vol. 97 No. SM5, pp. 733-751.

Poulos, H.G. (1971b). Behaviour of Laterally Loaded Piles: I - Single Piles, *JSMFD, ASCE*, Vol. 97 No. SM5, pp. 711-731.

Randolph, M.F. (1981). The response of flexible piles to lateral loading, *Géotechnique*, Vol. 31 No. 2, pp. 247-259.