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The Influence of Spatial Variability on the Design of Pile Foundations

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Summary This paper examines the influence of the spatial variation of a soil mass, that is, its natural or inherent variability, on the design of a single, statically-loaded pile foundation. The paper presents two hypothetical field problems, and employs the results of 223 vertical cone penetration tests performed in a 50 × 50 metre site within the Adelaide city area. The analyses are based on the LCPC Method of pile design (Bustamante and Gianeselli, 1982). It is shown that the spatial variation of the soil deposit can have important ramifications in relation to the design of pile foundations.

1. INTRODUCTION

Pile foundations are designed to transfer the applied loads at the surface to the underlying strata; both safely and economically. Ideally, full-scale pile load tests should be performed on each proposed pile to ensure safety. However, this would be both extremely expensive and impractical. An alternative approach is to use mathematical models to predict the behaviour of pile foundations under load. Many such techniques have been developed for this purpose, however, their accuracy depends on the quality and quantity of the available data. The cone penetration test (CPT) was originally developed to facilitate the design of pile foundations. A number of researchers have compared several CPT-based pile design methods with results of pile load tests. Of the techniques currently available, the *LCPC Method* (Bustamante and Gianeselli, 1982) has been shown to provide the best estimates (Briaud, 1988; Robertson et al., 1988). The following section briefly details this method.

2. LCPC METHOD

The LCPC (Laboratoire Central des Ponts et Chaussées, France) Method is used to predict the allowable design axial capacity of a statically loaded pile, Q_A, using the equation (Bustamante and Gianeselli, 1982):

$$Q_A = \frac{Q_B}{3} + \frac{Q_S}{2} \tag{1}$$

where: Q_B is the resistance due to the pile base;
Q_S is the resistance due to the pile shaft.

For a multi-layered soil, Bustamante and Gianeselli (1982) suggested that Q_B and Q_S may be determined by:

$$Q_B = q_{ca}\, k_c\, A_p \tag{2}$$

where: q_{ca} is the *clipped average cone tip resistance* at the level of the pile base;
k_c is the *penetrometer bearing capacity factor*;
A_p is the area of the base of the pile.

and:

$$Q_S = \sum_{i=1}^{n} q_{si}\, C_p\, t_i \tag{3}$$

where: q_{si} $= q_c/\psi$, the *limit unit skin friction* of the *i*th layer;
q_c is the cone tip resistance as measured by the CPT;
ψ is a constant which allows for the nature of the soil and the pile construction and placement technique;
C_p is the circumference of the pile shaft;
t_i is the thickness of the *i*th layer.

Bustamante and Gianeselli (1982) provided tabulated values for ψ dependent on the soil type and the pile construction method, as well as maximum values for q_{si}, to account for: the presence of localised hard elements; non-compliance with standard penetration rates; poor condition of cones; excess porewater pressures; and deviation of the CPT rods from the vertical.

The clipped average cone tip resistance, q_{ca}, is calculated using the following procedure :

1. The intermediate parameter, q'_{ca}, is determined by averaging the measured values of q_c over the length, $L_p - a_p$ to $L_p + a_p$, where: L_p is the length of the pile; a_p is equal to $1.5 \times D_p$; D_p is the width of a pile, or in the case of a circular cross-section pile, its diameter.

2. The measured values of q_c are then *clipped* to remove local irregularities, such that q_{si} is in the range: $0.7q'_{ca} \leq q_{si} \leq 1.3q'_{ca}$.

3. q_{ca} is then determined by averaging the *clipped* values of q_c, over the length, $L_p - a_p$ to $L_p + a_p$.

Two hypothetical field problems are used to assess the influence of spatial variability on the design of pile foundations, *driven* into a stiff, overconsolidated clay known as the *Keswick Clay* (Sheard and Bowman, 1987). Bustamante and Gianeselli (1982) recommended that, for precast piles driven into a compact to stiff clay, k_c = 0.55, ψ = 40, and $q_{si(max)}$ = 80 kPa (where there is minimal disturbance to the soil in contact with the pile shaft), otherwise $q_{si(max)}$ = 35 kPa. These values will be used when applying the LCPC Method to the field problems presented below.

3. HYPOTHETICAL FIELD PROBLEMS

In assessing the influence of the spatial variation of the undrained shear strength of clay soils on the design of pile foundations, two hypothetical field problems will be presented. Firstly, data obtained from more than 200 CPTs performed at the *South Parklands site* will be considered, and secondly, a series of simulated data will be examined. These are each presented separately below.

3.1 Problem 1: South Parklands Site

A relatively flat site, 50 × 50 metres in area and located within the South Parklands of the *Adelaide city area*, was chosen to assess the small-scale spatial variability of the Keswick and Hindmarsh Clays (Jaksa et al., 1993; Jaksa, 1995). In all, 223 vertical CPTs were performed at the site to a typical depth of 5 metres and in a grid layout as shown in Figure 1.

Two boreholes were drilled to provide samples for unconsolidated undrained triaxial testing, and a further eight continuous core samples were taken, at locations shown in Figure 1 by means of the dynamic push method. The depths to the surface of the Keswick Clay are also shown in Figure 1. Visual inspection of these cores indicated that the Keswick Clay is overlain by various soil layers consisting of red-brown and calcareous clays.

In order to minimise uncertainties associated with the measurements: (i) the CPTs were carried out during the period 3/7/92 to 14/8/92 (in order to limit climatic influences); (ii) an accurate micro-computer based data acquisition system was used to record and store the data (Jaksa and Kaggwa, 1994); (iii) a single electric cone penetrometer was used in accordance with ISOPT-1 (De Beer et al., 1988) and *AS 1289.F5.1* (Standards Association of Australia, 1977); (iv) a single drilling rig operator was used. Bureau of Meteorology data indicated that during the testing period weather conditions were relatively constant and, hence, it is appropriate to assume that the CPT measurements did not include variations as a result of climatic changes (Jaksa, 1995).

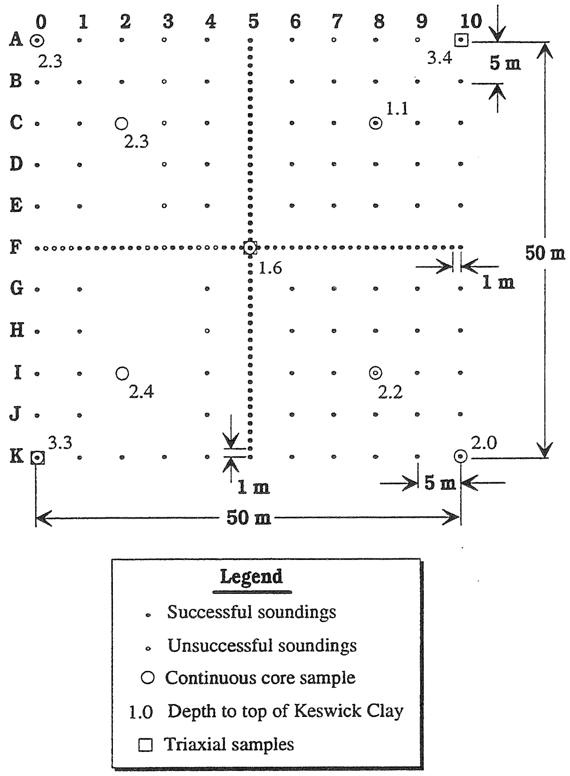

Figure 1. Layout of CPTs at the South Parklands site.

When designing a pile foundation it is necessary to obtain an estimate of the undrained shear strength, s_u, profile for all soil layers within the substrate. When based on laboratory testing, it is common practice to drill a borehole adjacent to the proposed pile, and to obtain sufficient undisturbed samples, from which s_u is determined. It is becoming increasingly more popular to design piles on the basis of CPT data adjacent to the proposed location of the pile. Often, however, limited budgets mean that piles are designed on the basis of CPT data obtained some distance away from the actual location of the pile.

The first hypothetical problem deals with a building to be constructed at the South Parklands site. A typical geotechnical investigation of the site would consist of a CPT at each of the corners of the building, and perhaps one in the centre; being a total of 5 CPTs. Suppose that the structural layout of the proposed building requires a pile to be located at the centre of the site; namely, at location F5. Six CPTs were performed in the vicinity of F5; these being, E54, F44, F51, F5, F501 and F5A, as shown in Figure 2. These 6 CPTs, when used together, provide a good description of the soils in the immediate vicinity of the proposed pile. Since each of the CPTs was driven to a maximum depth of 5 metres below the ground surface, it is only advisable to design piles less than 5 metres in length.

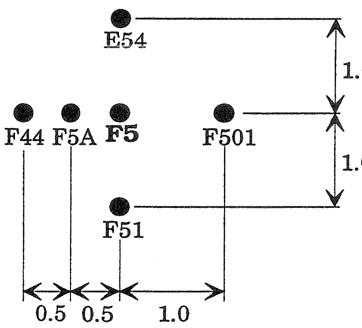

Figure 2. CPTs adjacent to F5.

Suppose that the proposed pile is a driven, precast concrete pile, 0.3 metres in diameter and 4.5 metres long. The LCPC Method recommends that to evaluate the base capacity of the pile (Q_B), CPT data over the depth range $1.5 \times D_p$, above and below the base of the pile, are to be used. However, little information is available regarding the lateral extent over which s_u of a clay soil contributes to the shaft capacity of a pile (Q_S).

Since pile driving is comparable to the insertion of a cone penetrometer into the subsoil profile, results of cone penetration analyses can be used to estimate the radius of the cylinder of soil which contributes to the axial capacity of a pile. Teh and Houlsby (1991) provided a chart for estimating the location of the boundary between the elastic and plastic regions of a subsurface profile as the result of a CPT. Using: a pile radius of 0.15 metres; a cone angle, $\beta = 60°$; and $I_r = 67.4$ (the rigidity index $= E_u/3s_u$, and determined from unconsolidated undrained triaxial tests) as input, the radius of the elastic zone, $r_p = 1.0$ metre, and the depth of the base of the elastic zone, below the tip of the pile, $z_p = 0.54$ metres. The value of r_p compares well with the diameter of 2 metres suggested by Poulos (1995). In addition, the value of 0.54 metres for z_p compares favourably with the $1.5 \times D_p = 0.45$ metres, suggested by the LCPC Method. Thus, the axial capacity of a 0.3 metre diameter by 4.5 metre deep pile is assumed to be influenced by a cylinder of soil, 2 metres in diameter and 4.95 metres deep. Hence, since the 6 CPTs shown in Figure 2, are contained within this soil cylinder, it would be expected that the average of these 6 CPTs would provide a very good representation of the axial capacity of the proposed pile at F5, with a relatively low level of uncertainty. Figure 3 shows the measurements of q_c for the 6 CPTs, as well as their mean.

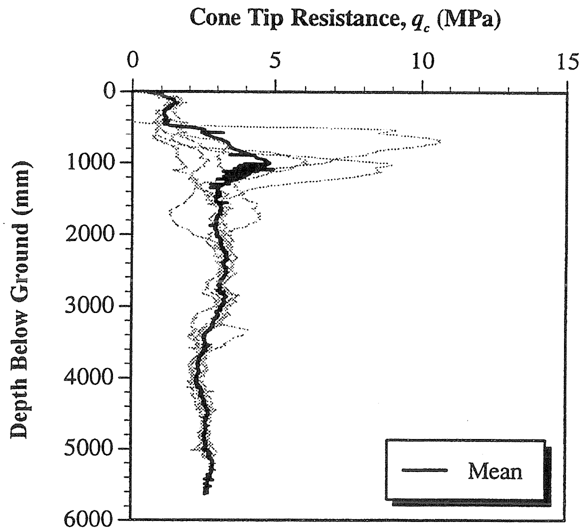

Figure 3. Data from the 6 CPTs adjacent to F5, including their mean.

By entering the mean q_c profile of the 6 CPTs into an *Excel®* spreadsheet, Q_A was found to equal 106.4 kN. Remember that a typical geotechnical investigation would involve, at most, a single CPT adjacent to the proposed pile. Suppose that such a CPT was performed along the centreline of the pile, that is, at F5 itself. Substituting the measurements of q_c, from CPT F5, into the *Excel* spreadsheet yields an estimate of Q_A equal to 102.8 kN, a 3.4% underestimate; assuming 106.4 kN to be the 'true' value.

In order to assess further the influence of spatial variability on pile design, let us suppose that the budget for the geotechnical investigation did not allow a CPT to be performed at F5, but at some distance away from the pile location. How does the distance of the CPT, away from the location of the proposed pile, influence its design estimate? It is possible to assess this by examining the other CPTs performed at the South Parklands site. For example, for a radial distance of 1 metre, 4 CPTs; that is, E54, F501, F51 and F44, were drilled at the site, and by entering these into the *Excel* spreadsheet, it is possible to determine the error.

Figure 4 shows the envelope of the maximum percentage error between the value of Q_A obtained by using a single CPT, as a function of the radial distance from F5, compared with the 'true' value of Q_A, given by the mean of the 6 CPTs, as detailed above. A positive percentage error indicates an overestimation, whereas a negative percentage error indicates an underestimation. It can be seen from Figure 4, that a maximum overestimation of 3%, and a maximum underestimation of 8%, may be obtained when the location of the CPT, used to design a pile, is some 35 metres away from the location of the proposed pile itself. Such errors would be quite acceptable to a geotechnical design engineer.

However, much of the spatial variability of the CPT data is 'masked' by the LCPC Method itself. For instance, Figure 4 was obtained using $q_{si(max)}$ = 35 kPa, which tends to limit the variability of Q_S, and as a result, Q_A. Figure 5, on the other hand, presents the same results as Figure 4, with $q_{si(max)}$ set to 80 kPa; and Figure 6 shows the same results, but with an unlimited $q_{si(max)}$.

It is evident from Figure 5 that, with $q_{si(max)}$ = 80 kPa, the maximum overestimation is 6%, and the maximum underestimation is 36%. The 6% overestimation, again, would be of little concern to a geotechnical design engineer, and, while the underestimation error is relatively large, the resulting estimates of Q_A are conservative, and hence, would result in overdesign, rather than compromising safety. Figure 6 demonstrates the spatial variability 'masking effect', or the inherent conservatism, of the LCPC Method. By not using a $q_{si(max)}$, the variability in the CPT data indicates a maximum overestimation error of 45%, and a 38% underestimation error. An overestimation of 45% could result in an unsafe design. However, by setting $q_{si(max)}$ = 35 kPa, or 80 kPa, the LCPC Method reduces the impact of spatial variability on the allowable axial capacity of the pile, particularly in regards to overestimation, and hence, unsafe design values. However, in soft soils, where q_c, and hence q_{si}, will invariably be low and generally less than $q_{si(max)}$, the influence of $q_{si(max)}$ will be minimal. In soft soils, therefore, the LCPC Method will result in less conservative designs than those given by stiffer soils.

In summary, this hypothetical problem has demonstrated that the spatial variability of the undrained shear strength of soils has a relatively minor influence on the design of pile foundations. However, the influence of spatial variability has been greatly reduced by the LCPC Method itself, which incorporates a maximum limit unit skin friction, $q_{si(max)}$, of either 35 or 80 kPa.

3.2 Problem 2: Simulated Data

The inherent variability of a soil deposit or rock mass is generally quantified in terms of: (i) the mean; (ii) the standard deviation, variance or coefficient of variation; and (iii) the scale of fluctuation (Vanmarcke, 1977) or the range of influence, a (Journel and Huijbregts, 1978). The second field problem examines the influence of the parameter a on the design capacity of the same pile considered in the previous field problem.

In order to carry out such an assessment, it is necessary to generate a large number of realisations of three-dimensional data. The *turning bands method* (Journel and Huijbregts, 1978) is one such technique which is able to generate simulated data in one, two, or three dimensions. The geostatistical software library, *GSLIB* (Deutsch and Journel, 1991), provides generic FORTRAN code for the turning bands method via the program *TB3D*.

In order to assess the influence of the range, a, on the design axial capacity of the same pile examined in §3.1, q_c data were simulated using a grid layout of external dimensions: 2 × 2 × 5 metres. The extent of the grid was determined by: the dimensions of the

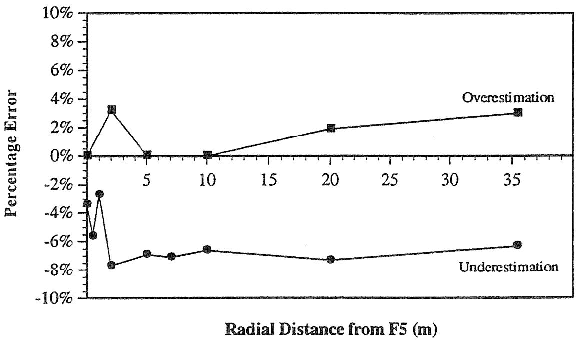

Figure 4. Relationship between the percentage error of Q_A and the radial distance of the CPT, used to determine the pile at F5, using $q_{si(max)}$ = 35 kPa.

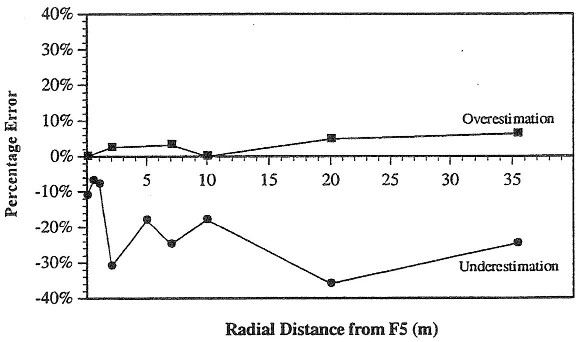

Figure 5. Relationship between the percentage error of Q_A and the radial distance of the CPT, used to determine the pile at F5, using $q_{si(max)}$ = 80 kPa.

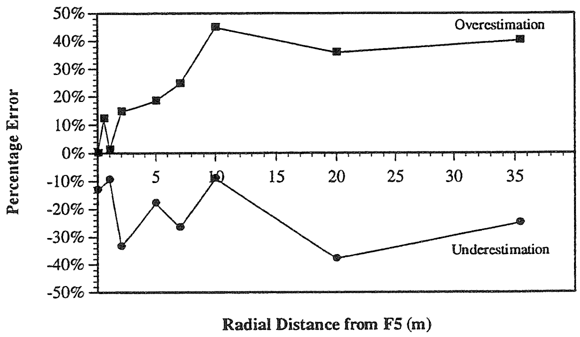

Figure 6. Relationship between the percentage error of Q_A and the radial distance of the CPT, used to determine the pile at F5, using an unlimited $q_{si(max)}$.

pile; the requirements of the LCPC Method; and the 2 metre lateral extent of the cylinder of soil, explained previously. The spacing between adjacent data points was chosen as a compromise between the following criteria: (i) to provide sufficient data to enable reliable modelling to be carried out; (ii) to ensure that the total number of simulated data did not exceed computer array and memory limitations; and (iii) to provide reasonable computer solution times. The resulting grid resolution was set at 0.1 metres in each of the three directions, which resulted in a $21 \times 21 \times 50$ grid, representing a total of 22,050 data points.

As mentioned previously, at best, a geotechnical investigation may include a single CPT performed along the centreline of each proposed pile. The uncertainty arises as to how well this single CPT represents the 'true' strength of the soil mass associated with the pile, and whether this CPT is influenced by the variability of the soil mass itself. In addressing these concerns, the field problem presented in this section is based on the following procedure:

1. The 'true' strength of the soil mass, which influences the behaviour of the pile, is assumed to be the *spatial average* of the simulated data within the $21 \times 21 \times 50$ grid. (The behaviour of many geotechnical engineering systems is governed, not by local parameters, but by spatially averaged characteristics - Vanmarcke, 1977). As a result, this spatially averaged strength is obtained by averaging the values associated with each depth level. That is, each horizontal plane, which consists of a 21×21 point grid, is averaged to provide an estimate of the 'true' spatially averaged strength corresponding to that particular depth. The end result is 50 averaged values of q_c, with each one corresponding to a depth from 0.1 metres to 5.0 metres below the ground surface.
2. The 'true' design axial capacity of the pile, Q_A, is then determined by substituting these data into the LCPC Method.
3. The centreline CPT is obtained by identifying the simulated values associated with the central grid point corresponding to each depth level.
4. An estimate of the design axial capacity of the pile, Q_A^*, based on the CPT measurements, is then made by substituting these centreline data into the LCPC Method.
5. The two values of Q_A and Q_A^* are then compared, and the percentage error, E_{Q_A}, is determined by:

$$E_{Q_A} = \frac{Q_A^* - Q_A}{Q_A} \times 100\% \qquad (4)$$

6. This procedure is subsequently repeated for several different ranges, a.

In order to generate random realisations, *TB3D* requires the mean and standard deviation of the simulated data set, the type of *semivariogram* (Journel and Huijbregts, 1978), as well as a seed which is used to randomise the simulation process. The mean, m, and standard deviation, σ^2, of the q_c measurements from the 223 CPTs performed at the South Parklands site are: 2.953 MPa and 3.017 MPa^2, respectively. Specifying these values, as well as an isotropic *spherical* semivariogram model (Journel and Huijbregts, 1978), *TB3D* was used to generate 100 simulations at each of the ranges, a: 0.001; 0.01; 0.1; 1.0; 10; 100; 1,000; 1,500 and 2,000 metres (where a = 0.001 metres represents a completely random soil, whereas a = 2,000 metres represents a perfectly correlated deposit). The results of the simulations are summarised in Figure 7.

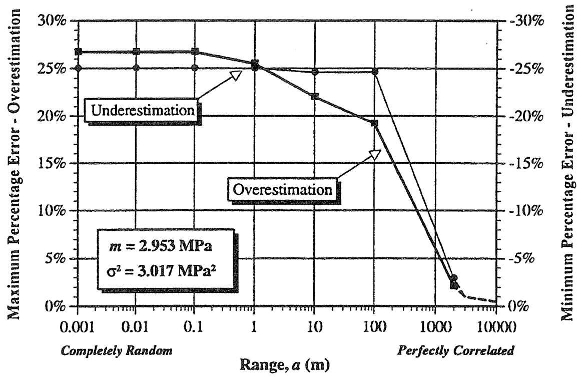

Figure 7. Relationship between the range, a, and the maximum and minimum percentage errors, E_{Q_A}, assuming the pile is founded within soils similar to those encountered at the South Parklands site (100 simulations / range).

Several conclusions can be inferred from Figure 7:

- Unlike the previous field problem, considerable overestimates, that is, up to 27%, as well as significant underestimates, up to –25%, can be observed. This implies that the conservatism, which is part of the LCPC Method, has less of an effect when the range decreases; that is, when the randomness of the material increases.
- The maximum and minimum errors indicated in Figure 7, are exactly that, and in many individual simulations the observed error was significantly lower. As a consequence, one must be aware that there is a probability of occurrence associated with each of the maxima and minima shown in Figure 7. By performing many more simulations than have been carried out in this study; that is, many hundreds of simulations, it would be possible to quantify these probabilities, and associate a risk to each of them.
- There is a strong relationship between the range, a, and the observed error, E_{Q_A}. For a very homogenous or highly correlated material, the error between the design axial capacity of the

pile (as indicated by the central test) and the 'true' pile capacity is relatively low; both in overestimation and underestimation. However, as the randomness of the material increases, represented by a decreasing range, the central test measurements fail to adequately represent the spatial average of the soil mass, and consequently, the observed error increases. What is surprising from Figure 7 is the magnitude of the errors and the degree of homogeneity associated with these errors. Several researchers have measured ranges, or scales of fluctuation, for s_u, between approximately 0.1 and 50 metres (Jaksa, 1995). The errors associated with these ranges can be as large as 20% or more, in both underestimation and overestimation, which is a significant error and compromise of safety, with respect to engineering structures.

- Since Figure 7 indicates that the observed error increases with increasing randomness of the soil mass, more testing is needed to reduce this error. For example, more than one CPT may be required to satisfactorily estimate the spatial average of a relatively heterogeneous soil mass.

4. CONCLUSIONS

This paper has examined the influence of the spatial variation of the soil deposit on the design axial capacity of a single, relatively shallow, pile. It has been observed that spatial variability can have a considerable effect on the results obtained. This is particularly so for materials with ranges of influence less than approximately 100 metres. The observed error between the 'true' design capacity of the pile, and that indicated by measurements, can be as high as 25%, both in underestimation - resulting in a more costly design; or more significantly in overestimation - resulting in underdesign, which compromises the safety of the overall structure.

In addition, it has been observed that the LCPC Method of pile design, regarded by many as being the most reliable CPT-based pile design technique, substantially reduces the effect of spatial variability, and as a consequence, results in a more conservative design solution. However, significant errors, both in underestimation and overestimation, can occur when the LCPC Method is applied to soft soils, or to soils which are moderately to highly uncorrelated; that is, $a < 200$ metres.

It remains to be seen whether the same results would be observed at other sites, using different pile dimensions, and different pile design criteria, such as settlement. What is necessary in a changing world is to maintain the present safety margins through large safety factors.

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