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Physical and Numerical Modelling of Combined Sedimentation/Consolidation of Coal Tailings

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Summary The combined processes of sedimentation and self-weight consolidation of coal mine tailings have been modelled physically in 1.0L measuring cylinders and a square glass settling column. Slurry densities were measured using a gamma-ray adsorption method. It was found that the sedimentation process dominates in the early stages of the tests and is controlled by factors such as the initial solids concentration and the suspension medium, but is not sensitive to the initial slurry height. Particle segregation occurs in slurries with low initial solids concentrations. Self-weight consolidation starts from the base and proceeds upwards for slurries with low initial solids concentrations, but this process is more complicated in slurries with high initial solids concentrations. The combined sedimentation/consolidation processes were modelled numerically using a coupled equation and an implicit finite difference technique. Good agreement was obtained between the numerical and experimental results for slurries where particle segregation is negligible.

1. NOTATION

c_v	Coefficient of consolidation
g	Gravitational acceleration
H	Height of slurry-water interface
k	Coefficient of permeability
So	Initial solids concentration by mass
t	Time
v_s	Particle settling velocity
α	Volumetric solids concentration
α_0	Initial volumetric solids concentration
α_m	Volumetric solids concentration at which sedimentation stops
ρ_l	Density of liquid
ρ_s	Density of solids
σ'	Effective stress
ξ	Eulerian space co-ordinate
$\vert_z$	refers to a material co-ordinate z

2. INTRODUCTION

The disposal of coal tailings is now a significant economic and environmental issue. Coal mine tailings are conventionally disposed of as an aqueous slurry by sub-aerial deposition, during which the tailings particles undergo sedimentation and self-weight consolidation under water. These two processes occur simultaneously, with sedimentation dominating towards the top of a recently placed layer, and self-weight consolidation dominating towards the base of the layer. In order to find new tailings disposal methods aimed at minimising costs and complying with environmental constraints, the combined sedimentation and self-weight consolidation behaviour of tailings in settling ponds must be understood. However, little research has been undertaken on combined sedimentation and consolidation although there has been extensive study of each separately.

The purpose of the study described in this paper was to obtain data on sedimentation and self-weight consolidation behaviour of coal mine tailings and to seek a better qualitative and quantitative understanding of the combined processes.

3. MATERIALS AND METHODS

The combined processes of sedimentation and self-weight consolidation of coal mine tailings were modelled physically in small cylindrical and square settling columns. The cylindrical columns were 1.0L measuring cylinders commonly used for particle size distribution analysis. The square column was made of glass and 1.4 m high with the cross section area of 0.0256 m^2.

3.1 Material

The coal mine tailings and tailings water used for the tests were obtained from New Hope Colliery in

South-East Queensland, Australia. The tailings comprised about 22.5% particles larger than 75μm, 23.3% of clay-size particles and 54.2% silt-size particles between 2μm and 75μm in size. The average liquid limit and average plastic limit of the tailings were 35.4% and 20.4%, respectively, giving an average plasticity index of 15. The tailings were thus classified as a low plasticity clay (CL) under the Unified Soil Classification system. The average specific gravity was 1.725. The mineral composition of the tailings was determined by the X-ray diffraction method. The major minerals present were quartz and kaolinite. The tailings comprised 51.7% pure coal and 48.3% mineral matter (42.1% ash and 6.2% volatiles), based on the results of proximate, ultimate, and ash analyses.

Tailings water used was collected directly from a tailings pond near the washery at New Hope Colliery. The flocculant added in the coal washing process was a polymer at a dosage of about 0.05% by mass. The pH value of the fresh tailings water was about 7.3.

3.2 Experimental Procedure

Slurries were prepared by mixing air-dried tailings and the requisite amount of tailings water for the desired solids concentrations. The slurries were then left to stand in buckets overnight and were covered to prevent evaporation. They were re-mixed thoroughly just prior to the start of a test, and poured directly into the settling columns.

In the small column tests, only the height of the interface formed between the slurry and supernatant water was monitored during the testing. At the end of some tests, a number of samples were taken from the sediment at different heights above the base of the column and were analysed for particle size distribution.

In the square column tests, the height of the slurry-water interface, slurry density, and pore water pressures were regularly monitored during the course of the settling test. The measuring system has been described in detail by Li (1994). Slurry densities were determined by a gamma-ray adsorption method. A minimum change of 20 mm was required in the height of the slurry-supernatant water interface for detection of a significant change in the density profile. From 20 to 50 minutes were required to determine a full density profile.

4. EXPERIMENTAL RESULTS

4.1 Small Column Test Results

Forty-six small column tests in measuring cylinders were conducted to investigate the effect of factors such as initial solids concentration, suspension medium, and initial slurry height on sedimentation and self-weight consolidation. Solids concentration is defined as the ratio of the mass of solids to the total mass of solids and water. Particle segregation was also investigated. The slurries had initial solids concentrations ranging from 2.3% to 60% by mass. Tests ranged in duration from 1 day to about 7 months.

4.1.1 Effect of Initial Solids Concentration

Slurries with different initial solids concentrations behaved differently (Figure 1). If the initial solids concentration (So) was very low, less than 5% by mass, Stokesian free settling was observed at the beginning of the test. As the test continued, the lower part of the slurry column became denser and a slurry-water interface formed. The interface settled very rapidly until the slurry reached a state in which particles were in a close contact and self-weight consolidation became dominant.

For intermediate solids concentrations, the settling rate of the slurry-water interface increased with time after a short period of floc formation. This applies to most of the curves in Figure 1 with initial solids concentrations between 15% and 40%. At very high solids concentrations, the slurry might directly enter the consolidation stage without first undergoing sedimentation. This applies to slurries with initial solids concentrations greater than 50%. The overall rate of settling was then very slow and initial settling was difficult to observe.

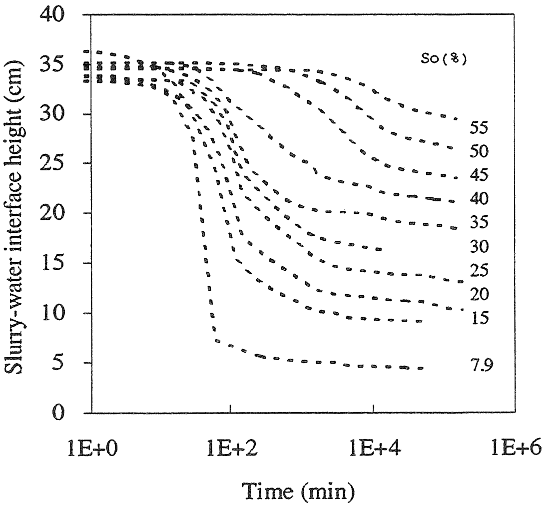

Figure 1. Slurry-water interface height versus time from small column settling tests

Also, as the solids concentration increases, the amount of solids present and hence the height of material also increases. A higher material height gives higher self-weight stress. Hence a slurry with a higher initial solids concentration ultimately results in a lower average specific volume but takes longer to reach the equilibrium state.

4.1.2 Effects of Other Factors

Suspension media greatly affected the sedimentation of slurries with initial solids concentrations of less than about 45%. In the early stages of settling, slurries with tailings water as the suspension medium settled faster than those using distilled water. Adding a flocculant such as Alfloc 625 also increases the settling velocity. However, the suspension medium and dosage of flocculant had no effect on the final average specific volume of the sediments. For slurries with an initial solids concentration of 60%, the suspension medium had no effect on the rate of settling.

The test results show that the initial slurry height has no effect on the rate of sedimentation and that different initial slurry heights give different average specific volumes at the end of consolidation. The greater the initial slurry height, the lower the final average specific volume. Test results also show that slurries with lower initial slurry heights reach the equilibrium state faster, because of their shorter drainage path lengths.

4.1.3 Particle Segregation

The test results for slurries with initial solids concentrations less than 30% show that a high proportion of the coarse silt particles settled to the lower part of the slurry, leaving the upper part with a high percentage of fine material. However, the particle size distributions from different column heights were very similar in slurries with initial solids concentrations equal to or greater than 40%. This shows that particle segregation can be ignored for those slurries.

4.2 Square Column Test Results

Slurry samples were prepared with initial solids concentrations of 25%, 30%, 35%, and 40% for Tests 1 to 4, respectively. The initial slurry heights for all tests were about 1.25 m, and the slurry densities were initially uniform throughout the column. The duration of the tests ranged between about 12 days and more than 2 months. The figures shown below demonstrate both experimental results and those predicted by numerical modelling. Only the experimental results are discussed in this section and the predicted results are discussed in Section 6.

4.2.1 Density Profiles

The density profiles of Tests 1 to 4 show a range of behaviour. The extremes are typified by Tests 1 and 4, while Tests 2 and 3 represent intermediate behaviour. In Test 1 (Figure 2), at an early stage of the test, a thin layer of sediment formed at the base of the column. Its density increased rapidly from 1.12 Mg/m^3 to 1.30 Mg/m^3, but changed little in the later stages of the test. In contrast, the density of the upper part of the slurry column decreased almost uniformly. This reduction was caused by particle segregation, where the coarse particles settled rapidly leaving the fines still in suspension.

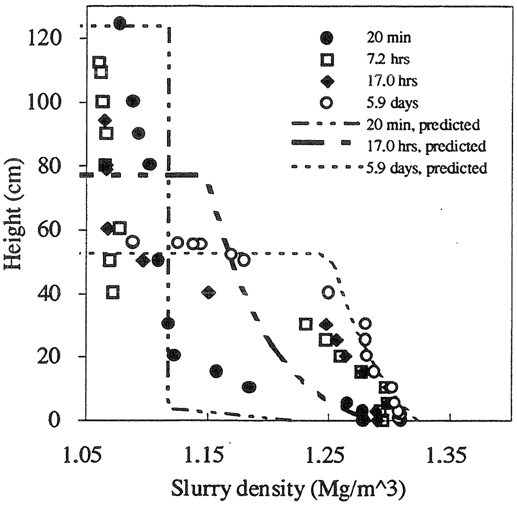

Figure 2. Experimental and predicted slurry density profiles from Test 1

As the settling continued, particle segregation effects weakened and a zone of almost constant density was formed in the upper part of the settling column. This zone progressively decreased in height until it vanished. At the same time, the thin layer of sediments at the base of the column grew in thickness. During the early stages, the density of the upper zone was quite different from that of the lower zone, and there existed a very narrow transition band to distinguish these two zones. Later this band became vague, and eventually vanished. The density of the slurry then decreased gradually from the base to the top.

In Test 4 (Figure 3), as the slurry-water interface settled, the density increased throughout the slurry, with a peak at the base of the column. However, the increase in density near the base was slow and the zone of increasing concentration moved upwards slowly. At the uppermost part of the slurry column, there was another zone of high concentration in which the density increased at a slower rate than at the base. However, in the early stages of the test, this zone spread downwards at a greater rate than the base layer propagated upwards. When these two high concentration zones met, the slurry density decreased gradually from the base to the top. This settling pattern lacks a constant-density zone.

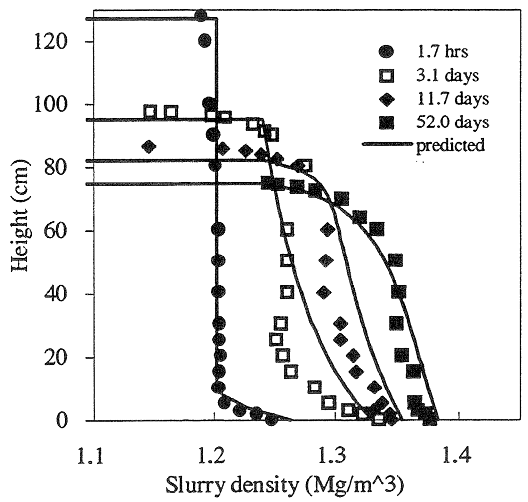

Figure 3. Experimental and predicted slurry density profiles from Test 4

4.2.2 Excess Pore Water Pressure Profiles

The isochrones of excess pore pressures for Tests 1 to 4 had similar trends with time, as shown in Figures 4 and 5. The excess pore pressures built up immediately as the slurry was poured into the settling column. The excess pressures were initially quasi-hydrostatic, varying from zero at the slurry-water interface to a value at the base equal to the total pressure. This implies that effective stresses had not developed. This situation existed for only a short period of time. As time went on, the excess pore water pressures decreased monotonically over the whole depth of the slurry column, and, if the test had been continued long enough, would ultimately have vanished.

4.2.3 Effective Stress Profiles

Effective stresses were calculated based on the slurry density and pore water pressure profiles. In the very early stages of all the tests, no effective stresses were detected, suggesting that sedimentation process had dominated during the early stages of the tests. This period was short and depended on the initial solids concentration. The lower the initial solids concentration, the shorter the period.

In Tests 1 to 3, effective stresses developed from the base of the slurry column and spread upwards. This shows that self-weight consolidation started at the base and proceeded upwards due to the sediment first forming at the base. However, this was not the case for Test 4. Early in the test (between 2 and 5 days), the effective stresses did not tend to decrease monotonically with the decreasing height and at some positions local values even exceeded those at greater depth. This supports neither the previous observation (that the consolidation proceeds upwards monotonically from the base) nor the observation by Scott et al. (1986) that consolidation spreads downward from the surface. Instead, it confirms Kearsey and Gill's claim (1963) that some local high effective stresses may develop. Clearly, further study is required to clarify this.

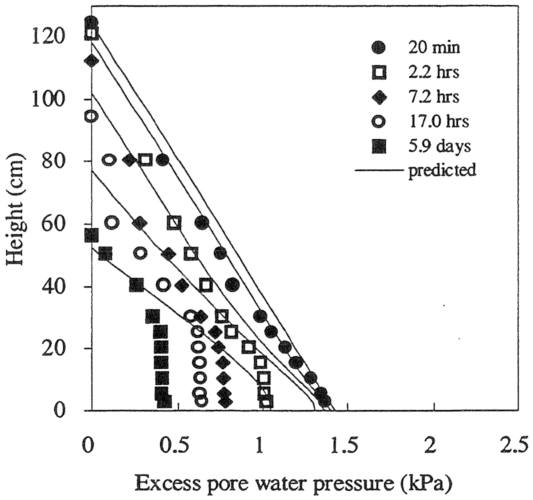

Figure 4. Experimental and predicted excess pore water pressure profiles from Test 1

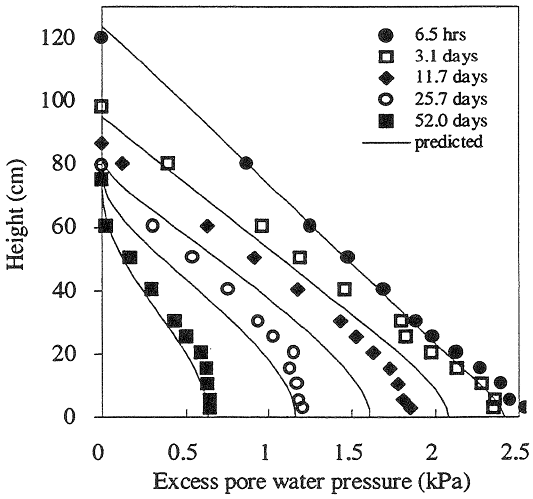

Figure 5. Experimental and predicted excess pore water pressure profiles from Test 4

The slurry density profiles from Test 1 at 7.2 hrs and 17.0 hrs (Figure 2) show a narrow transition band at about 400 mm from the base of the slurry column. This divides the slurry column into two zones. The upper zone might be considered a sedimentation zone and the lower zone a consolidation zone. However, the corresponding effective stress profiles show that effective stresses existed in the slurry of the upper zone. Thus there is no sharp boundary between the sedimentation and consolidation processes. This is consistent with the results obtained by Imai (1981) and Been and Sills (1981).

5. NUMERICAL MODELLING

5.1 Mathematical Model

For the development of a mathematical model, the coal mine tailings slurry is taken as a two-phase incompressible medium. A Eulerian coordinate system is used, with the space co-ordinate datum at the bottom of the column and its positive direction taken against gravity. The governing equation for the one-dimensional flocculated sedimentation and consolidation of a slurry column, due to mass conservation, momentum conservation, and Darcy's law, is given by (Li, 1994)

$$\frac{\partial \alpha}{\partial t} = \frac{\partial}{\partial \xi}\left[c_v(\alpha)\frac{\partial \alpha}{\partial \xi}\right] + \frac{dK(\alpha)}{d\alpha}\frac{\partial \alpha}{\partial \xi} \qquad (1)$$

where

$$c_v(\alpha) = \frac{k\alpha}{\rho_l g}\frac{\partial \sigma'}{\partial \alpha} \qquad (1a)$$

$$K(\alpha) = \frac{(\rho_s - \rho_l)}{\rho_l}k\alpha^2 \qquad (1b)$$

The initial and boundary conditions are

$$\alpha = \alpha_0(\xi) \qquad \text{at } t=0 \qquad (1c)$$

$$\frac{\partial \alpha}{\partial \xi} = -\frac{K(\alpha)}{c_v(\alpha)} = -\frac{\rho_s - \rho_l}{\partial \sigma' / \partial \alpha}\alpha g \qquad \text{at } \xi=0 \qquad (1d)$$

$$\left.\frac{\partial \alpha}{\partial t}\right|_z = -\alpha\frac{\partial}{\partial \xi}(v_s) \qquad \text{at } \xi=H(t),\ \alpha < \alpha_m \qquad (1e)$$

$$\alpha = \alpha_m \qquad \text{at } \xi=H(t) \text{ for pure consolidation} \qquad (1f)$$

$$\frac{dH}{dt} = -v_s = -\frac{1}{\alpha}\left(c_v\frac{\partial \alpha}{\partial \xi} + K\right) \qquad \text{at } \xi=H(t) \qquad (1g)$$

In this model, there is no sharp discontinuity between sedimentation and consolidation. Consolidation may initiate before sedimentation ceases.

5.2 Numerical Solution

A central difference scheme was used to approximate the governing equation. The resulting implicit finite difference equations are tri-diagonal matrix equations and can be conveniently solved using the Thomas algorithm (von Rosenberg, 1969). If $\frac{dK}{d\alpha} = 0$, the numerical method reduces to the Crank-Nicolson method (von Rosenberg, 1969) for solving the one-dimensional diffusion equation. Also, when $c_v = 0$, that is, there is no consolidation, then it becomes an implicit method used to solve the one-dimensional convection equation. If consolidation is dominant, there is no restriction on the size of the time increment and large grid spacings can be used. However, small grid spacings should be used if sedimentation is dominant.

Before applying the numerical algorithm, it is necessary to determine the slurry height H(t) and the volumetric solids concentration just below the water-slurry interface. Since H(t) varies with time, this is a moving boundary problem. To handle the moving boundary, a moving grid system developed by Crank and Gupta (1972) was adopted. This involves a grid and upper boundary conditions which change with time. The values of the variable, α, on a new grid are obtained using the spline-under-tension interpolation technique (Cline, 1974).

5.3 Parameter Determination

A value of α_m, a permeability relationship, and a compressibility relationship are required to solve the governing equation (1).

5.3.1 Determination of α_m

The volumetric solids concentration α_m is also the final solids concentration just below the slurry-water interface of a slurry with an initial solids concentration less than α_m. At the slurry-water interface, there is no effective stress. Hence, the value of $1/\alpha_m$ can be read from a plot of final specific volume ($1/\alpha$) versus effective stress at the point where the effective stress is zero. The value of $1/\alpha_m$ for the coal mine tailings tested was determined to be about 3.0. Thus α_m is about 1/3.

A few direct measurements of the slurry void ratio near the slurry-water interface confirmed this value of α_m for the coal mine tailings.

5.3.2 Compressibility Relationship

If it is assumed that there is a unique relationship between the effective stress and the volumetric solids concentration, the compressibility relationship for the coal mine tailings can be established directly from the solids concentrations and effective stresses obtained from the square column settlings tests. Such a relationship is given by

$$\sigma' = 0.646 \times 10^3 \alpha^{9.11} \qquad (2)$$

5.3.3 Permeability Relationship

If it is assumed that there is a unique relationship between the coefficient of permeability and the volumetric solids concentration, the permeability relationship for the coal mine tailings can be obtained using the measured values of solids concentrations and excess pore water pressures from the square column settlings tests, and Darcy's law. Whence

$$k(\alpha) = 1.547 \times 10^{-3} \exp(-26.667\alpha - 0.3\alpha_0) \quad (3)$$

6. NUMERICAL RESULTS

6.1 Slurry Density

The experimental and predicted slurry densities for Test 1 and 4 are compared on Figures 2 and 3, respectively. Figure 3 shows that, in general, the experimental and predicted slurry densities for Test 4 agree reasonably well.

However, Figure 2 shows that, except at very late stages when consolidation is dominant, the predicted slurry densities poorly match the experimental values. Slurry densities were overestimated in the upper part of the column and underestimated in the lower part. This is attributable to particle segregation which is not allowed for by the mathematical model for sedimentation and consolidation given by (1). In particle segregation, larger and heavier particles settle faster, leaving the upper part of the slurry column less dense and the lower part of the slurry more dense. The slurry for Test 1 with an initial solids concentration of 25% by mass clearly underwent particle segregation during sedimentation.

6.2 Excess Pore Water Pressure

The experimental and predicted excess pore water pressures for Tests 1 and 4, are compared on Figures 4 and 5, respectively. In Figure 4, in the early stages of the test and for the upper part of the slurry, the predicted excess pore pressures agree reasonably well with the experimental values. However, for the lower part of the slurry column, the discrepancies between the predicted and measured excess pore pressures increase with increasing time. This is attributable to the rapid dissipation of the excess pore water pressures in the lower part of the slurry, which results from particle segregation. Coarse particles concentrated in the lower part of the slurry, increasing the permeability.

Figure 5 shows that the predicted excess pore water pressures generally agree well with the experimental values for Test 4.

7. CONCLUSIONS

The sedimentation dominates the early stages of settling in slurries and is controlled by such factors as the initial solids concentration and the suspension medium, but is not sensitive to the initial slurry height. Particle segregation occurs in slurries with low initial solids concentrations. Self-weight consolidation starts at the base of the slurry column and proceeds upwards for slurries with low initial solids concentrations. However, this process is more complicated for slurries with high initial solids concentrations. The way that effective stresses develop in dense slurries is unclear and should be investigated further. The mathematical model and the numerical solution are capable of predicting the behaviour of tailings slurries with high initial solids concentrations during combined (flocculated) sedimentation and self-weight consolidation. However, particle segregation must be taken into account for slurries with low initial solids concentrations if sedimentation is to be modelled accurately.

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