

Permeability Anisotropy of Fibrous Peat in a Permeameter

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SYNOPSIS Peat consisted of undecomposed vegetable fibers has a large anisotropical characteristic. The permeability anisotropy of undecomposed fibrous peat is discussed in this study. As the assumption that both hydraulic gradient and discharge velocity are vectors, a relation governing the directional variation of coefficient of permeability of anisotropic fibrous peat is represented as a function of maximum and minimum coefficients of permeability and direction of resultant hydraulic gradient. It is shown that Mohr's circle for coefficient of permeability can be drawn using the maximum and minimum coefficients. From the circle, k_h and k_v can be referred to as the principal coefficient of permeability and the coefficient of permeability along any direction can be obtained. The maximum, minimum and other permeability of fibrous peats are measured by a laboratory constant head permeameter. It is shown that the experimental results show good agreement with the theoretical one and the anisotropy factor k_h/k_v is 2.7 and 3.0 for the peats used in this study.

1 INTRODUCTION

Permeability anisotropy of sedimentary soils is caused by three facts: (1) Macro stratification; (2) micro stratification; (3) orientation of flat particles [Witt et al, 1983]. In cases of macro stratification the overall factor of anisotropy can easily be assessed, provided information on the thickness and permeabilities of the soil layers are available. It is very difficult, however, to quantify the anisotropy of a soil layer which shows micro stratification. In such soils, in site pumping tests are necessary to obtain information on the anisotropy [Witt et al, 1983]. The third reason for permeability anisotropy is the flatness of soil particles.

In the case of fibrous peat, the cause of permeability anisotropy is the shape of undecomposed vegetable fibers plus the orientation of fibers. It is very difficult, however, to obtain the information on the shape and the orientation of the undecomposed fibers. The physical characterization of the disorder system of void in fibrous peat is a problem of considerable difficulty. However, the permeability characteristic of peat is of vital importance in many problems in construction at peatland.

Some experimental considerations of permeability anisotropy of undecomposed fibrous peat have been performed in order to obtain the basic data [Maeda, 1955].

In this study, according to the assumption that both hydraulic gradient and discharge velocity are vectors, a relation governing the directional variation of coefficient of permeability of fibrous peat is represented. It is shown that Mohr's circle for the coefficient of permeability can be using an anisotropic peat and two directions are defined as the principal direction of flow. From the circle, k_h and k_v can be referred to as the principal coefficient of permeability and the coefficient of permeability along any direction can be obtained. The maximum, minimum and other permeability of an undecomposed fibrous peats are measured by a laboratory constant head permeameter. It is shown that the measured coefficient of permeability along the direction of 45° with the horizontal show good

agreement with the theoretical and the anisotropy factor k_h/k_v is 2.7 and 3.0 for the undecomposed fibrous peats used in this study.

2 DIRECTIONAL VARIATION OF COEFFICIENT OF PERMEABILITY

In an anisotropic medium the coefficient of permeability varies with direction. The direction of the resultant macroscopic hydraulic gradient and the direction of the resultant discharge velocity are collinear for flow through isotropic media, but for flow through anisotropic soil, they are no longer collinear. The directions of maximum and minimum permeabilities are orthogonal, and this assumption will be made for the purpose of analysis.

Consider the two dimensional flow through an anisotropic medium with the directions of the resultant hydraulic gradient and of the resultant discharge velocity as shown in Figure 1. The coordinate directions x and z are selected in the directions of k_{max} and k_{min} . In the case of Figure 1, the generalized Darcy law is represented as follows;

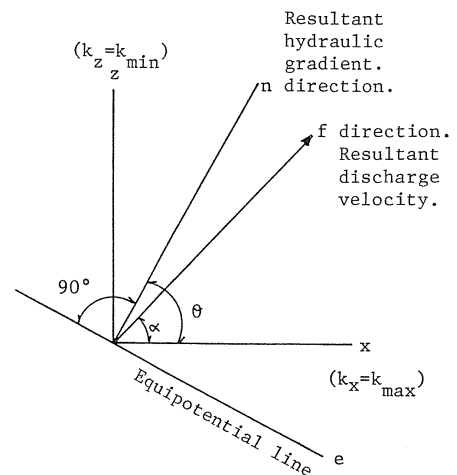


Figure 1 Two dimensional flow through anisotropic media

$$\begin{aligned} v_x &= -k_x \frac{\partial h}{\partial x}, & v_z &= -k_z \frac{\partial h}{\partial z}, \\ v_f &= -k_\alpha \frac{\partial h}{\partial f}, & v_n &= -k_\theta \frac{\partial h}{\partial n}, \end{aligned} \quad (1)$$

where, v_x , v_z , v_f and v_n are components of the discharge velocity along the directions x , z , f and n definitely oriented in the peat, h is the total head existing at a point, and k_x , k_z , k_α and k_θ are the coefficient of permeability along x , z , f and n directions respectively. Eq.(1) imply that the velocity of flow along each of the two directions x and z depends only on the hydraulic gradient in that direction. These two perpendicular directions will be called the principal directions of flow. k_x and k_z will be referred to as the principal coefficients of permeability k_1 and k_2 respectively. x and z axes are the principal directions of flow.

As both hydraulic gradient and velocity are vectors, the gradient in the f direction is

$$\begin{aligned} \frac{\partial h}{\partial f} &= \frac{\partial h}{\partial x} \cos \alpha + \frac{\partial h}{\partial z} \cos(90 - \alpha) \\ &= \frac{\partial h}{\partial x} \cos \alpha + \frac{\partial h}{\partial z} \sin \alpha \end{aligned} \quad (2)$$

Substituting eq.(1) for $\partial h/\partial x$ and $\partial h/\partial z$ into eq.(2), and as $v_x = v_f \cos \alpha$, $v_z = v_f \sin \alpha$,

$$\begin{aligned} -\frac{1}{k_\alpha} v_f &= -\frac{1}{k_x} v_f \cos^2 \alpha - \frac{1}{k_z} v_f \sin^2 \alpha \\ \text{or} \quad \frac{1}{k_\alpha} &= \frac{\cos^2 \alpha}{k_x} + \frac{\sin^2 \alpha}{k_z} \end{aligned} \quad (3)$$

Thus the generalization of Darcy's law requires that the directional variation of the coefficient of permeability be governed by eq.(3) [Leonards, 1962]. An example calculated from eq.(3) is shown as curve in Figure 2, with $k_x = 2 \times 10^{-3}$ and $k_z = 1 \times 10^{-3}$ cm/s, that is, $k_x/k_z = 2.0$.

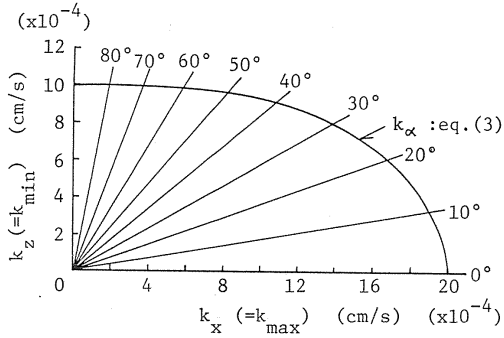


Figure 2 Directional variation of coefficient of permeability k_α (for $k_x/k_z = 2.0$)

As hydraulic gradient and discharge velocity are assumed to be vector, their resolution in the direction of the resultant hydraulic gradient should lead to the same result as resolution in the direction of the resultant velocity. Therefore,

$$v_n = v_x \cos \theta + v_z \sin \theta \quad (4)$$

On the other hand,

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial n} \cos \theta, \quad \frac{\partial h}{\partial z} = \frac{\partial h}{\partial n} \sin \theta \quad (5)$$

Substituting eq.(1) for v_n , v_x , v_z and eq.(6) for $\partial h/\partial x$, $\partial h/\partial z$ into eq.(4),

$$-k_\theta \frac{\partial h}{\partial n} = -k_x \frac{\partial h}{\partial n} \cos^2 \theta - k_z \frac{\partial h}{\partial n} \sin^2 \theta \quad (6)$$

or

$$k_\theta = k_x \cos^2 \theta + k_z \sin^2 \theta \quad (7)$$

The relationship between k_θ and k_x , k_z is shown as curve in Figure 3, with $k_x = 2 \times 10^{-3}$ and $k_z = 1 \times 10^{-3}$, that is, $k_x/k_z = 2.0$.

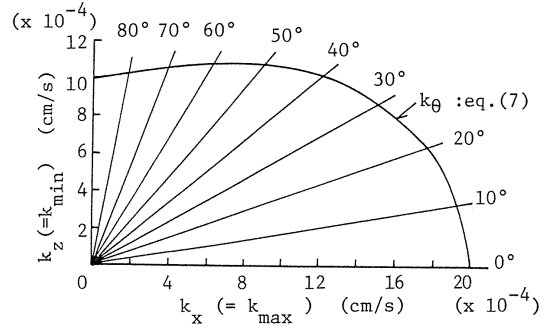


Figure 3 Directional variation of coefficient of permeability k_θ (for $k_x/k_z = 2.0$)

Eq.(3) will then give the value of k_α in terms of $k_x (=k_{max})$ for any direction α of the resultant discharge velocity with respect to $k_x (=k_{max})$. Eq.(7) will then give the value of k_θ in terms of $k_x (=k_{max})$ and $k_z (=k_{min})$ for the direction θ of resultant hydraulic gradient with respect to $k_x (=k_{max})$ [Leonards, 1962]. Scheidegger [1957] showed that the discrepancy between eqs.(3) and (7) can never exceed 25 per cent. Measurement results of the directional permeability of rock samples is shown that the discrepancy between eqs.(3) and (7) is of little practical significance [Johnson et al, 1948].

3 MOHR'S CIRCLE FOR PERMEABILITY

As shown in eq.(1), the coefficient of permeability k_θ normal to the equipotential line was defined by $v_n = -k_\theta (\partial h/\partial n)$. The value of k_θ represents the component of the discharge velocity normal to the equipotential line when the resultant hydraulic gradient is unity. That is, when $\partial h/\partial n = 1$, k_θ is equal to v_n . k_x and k_z represent the maximum and minimum coefficients of permeability respectively and k_x and k_z will be referred to as the principal coefficients of permeability k_1 and k_2 respectively. From the above considerations, eq.(7) can be written as follow;

$$\begin{aligned} k_\theta &= k_1 \cos^2 \theta + k_2 \sin^2 \theta \\ &= k_1 \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) + k_2 \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right) \\ &= \frac{1}{2} (k_1 + k_2) + \frac{1}{2} (k_1 - k_2) \cos 2\theta \end{aligned} \quad (8)$$

The component v_e of the velocity v along the equipotential line is

$$v_e = v_x \sin \theta - v_z \cos \theta \quad (9)$$

Substituting eq.(1) for v_x , v_z and eq.(5) for $\partial h/\partial x$, $\partial h/\partial z$ into eq.(9),

$$\begin{aligned} v_e &= -k_x \frac{\partial h}{\partial x} \sin \theta + k_z \frac{\partial h}{\partial z} \cos \theta \\ &= - \left[(k_x - k_z) \sin \theta \cos \theta \right] \frac{\partial h}{\partial n} \end{aligned} \quad (10)$$

Here, the coefficient of permeability K along the equipotential line may be introduced by following relation;

$$v_e = -K \frac{\partial h}{\partial n} \quad (11)$$

K represents the component of the discharge velocity along the equipotential line when the maximum hydraulic gradient is unity. With eq.(11), eq.(10) is written as follow;

$$K = \frac{1}{2} (k_x - k_z) \sin 2\theta \quad (12)$$

Substituting $k_x = k_1$ and $k_z = k_2$ into eq.(12),

$$K = \frac{1}{2} (k_1 - k_2) \sin 2\theta \quad (13)$$

Eqs.(8), (12) and (13) show that the variation of k_θ and K with respect to θ is the same as the variation of normal and shear stresses at a point as given in mechanics of materials. Therefore, Mohr's circle can be drawn for k_θ and K to show their variation with respect to θ [Yang,1953].

Mohr's circle for coefficient of permeability is shown in Figure 4. As shown in Figure 4, OA is k_1

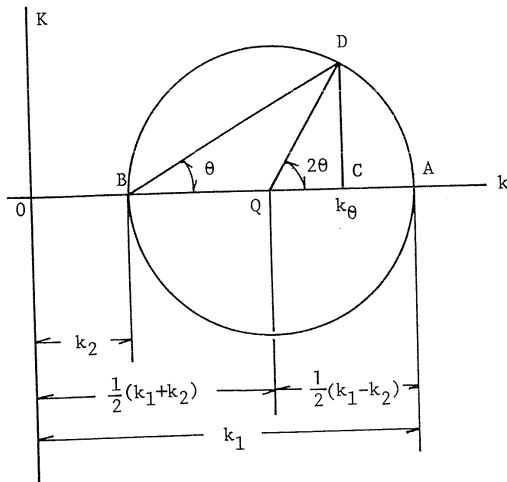


Figure 4 Mohr's circle for coefficient of permeability

and OB is k_2 . The diameter of Mohr's circle is AB ($= k_1 - k_2$). For any direction n of the resultant hydraulic gradient making an angle θ with the x axis, draw the radius QD making an angle 2θ with the abscissa(k axis). Then the coordinates of D represent k_θ and K. That is, $OC = k_\theta$, $DC = K$.

4 EXPERIMENTS

4.1 Index Properties of Peats

The laboratory permeability tests have been performed with the undecomposed fibrous peats. The peat samples were obtained by the thin-wall piston sampler from the peatland in Hokkaido, Japan. The peats was sedge peats and the major botanical constituent is one or more species of sedge. The constituent is nonwoody. According the descriptions of peat categories by Radforth, the peat used in this study falls within category 13. That is, the peat consists of coarse fibers criss-crossing fine-fibrous peat [Radforth,1969].

Table 1 Index properties of peat samples

	sample A	sample B
specific gravity	1.57	1.60
water content (%)	720	760
wet density (g/cm^3)	1.03	1.07
dry density (g/cm^3)	0.126	0.124
void ratio	11.5	11.9
ignition loss (%)	91	93

Two samples of fibrous peat were used in this study. The index properties of the peat samples were shown in Table 1.

The samples for permeability tests were obtained from the three directions as shown in Figure 5.

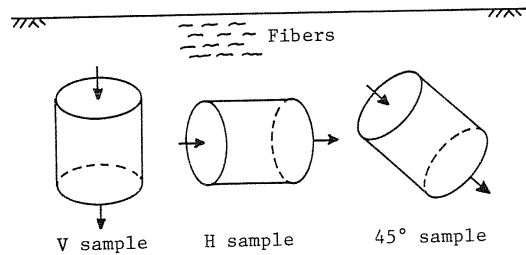


Figure 5 Schematically diagram of peat samples for permeability test

The arrows in Figure 5 indicate the direction of flow of water in the permeability tests. The coefficients of permeability obtained from the V and H samples show k_v and k_h , and the minimum and maximum coefficients, respectively. k_h and k_v represent the principal coefficients of permeability k_1 and k_2 respectively.

4.2 Permeameter

The constant head permeameter was used in this study. The schematic diagram of the cross section of the permeameter was shown in Figure 6.

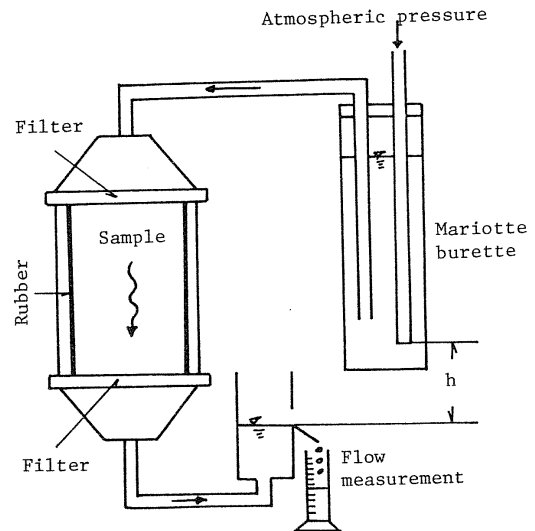


Figure 6 Test setup for constant-head permeability apparatus

The difference between the head and tail water levels in the permeameter was given by h as shown in Figure 6. The constant head was kept due to the Mariotte burette. The size of specimen was 75 mm in diameter and 70 mm in length. Therefore, the hydraulic gradient was given as $h/70$, where the unit of h is cm.

4.3 Procedures of Experiments

The sample of fibrous peat was set in the permeameter. All the tests were conducted on completely saturated specimens. The hydraulic gradient was varied from 0.15 to 2.0. The quantities of flow for different hydraulic gradient were measured by

the measuring cylinder for all specimens and the average discharge velocity was calculated from the corresponding quantities of flow.

5 RESULTS AND DISCUSSIONS

The log-log plots of the hydraulic gradient i versus the average discharge velocity v were shown in Figures 7 and 8. It is seen that the slopes of these log-log plots are unity. The results show that the average discharge velocity is proportional to the hydraulic gradient and the flow of water through the fibrous peat is laminar flow.

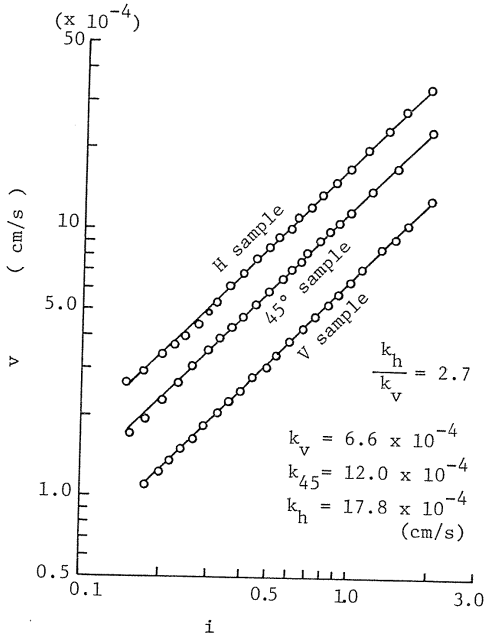


Figure 7 A typical result of log-log plot of hydraulic gradient i versus average discharge velocity v (Sample A)

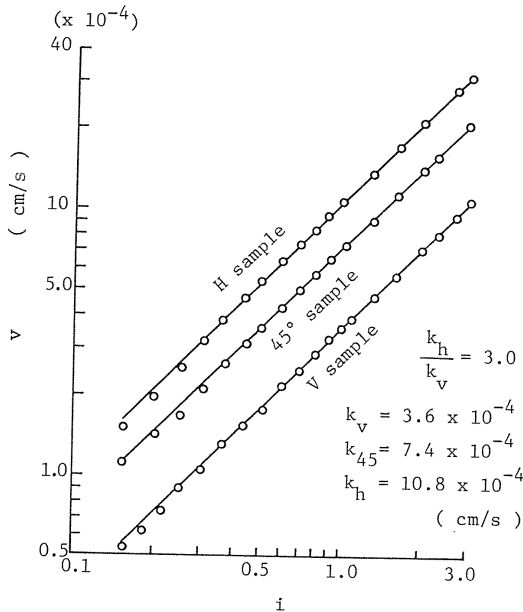


Figure 8 A typical result of log-log plot of hydraulic gradient i versus average discharge velocity v (Sample B)

The values of k_v , k_h and k_{45} are shown in Figures 7 and 8, where, k_{45} shows the coefficient of permeability of the 45° sample in Figure 5. The values of the anisotropy factor k_h/k_v are also shown in Figures 7 and 8, and are about 2.7 and 3.0 for the samples A and B respectively.

For the sample A, the values of k_h and k_v were 17.8×10^{-4} and 6.6×10^{-4} cm/s respectively. The variation of k_θ calculated by eq.(7) using k_v and k_h was shown in Figure 9 as curve. The measured values of k_{45} were also plotted in Figure 9 and nearly agree with the calculated one.

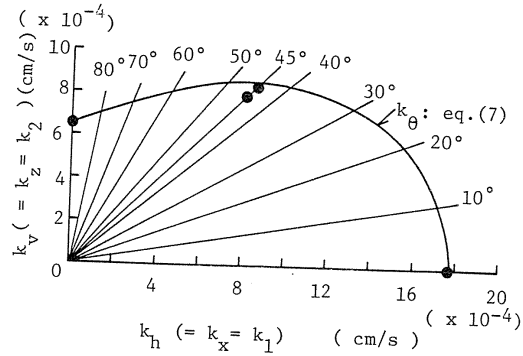


Figure 9 Directional variation of k_θ and experimental results for peat sample A

For the sample B, the values of k_h and k_v were 10.8×10^{-4} and 3.6×10^{-4} cm/s respectively. The variation of k_θ calculated by eq.(7) was shown in Figure 10 as curve. The plotted points in Figure 10 show the values of k_{45} measured in the permeability test. It is seen that the measured results show good agreement with the calculated one.

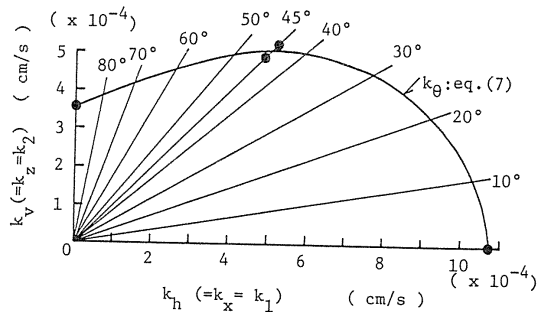


Figure 10 Directional variation of k_θ and experimental results for peat sample B

Mohr's circles for the coefficient of permeability of the samples A and B were shown in Figures 11 and 12. The Mohr's circles in Figures 11 and 12 were drawn using $k_h (= k_1 = k_x)$ and $k_v (= k_2 = k_z)$ according to eq.(8) and eq.(13).

The measured results of the coefficient of permeability were plotted in Figures 11 and 12. It is seen that the measured results show good agreement with the theoretical and the coefficient of permeability along any direction can be obtained from the Mohr's circle.

6 CONCLUSIONS

The permeability anisotropy of the undecomposed fibrous peats was discussed. It was shown that the relation governing the directional variation of the coefficient of permeability of fibrous peats

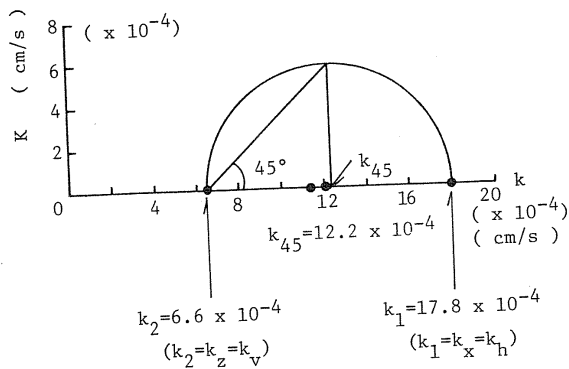


Figure 11 Mohr's circle for coefficient of permeability and experimental results for peat sample A

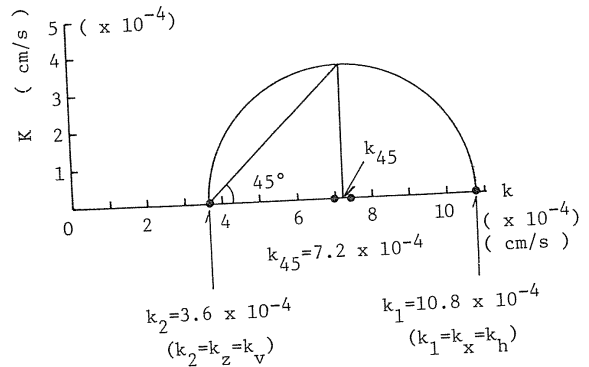


Figure 12 Mohr's circle for coefficient of permeability and experimental results for peat sample B

is represented by a function of the two principal coefficients of permeability and the Mohr's circle for the coefficient of permeability can be drawn using the two principal coefficients of permeability. The coefficient of permeability along any direction can be obtained from the Mohr's circle for the coefficient of permeability. It was shown from the results of the permeability tests that the experimental results showed good agreement with the theoretically calculated results. The anisotropy factors of permeability of the peats used in this study were about 2.7 and 3.0. For the permeability anisotropy of undecomposed fibrous peat, the effect associated with the shape of undecomposed fibers and these orientation will be investigated in the next stage.

6 REFERENCES

- Johnson, W.E. and Hughes, R.V. (1948). Directional Permeability Measurements and their Significance. *Producers Monthly*, Vol. 13, No. 1, P. 17.
- Leonards, G.A. (1962). *Foundation Engineering*. McGraw Hill Book, pp. 107-139.
- Maeda, K. (1955). Permeability Test in Peatland of Kushiro. *Thuchi to Kiso*, Vol. 3, No. 10.
- Radforth, N.W. (1969). Classification of Muskeg, *Muskeg Engineering Handbook*. The Muskeg Subcommittee of NRC, Univ. of Toronto Press, pp. 31-52.
- Scheidegger, A.E. (1957). *The Physics of Flow through Porous Media*. The Macmillan Company, New York.
- Witt, K.J. and Brauns, J. (1983). Permeability Anisotropy due to Particle Shape. *ASCE, Jour. of Geot. Engr.*, Vol. 109, No. 9, pp. 1181-1187.
- Yang, S.T. (1953). On the Permeability of Homogenous Anisotropic Soils. *Proc. of 2nd ICSMFE*, Vol. 2, pp. 317-320.