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Young's Moduli of a Soft Rock in Compression Bending and Tension

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SUMMARY: The range of values of unconfined Young's modulus of a synthetic soft rock for a variety of saturated water contents was measured in the laboratory by a series of uniaxial compressive, bending and tensile tests. The results obtained showed that the tensile moduli were about an order of magnitude greater than the compressive moduli, with the bending moduli falling in between.

1. INTRODUCTION

In order to analyse the results of various forms of laboratory model tests conducted on a synthetic soft rock, it was necessary to establish the strength and deformation properties of the soft rock with some precision. Therefore, a range of different tests were carried out to establish these properties and the factors which could influence their individual values.

One of the principal properties requiring evaluation was the Young's modulus of the soft rock so that the initial portions of the model test results could be analysed by elastic methods. However, since the response of soft rock can be greatly influenced by the development of tensile stresses (Johnston and Choi, 1985; Haberfield, 1987), it seemed appropriate that in addition to the more conventionally encountered compressive modulus, the Young's modulus for tensile stress should be investigated also.

As the overall test programme required the determination of the plane strain fracture toughness of the synthetic rock, and since notched beams were used for this purpose, it was convenient to establish the tensile modulus from the notch opening characteristics of these tests. Furthermore, since beams were being used, it was a simple matter to measure also the Young's modulus in bending for both notched and un-notched beams.

This paper, therefore, examines the variability of the unconfined Young's modulus of a synthetic soft rock under conditions of compression, bending and tension.

2. TEST MATERIAL

The various model studies referred to above were undertaken to establish the mechanisms of behaviour for soft rocks so that appropriate means of numerical modelling could be developed for the purposes of design. Originally, natural rock specimens were used, but it was soon realised that these materials possessed properties and characteristics which were highly variable and unpredictable leading to significant experimental scatter with the model studies. In order to overcome these problems, a synthetic soft rock was manufactured. This material, known as Johnstone, was found to be homogenous and isotropic, and could be reliably and simply reproduced with a range of properties. These properties are similar to naturally occurring soft rocks, and can be controlled and accurately determined. In addition, the Johnstone can be easily cast or machined in a wide range of shapes to suit specific applications.

The synthetic rock is formed from a specific mixture of mudstone powder of a prescribed grainsize distribution, a small quantity of cement, water and set accelerator. The mixture is placed in a mould and is compressed under load allowing full dissipation of porewater pressures. Once equilibrium is attained, the load is removed and the mould is stripped. The resulting specimen is then allowed to cure for at least three weeks after which the material properties become constant.

It was found that the stress applied during the above compression process determines the final properties of the Johnstone, with the higher stresses producing lower voids ratios and therefore a product of greater strength. The saturated water content of the Johnstone was found to be an excellent indicator of its material properties. In general, the Johnstone was manufactured at saturated water contents of between about 11% and 21%, which corresponded to uniaxial compressive strengths of between about 8 and 2 MPa. Full details of the manufacturing process and the properties of this synthetic rock may be found in Johnston and Choi (1986).

3. COMPRESSIVE MODULUS

The compressive modulus of the synthetic rock was established by conducting a large number of uniaxial (Choi, 1984) and triaxial (Novello, 1987) compression tests on specimens of 54 mm diameter. Since it was the unconfined modulus which was required in this specific investigation, only the triaxial tests conducted with very low confining pressures (< 50 kPa) were considered. In general, the compressive tests were carried out to determine the drained moduli and the procedures adopted were as recommended by Chiu et al. (1983). In the case of the uniaxial test results obtained by Choi, the secant moduli at half peak deviator stress were measured, whereas, for the low pressure triaxial tests conducted by Novello, the initial tangent moduli were derived. However, since all tests showed effectively linear stress-strain responses until at least half the peak load, there was no apparent difference between the compression moduli derived from the two sources.

The range of variation of compressive Young's modulus for the Johnstone is given in Figure 1. The line drawn through these points represents the line of best fit for these test results.

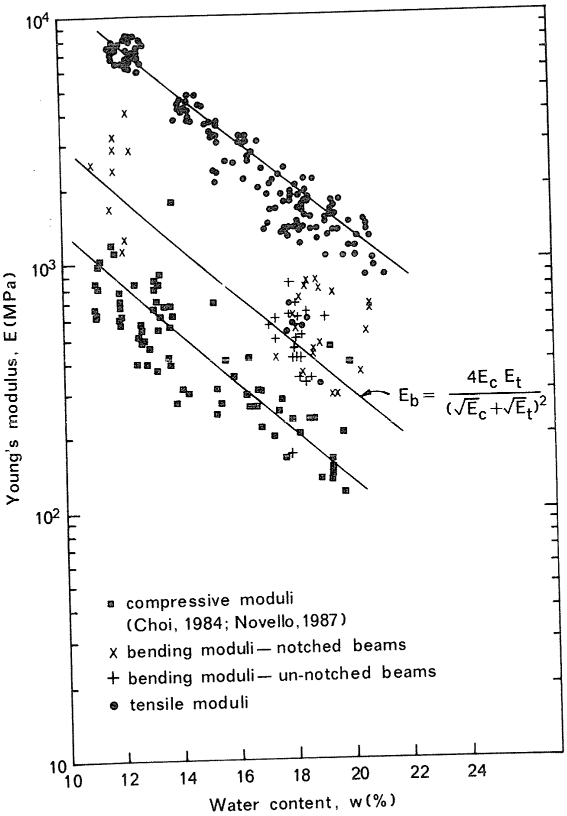

Fig. 1 Measured variation of Young's moduli with saturated water content.

4. TENSILE MODULUS

As part of a major investigation into development and propagation of stress and strain induced cracking in soft rocks (Haberfield, 1987), methods of establishing the fracture toughness of soft rocks were examined. After careful consideration of the various factors which might influence the results obtained, one of the principal techniques finally adopted involved the use of three point loading of single edge cracked beams (SECB) as shown in Figure 2. The beams had overall dimensions of depth W, thickness B and length of a little more than the span S, and were cut and machined from larger manufactured blocks of synthetic soft rock of constant saturated water content. At the mid-point of each beam, a notch of width δ, and length a, was carefully cut into the underside of the beam by means of a specially set-up bandsaw. In order to obtain valid values of plane strain fracture toughness for the Johnstone, the minimum size of specimen tested had the dimensions W=60mm, B=30 mm and S=240 mm. In addition, the notch length was always between 0.4W and 0.6W and the loading ram rate of displacement applying load P was always less than 0.05 mm/min to ensure a fully drained response. A full discussion of the details of these tests may be found in Haberfield (1987).

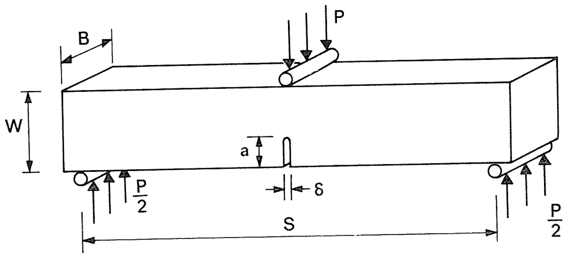

Fig. 2 Test arrangement of three point loading of single edge cracked beams.

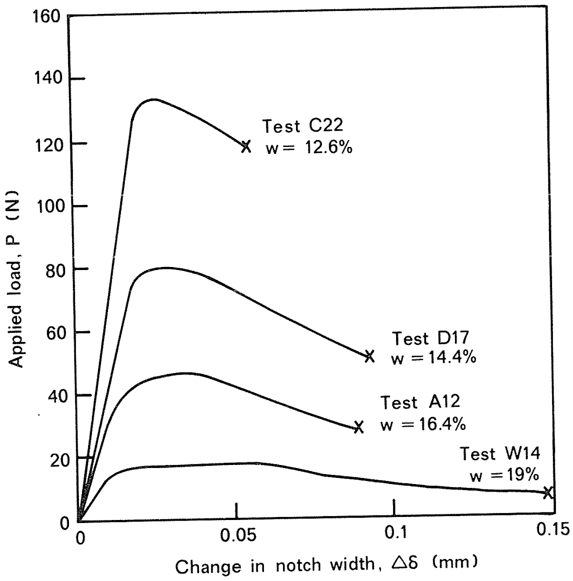

Fig. 3 Examples of plots of applied load against change in notch width for tests on single edge cracked beams.

When analysing the results of a fracture toughness test, the load P is plotted against the change in notch width $\Delta\delta$ at the bottom of the beam. This latter measurement was made possible by means of a clip gauge attached at the opening of the notch. Examples of these plots for a range of different saturated water contents, w, are shown in Figure 3. In addition to providing a means of calculating fracture toughness, each of these curves may also be used to estimate a tensile Young's modulus by measuring the initial slope and applying the following relationship (Tada et al., 1973).

$$E_t = \frac{P}{\Delta\delta} \frac{6Sa}{BW^2} V_1\left(\frac{a}{W}\right) \qquad (1)$$

where $\Delta\delta$ is the change in notch width for an applied load P, and

$$V_1\left(\frac{a}{W}\right) = 0.76 - 2.28\left(\frac{a}{W}\right) + 3.87\left(\frac{a}{W}\right)^2 - 2.04\left(\frac{a}{W}\right)^3 + \frac{0.66}{\left(1 - \frac{a}{W}\right)^2} \qquad (2)$$

The values of initial tensile tangent modulus obtained from the fracture toughness tests are shown in Figure 1 along with the compressive moduli discussed above. Again, a line of best fit has been drawn through the tensile moduli results to show that these results are generally an order of magnitude larger than the moduli obtained from compressive testing.

5. BENDING MODULUS

5.1 Bending Tests with a Notched Beam

As described above, the fracture toughness of the synthetic rock was established by means of three point loading of single edge cracked beams. Furthermore, by considering the change in notch width with applied load, it was possible to calculate the tensile Young's modulus for each tested specimen. In addition to the measurements already described, the vertical displacement $\Delta\ell$, of the beam at the point of application of the mid-span vertical load was also measured during these tests. With these results, it was possible to calculate the modulus in bending for each tested specimen from the relationship (Tada et al., 1973).

$$E_b = \frac{P}{\Delta\ell}\,\frac{S^2}{4BW^2}\left[\frac{S}{W} + 6V_2\left(\frac{a}{W}\right)\right] \qquad (3)$$

where

$$V_2\left(\frac{a}{W}\right) = \left(\frac{\frac{a}{W}}{1-\frac{a}{W}}\right)^2 \left[5.58 - 19.57\left(\frac{a}{W}\right) + 36.82\left(\frac{a}{W}\right)^2 - 34.94\left(\frac{a}{W}\right)^3 + 12.77\left(\frac{a}{W}\right)^4\right] \qquad (4)$$

The results of these bending modulus determinations are shown in Figure 1.

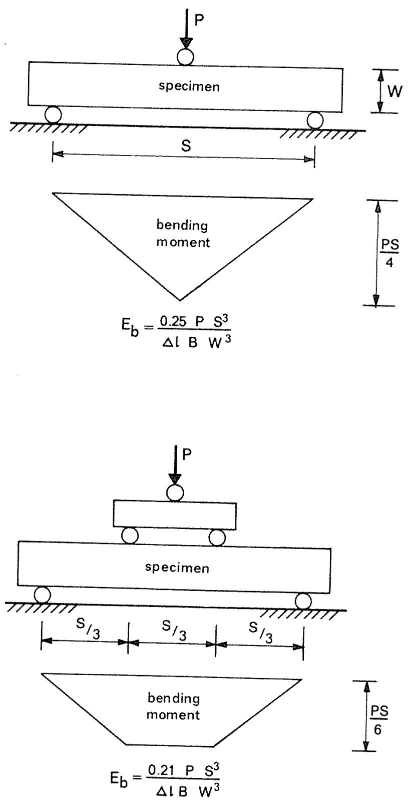

Fig. 4 Three and four point loading tests on beams for measurement of bending modulus.

5.2 Bending Tests Without a Notch

The tensile strength of the synthetic rock is a property that has been measured by a number of different methods (Choi et al., 1987), including the direct method, the Brazilian method, the ring method and various bending methods. The latter bending methods make use of three and four point loading techniques as shown in Figure 4. As these tests on the synthetic soft rock involved measurements of both applied load P and the vertical deflection of the point of application of the load $\Delta\ell$, it was possible to make estimations of the Young's modulus in bending from the expressions shown in Figure 4 for the elastic portions of the loading response.

The results obtained from these tests are presented in Figure 1 along with the results obtained from the other methods of determination. It can be seen that the range of results from the two forms of test which reflect the bending modulus are in reasonable agreement with each other and their range is intermediate to the respective ranges of compressive and tensile moduli.

6. EXAMINATION OF TEST AND INTERPRETATION METHODS

On the basis of the results presented in Figure 1, there appears to be a significant difference between the compressive, bending and tensile Young's moduli for all saturated water contents of the soft rock tested. It was important, therefore, to establish whether this result reflected reality or could these differences have been brought about by the methods of testing and interpretation which were used.

A careful examination of the means and accuracy of displacement measurement for all test techniques, including indentation displacement in the bending tests, revealed no significant anomalies. Also, a detailed consideration of the load-displacement results for each test type showed that the response was linear elastic even down to the very small initial displacements. Therefore, differences in the range of displacement measurements for the test types could not explain the results shown in Figure 1.

It was felt that a more practical method of evaluating the tests and their interpretation would be to repeat them but with a material of known and constant modulus in compression, bending and tension. Should any significant variation in modulus be encountered, then the test methods and their interpretation would have to be questioned. The material chosen for this examination was aluminium. It should be noted that while all other tests were effectively identical to those used with the Johnstone, the uniaxial compression tests had to adopt a different format. These tests were not carried out with the test specimen between loading platens as is normal for geotechnical materials. While measurements of axial contraction between the platens for soft rocks do not appear to be influenced to any great extent by end effects at the platens (provided, of course, a correction is applied for any platen compression), for materials such as aluminium which are much stiffer, these end effects can significantly influence the calculated modulus (Chiu et al., 1983). Therefore, the aluminium specimen was tested in a universal testing machine by gripping the ends and measuring axial deflections on the specimen itself. Both compressive and tensile moduli were determined in this manner and found to be identical.

Table I Young's modulus of aluminium as measured by the test techniques used for Johnstone

Load displacement rate (mm/min)	Young's modulus (GPa)			
	Uniaxial test, compression and tension	Bending test		Tensile test, notch width measurement
		Notched beam	Un-notched beam	
0.05	65.6	66.0	65.0	62.2
0.5	66.0	65.0	65.5	65.5
5.0	65.6	65.5	65.0	65.5

The above tests on aluminium specimen were conducted at three different load displacement rates to see if there was any rate effect.

From the results of these investigations which are presented in Table I, it would appear that the Young's modulus of aluminium was not influenced by test techniques nor their methods of interpretation. It would seem reasonable to suggest that the results shown in Figure 1 are a true reflection of the variations between the compressive, bending and tensile moduli of the synthetic rock.

Adler (1970) proposed a relationship between the compressive E_c, bending E_b and tensile E_t moduli of a linear elastic material as

$$E_b = \frac{4\, E_c E_t}{(\sqrt{E_c} + \sqrt{E_t})^2} \quad (5)$$

Since the Johnstone appears to be linearly elastic (at least in so far as the material is subjected to only one of the loading modes of compression, bending or tension at any one time), then the results shown in Figure 1 should be represented by Equation (5). By considering the best fit lines for the compressive and tensile moduli, Equation (5) has been used to calculate the likely variation of bending modulus for the range of water contents employed. This line has been drawn in Figure 1 to show that the bending moduli actually measured seem to be in reasonable agreement with the measured ranges for compressive and tensile moduli. This result seems to add further support to the validity of the results shown in Figure 1.

7. CONCLUSIONS

On the basis of the detailed test results presented above, it would appear that the values of Young's modulus for the synthetic soft rock known as Johnstone, are highly dependent on whether the applied load induces compressive, bending or tensile stresses. The results seem to indicate that the range of tensile moduli are about an order of magnitude greater than the range of compressive moduli, with the bending moduli between the two as predicted by Adler (1970).

Although it is appreciated that the rock tested was synthetic, since its properties and performance are very similar to naturally occurring soft rocks, it is highly likely that similar characteristics will be displayed by these natural materials. Indeed, in the case of compacted clays, Ajaz and Parry (1975), have shown that the Young's modulus can be significantly greater in tension than compression.

The implication of these results is that it would appear important to ensure that the relevant modulus is used in analytical or numerical assessments of rock behaviour in the field, if reasonable estimations of performance are to be obtained. Should the compressive modulus be used in cases where the dominant stress mode is tensile, then since the compressive value is about an order of magnitude less than the tensile, estimations of elastic deflections could be an order of magnitude more than would actually occur.

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