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Subsidence Prediction Based on Multi-Membrane Model - An Analogue Approach

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SUMMARY Whenever a void is created underground by mining the minerals will lead to subsidence which may pose strata control problems and cause untold loss to the property. Hence a phenomena to predict the subsidence so that it can be controlled, will certainly help the mine planners and surface designers. Treating the rockmass as an elastic medium appears to be more logical assumption and based on this subsidence is predicted by using multi-membrane model with the help of three-dimensional electrolytic tank analogue. The predicted subsidence is in close conformity with the actual measured subsidence. Thus it can be presumed that this analytical approach for the prediction of subsidence may be better.

1 INTRODUCTION

In an era of severe energy crisis and concern to the environmental damage,it is imperative to extract the maximum amount of coal/minerals locked up underneath builtup areas, railway lines, civil constructions, rivers and roads etc. With least damage to the overall surface environment. The interaction of mine workings and the ground surface is of prime importance in the rehabilitation programmes. A proper forecasting of ground movements and subsidence is, therefore, an important element that can help the mine designer and town planner in designing and planning mine workings and civil constructions.

It is a well known fact that, whenever, minerals are won by underground mining methods, the overlying strata has a tendency to subside. This subsidence may get extended upto surface depending upon a large number of contributing factors responsible for subsidence. This results in the formation of hallows, trenches, open cracks in the earth's crust, and extensive subsidence trough posing ultimately a number of operational problems like damage to the surface property, damage to the canals, roads, communications, networks, valuable agricultural land and above all to the aesthetic beauty causing damage to the man's environment. However, for the development of the country, the increased mining activity is a must, side by side with the increasing residential and industrial development on the surface. It becomes, therefore obligatory for both mine and town planners to do everything possible to minimise this damage due to subsidence. The task, therefore, for a subsidence engineer is to develop procedures for:
a) Predicting strata and ground movements over mine workings.
b) Ascertaining the effects of such movements on builtup structures, mine shafts etc. and
c) Minimising subsidence damage by means of improvement and with proper regional and town planning.

It is therefore, the duty of a subsidence engineer to take necessary measures in relation to both mining and construction activity to protect surface structures within his mining lease areas from the damage that could impair their functioning or even rendering them dangerous or paralysed. The understanding of subsidence phenomena, therefore, is of utmost importance to both mine planners and surface designers in forecasting the possible effects of underground mine workings and other similar operations on the surface features and to take remedial steps to counteract its adverse effects.

A known age old technique to measure subsidence is surveying and its forecast by thumb rules. Research and experience has shown that subsidence is dependent on a number of factors and it is often difficult to incorporate all the interdependent variables in a single analytical or mathematical expression for varying nature of rock types and geological complexities.

Though many theories to predict rock movement have been suggested by a number of authors from time to time, treating the rock mass as an elastic medium appears to be logical in the process of understanding the behaviour of strata. Salamon (1963) has derived an analytical expression to determine the stresses and displacements at any point as a result of underground mining, treating the rockmass as an elastic medium in three dimensions and with irregular mining boundary. Based on this theory, Electrolytic Tank Analogue (ETA) was designed (Dhar, 1970) to predict the elastic response of the strata above an underground excavation. This theory has been used by various workers as well, to predict ground behaviour. From such studies it looks that treating the rock mass as an elastic medium appears to be logical attempt in the process of understanding the strata behaviour using analogy as an approach.

2 MULTI-MEMBRANE MODEL APPROACH

2.1 The Problem

Here an attempt has been made to predict surface subsidence for a given mining configuration, treating the strata above the seam as a multi-membrane model. This multi-membrane technique based on elastic theory and with some necessary assumptions and modifications has been extensively used for the prediction of strata movement as a result of subsidence using a 66 kilobyte DCM spectrum microcomputer which has the capacity and memory for solving Salamon's double integral formula for vertical displacements.

2.2 Basic Assumptions For The Approach

The basic assumptions in the application of linear elasticity to predict the strata movement are that,
a) The rocks have linear stress-strain relationships, i.e. they obey Hooke's law of elasticity
b) The strains produced due to mining are infinitesimal and,
c) Rock bodies within the requirements of individual models are continuous.

In the process of mining the only visible effects are deformations, and these movements are associated with stress changes through the equations of theory of elasticity. Therefore, an elastic solution should not only satisfy the basic mathematical equations of the idealized models, which were developed by Salamon (1963) to abate the problem of calculation of the displacements and stress fields to the determination of two axially symmetrical functions, but also the boundary conditions existing thereon. They are
I) the ground surface must be stress-free
II) at great depth both the stress and displacement induced by creation of mining excavations must be zero, and
III) since excavations are much wider than the thickness are considered, it is sufficient to specify conditions at the roof and floor of the seam.

2.3 Multi-Membrane Model Concept

As the study to predict the subsidence by the idealized models, developed by Salamon, like Homogenous isotropic model and homogeneous transversely isotropic model did not give a good matching predicted subsidence profiles with the measured profiles, the study is extended to multi-membrane model approach. The assumption in this model is that the strata contains a large number of horizontal membranes connected by a large number of vertical springs. And it is also assumed that the membranes are weightless,initially falt and tensioned uniformly. Then the equation of equilibrium for any one membrane is

$$\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial y^2} = - \frac{P\,(X,\,Y)}{T} \qquad (1)$$

where W is the deflection of the membrane, P(X,Y) is the applied transverse pressure and T is the uniform horizontal tension per unit length. In this model the vertical displacement at a point P(X,Y,Z) can be calculated by using the following formula,

$$W(X,Y,Z) = \iint_A \frac{S_{Z(X,Y,0)}}{\pi} \left[-\frac{a^2}{4} \left\{ \frac{Z_1}{(a^2r^2+Z_1^2)^{3/2}} - \frac{Z_2}{(a^2r^2+Z_2^2)^{3/2}} \right\} \right] dx\,dy \qquad (2)$$

where, S_z (X,Y,0) is the closure at the seam level and equal to 0.4 times the thickness of extraction multiplied by voltage measured on the analogue at a point (X,Y,0) and then divided by maximum voltage measured over the simulated analogue plate, and

$$Z_1 = Z - H \; ; \; Z_2 = Z + H \text{ and}$$

$$r^2 = (X - X')^2 + (Y - Y')^2$$

H is the depth of the seam, and Z is the depth from surface to a point under reference where subsidence is to be determined. Here, Z is taken to be zero as subsidence is predicted on the surface, and

$$a^2 = \frac{hE}{T}$$

where, h is the distance between two horizontal membranes and E is the Young's Modulus of the strata.

The equation (2) has, therefore, been used to predict the subsidence with the help of a model plate of the mine plan that is simulated on the Electrolytic Tank Analogue (ETA).

3 EXPERIMENTAL SET UP (ANALOGUE) TO MEASURE THE VOLTAGE VALUES OVER MODEL PLATES

3.1 General

The ETA was designed on Salamon's approach and the earlier phenomena that there exists a mathematical analogy between laws of theory of elasticity and equations governing the steady flow of elasticity. ETA approach was successfully used to obtain solutions to determine the elastic response of strata surrounding tabular mining excavations, then ETA was used to predict the pillar stresses in three dimensions. Subsequently ETA was built by Suryanarayana (1972) for the simulation of mining excavations in three dimensions. Since then, the set up with slight modifications from time to time was used by many workers and a number of mine problems were solved by many investigators. The theory of elasticity is the most convenient method to analyse the Strata behaviour Surrounding the openings, and Salamon's analogy is applicable to ex cavations of any configuration and complexity provided they lie within a plane and the height of extraction is relatively much smaller than the lateral extent of the workings. On the same principle ETA was developed.

3.2 Electrolytic Tank Analogue

Electrolytic Tank Analogue basically consists of a cubical tank body of perspex material, of 4'x4'x4' dimension, which is transparent. The front face is 1" thick rest faces being ½" in thickness. The ETA consists of two principal units, namely the electrolyte and electrodes.

The bottom electrode is made up of brass sheet and kept flat over the bottom of the tank. The top electrode is made up of copper sheet placed over the tank and the model plate on which mine plan is simulated, is fixed into the central square aperture of the electrode and the top electrode is properly earthed. Then the bottom electrode is energised by supplying voltage at a constant value of 10 volts by adjusting the control knobs of the power amplifier. An A.C. milli volt recorder at the full deflection scale of 0.3 volts was used to read the voltage on the probing holes of the model plates with the help of platinum Probe. Figure 1 shows the sectional view through Electrolytic Tank Analogue.

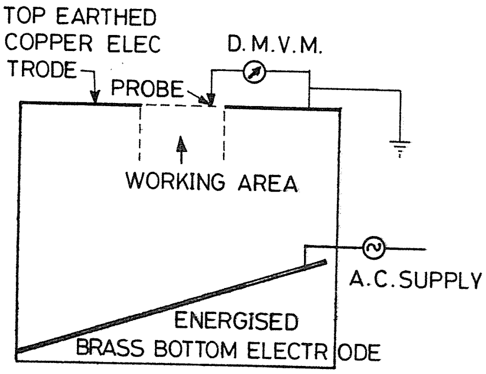

Figure 1 Sectional View Through Electrolytic Tank Analogue

4 SIMULATION OF MINE EXCAVATIONS

4.1 General

Mine excavations of Jharia Coal fields (India) were simulated on the copper model plates of 1'x1' by etching out the copper. The etched out portion represents the excavated area. Over the etched out area 1mm. diameter holes were drilled at a regular interval of 5mm on both the axes. Excavations of Mine I and Mine II of Jharia Coal fields over which surface subsidence was measured by usual means of survey techniques are simulated on the model plates. The voltage measurement as is the technique and practice in analogue simulation were measured over all the grid points of the model plates and the observed values were fed to the computer along with mine and analogue parameters, which are shown in Table I.

TABLE I

DETAILS AND PARAMETERS OF THE MINES

Sl. No.	Details	Mine I	Mine II
1	Depth of the seam(m)	46.0	47.0
2	Inclination of the seam (Degrees)	7.5	9.5
3	Assumed inclination of the seam	Horizontal	Horizontal
4	Thickness of extraction(m)	2.6	2.55
5	Scale of the Mine Plan one hole space=	7.9 m	8.75 m
6	Poison's Ratio	0.25	0.25
7	Young's Modulus(kg/cm^2)	80000	80000
8	Maximum convergence assumed at the centre of the panel (mm) in percentage of extraction	1040.0 40.0	1020.0 40.0
9	Distance between the top and bottom electrodes (m)	1.1	1.1
10	Supply voltage, V in Volts	10.0	10.0

Taking the parameters of the mines, surface subsidence is predicted by using the equation(2). The subsidence is predicted only over the lines of interest which are shown in the figure 2 and figure 3 and compared with the actual measured subsidence to analyse the descripancies, if any. A computer program in FORTRAN language to predict the surface subsidence over the lines of interest, and to plot the actual and predicted subsidence and to give the percentage of variance in the actual and predicted values at the desired co-ordinates, was developed by the authors. The mine plans of Mine I and Mine II along with sectional lines over which field subsidence values have been measured (Mozumdar, 1985) were shown in Figure 2 and 3 respectively. Brief description for each mine has also been given to further explain the working conditions, seam levels exploited, method and overall working conditions etc. to analyse the discrepancy in the actual and predicted subsidence profiles.

4.2 Mine I Details

Surface subsidence was measured while this mine was working No. III Seam which is 3.9 meters thick and lying at an average depth of 47.0 m. The IV seam lying over the III Seam was virgin but zero seam underlying the III seam was developed and standing on pillars. The overlying strata above III Seam was undisturbed and consisted of hard grey sandstone constituting about 80% of the strata while the remaining 20% was shale and Coal. The seam was developed by Bord and Pillar workings (Figure 2) to an height of 2.55 m and depillaring was done by conventional method, maintaining a diagonal line of extraction. Along two lines, one parallel and another perpendicular to the line of extraction, Subsidence observations were made.

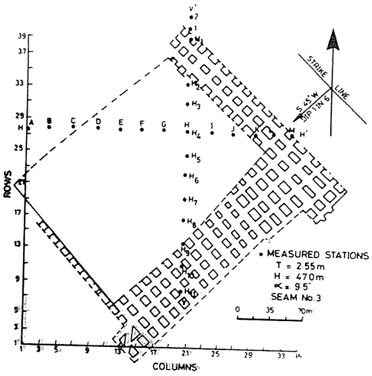

Figure 2 Plan showing the simulated excavations of Mine I

4.3 Mine II Details

In this mine observations were carried out on the Surface, when it was working number X Seam whose thickness is 9.75 m and lying at an average depth of 187.0 m. There are many seams overlying X Seam which were worked out by Caving/Stowing. The immediate roof above the working X Seam was fine grained sandstone. 0.3 to 1.3 m thick stone band splitted the X Seam into two sections. The top section of the splitted X Seam was developed by Bord and Pillar methods upto an height of 3 m. The bottom section of the splitted X Seam was Virgin. The depillaring was done by conventional method and the overall percentage of extraction was given to be 46.0. Subsidence profiles were drawn along two lines one parallel and the other perpendicular to the line of extraction as shown in figure 3.

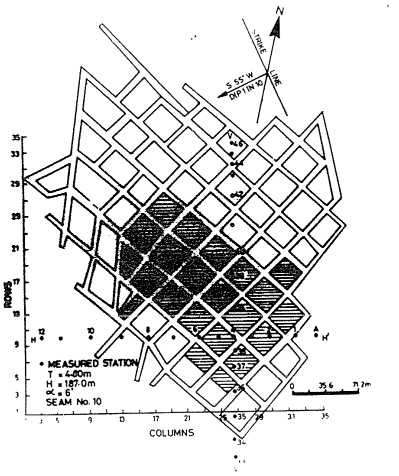

Figure 3 Plan showing the simulated excavations of Mine II

5 EXPERIMENTAL RESULTS

5.1 General

The predicted subsidence along the lines of interest H-H' and V-V' in Mine I and H-H' and V-V' in Mine II along with the measured actual subsidence is shown in figures 4, 5, 6 and 7 respectively.

It may be mentioned that one of the factors that enters into the subsidence Prediction is the maximum convergence which is expressed as a percentage of thickness of extraction as given in Table I. However, the value of a^2 is 8.0 in both the cases gave good results.

Figures given for the comparison of predicted and measured surface subsidence profiles shown a considerable agreement in general for all cases. However, the descripancies between the measured and predicted subsidence profiles could be due to several factors depending upon strata and Seam working conditions. Given below is an attempt to explain one by one where the agreements in general are not satisfactory.

5.2 Sectional Line H-H' of Mine I

Figure 4 shows the comparison of the predicted and measured surface subsidence profiles. It is very interesting to observe that both the profiles are more or less matching with an exception at the right edge. This descripancy probably due to the presence of barrier pillar. The maximum predicted subsidence point is almost coinciding with that of the measured one.

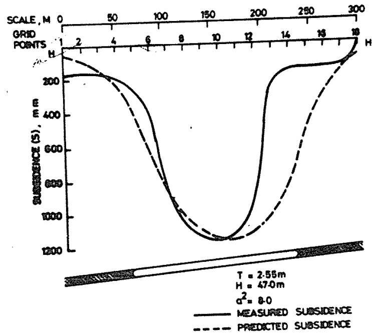

Figure 4 Comparison of predicted surface subsidence profile with the measured profile along line H-H' of Mine I

5.3 Sectional Line V-V' of Mine I

The measured Subsidence Profile is not symmetrical (Figure 5) and the subsidence is continuous at the dip side and probably this is due to the continuous mining activity in the adjoining panel. Sudden fall in the measured profile may be due to collapse of Pillars. The shift in maximum subsidence point towards dip side may be due to the seam inclination and the absence of panel barrier.

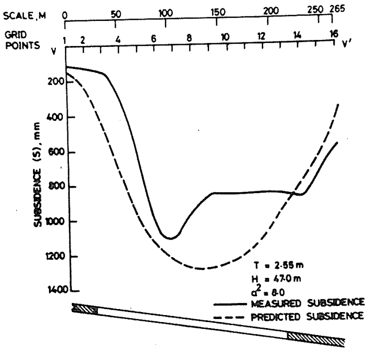

Figure 5 Comparison of predicted surface subsidence profile with the measured profile along line V-V' of Mine I

5.4 Sectional Line H-H' of Mine II

The descripancy in the location of maximum subsidence point is probably due to the inclination of the Seam. The measured subsidence profile is comparatively smooth. From the figure 6 it is logical to conclude that at greater depths the influence of the ribs on the surface subsidence will be less.

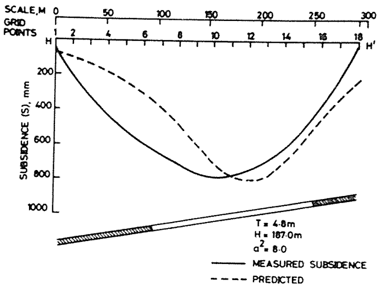

Figure 6 Comparison of predicted surface subsidence profile with the measured profile along line H-H' of Mine II

5.5 Sectional Line V-V' of Mine II

The measured profile is comparatively smooth in dip side (Figure 7) and considerably in agreement with the predicted profile. The descripancy in the two profiles on rise side could be due to the presence of pillars.

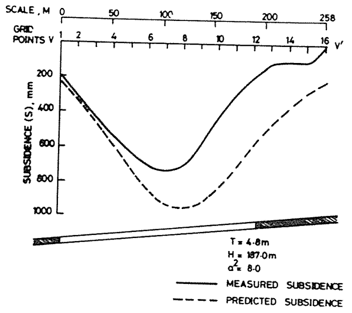

Figure 7 Comparison of predicted surface subsidence profile with the measured profile along line V-V' of Mine II

6 CONCLUSIONS

From the investigations for the prediction of subsidence by treating the ground as a multi-membrane model revealed the following

1 Satisfactory results were obtained by assuming the maximum convergence value to be 40 percent of the thickness of extraction.
2 A close agreement has been observed between the measured and predicted subsidence for the value of 8.0 to a^2.
3 From the profiles along sectional line H-H' of Mine II it can be concluded that at comparatively greater depths the influence of the ribs on the surface subsidence will be less.
4 In general the mentioned investigations seems to be useful and quite logical where scientific technique that can be used for the prediction of Subsidence prior to the actual extraction of coal underground.

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