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Face Stability Analysis in Fractured Rock by the Statistical Simulation of Rigid Block Failure

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SUMMARY A new approach, and computer program, for the analysis of face stability in jointed rock masses is presented. The geometrical properties of discontinuities are characterised by statistical distributions which are then used to generate random, three-dimensional realisations of the rock structure. Each realisation is interrogated to identify, and then determine the translational and rotational stability of all kinematically feasible rigid tetrahedral blocks. Face stability is assessed by determining the volume of rock, per unit area of face, that is predicted to be unstable. The method is illustrated by a case example based on slope stability analysis in an open pit iron ore mine in Western Australia. The case example shows how, for a given group of discontinuity characteristics, water pressure and face orientation control stability. When several different random realisations were generated for a single face, from the same discontinuity characteristics, a wide range of stability indices was observed.

1 INTRODUCTION

In many mining and civil engineering operations it is necessary to create free faces in discontinuous rock masses that will remain stable for a period ranging from a few months to many decades. The term 'free face' is used here to describe any natural or artificial, overhanging or non-overhanging, planar rock face located in an open excavation or below ground. The behaviour of a discontinuous rock mass adjacent to such a rock face will, at relatively low stress levels, be controlled by the geometrical and mechanical properties of the discontinuities. The traditional approach to face stability analysis under these conditions, particularly for non-overhanging rock slopes, is to postulate a failure mechanism that could develop under the observed discontinuity characteristics. The most common mechanisms include sliding or rotation of tetrahedral or polyhedral blocks, multiple interactive block movement and circular or multi-linear slip. Analysis of the mechanism provides an index of face stability such as factor of safety, required equilibrating force, or expected volume of failure. There is a wide range of discontinuity characteristics that control face stability, these are: (i) discontinuity cohesion and friction (including the effects of surface roughness, aperture and infill) (ii) orientation, (iii) size, (iv) location, and (v) frequency (Piteau, 1970). Other factors influencing face stability are face orientation, size and attitude (ie whether overhanging or not) and water pressure. Although the traditional approaches referred to above are good at modelling the influence of discontinuity shear strength, discontinuity orientation and water pressure, they are not suited to modelling the influence of discontinuity location, size and frequency or the variability of any of the input parameters. The traditional methods are also poor at modelling the influence of free face orientation and size.

If the discontinuity frequency is relatively low, and rock material stiffness and strength are high, it is possible to address some of the deficiencies in the traditional methods by modelling the influence of discontinuities explicitly. This can be achieved by means of analytical or numerical models of the displacements and stability of deformable and rigid rock blocks defined by the observed discontinuity geometry. Numerical methods for modelling the behaviour of idealised two-dimensional block assemblages have been developed by Cundall (1971) and more recently by Lorig and Brady (1984). Modelling three-dimensional assemblages of complex polyhedra is analytically and computationally more demanding so that, to date, such work has been limited to the analysis of individual rigid tetrahedral blocks (Priest, 1985) and rigid polyhedra (Warburton, 1981) bounded by infinitely stiff discontinuities in a rigid rock mass.

In 1980 Hudson and La Pointe developed the idea of generating random realisations of discontinuity geometry for the analysis of fluid flow through fractured rock masses. This approach, which was also adopted by Long (1983), and Samaniego (1985) involves characterising the distributional properties of each discontinuity set occurring in the rock mass under study and then selecting a series of random values from each distribution to produce a random two- or three-dimensional realisation. The fluid transport properties of this realisation are then determined numerically and taken to predict the properties of the rock mass under study. In most cases each discontinuity set is characterised in terms of the distribution type, mean and standard deviation for orientation, size and aperture. The discontinuity centres are usually assumed to obey a homogeneous two- or three-dimensional Poisson process governed by a constant linear, areal or volumetric frequency.

The aim of this paper is to describe a new method for the analysis and design of rock faces in discontinuous rock masses subject to relatively low stress levels. The method combines the idea of stability analyses of individual rigid tetrahedral blocks with the concept of random realisations of discontinuity geometry described above. The method is implemented in the form of a FORTRAN 77 program BLOCK which carries out a Monte Carlo type analysis of block stability by generating a three-dimensional random realisation of discontinuity planes intersecting a specified rock face. Each kinematically feasible tetrahedral block in the realisation is processed to determine its translational and rotational stability. The assumptions and solution techniques for the program BLOCK are described in the first part of the paper; the second part is a case example demonstrating an application of the program to the analysis of a non-overhanging rock face in an open pit mine in Western Australia.

2 DATA COLLECTION AND ANALYSIS

Representative data on the geometrical and mechanical properties of discontinuities in the rock mass under study must be obtained for inputting to the random realisation routines of BLOCK. The discontinuities are assumed to occur in up to 10 sub-parallel groups, or sets, with orientations obeying an isotropic Fisher distribution. Although more sophisticated anisotropic orientation models could be incorporated, they have not yet been needed. The following data are required for each set:

(i) mean orientation and Fisher's constant, the latter being a measure of the degree of preferred orientation within the set,
(ii) the nature of the trace length distribution, its mean and, if applicable, the standard deviation,
(iii) the frequency along the mean normal,
(iv) the Coulomb shear strength parameters - cohesion and angle of friction.

The geometrical data can be obtained by the examination of borehole core or by taking measurements at exposed rock faces, adopting scanline or photographic sampling techniques. Graphical methods for determining discontinuity orientation from borehole core, and vectorial methods for analysing discontinuity data to determine mean orientation and Fisher's constant are explained by Priest (1985). Trace length data obtained from scanlines can be processed to eliminate sampling bias and to estimate distributional properties by following the methods explained by Priest and Hudson (1981). Negative exponential, uniform, normal or log-normal distributions of trace length can be specified in the input data. Alternative methods for determining mean trace length, from photographs or window samples of the rock face, are discussed by Baecher (1983) and Pahl (1981). The analysis of discontinuity frequency data, to obtain the normal frequency for each set, is discussed by Hudson and Priest (1983).

The Coulomb shear strength parameters for discontinuities from each set can be determined from direct shear tests, following the methods described by Hoek and Bray (1981). At present only single deterministic values of cohesion and angle of friction for each set can be input. It would be a relatively simple task to modify the shear strength model to allow for stress dependent or anisotropic shear behaviour. Similarly a stochastic shear strength model could be incorporated if data on the variability of the shear strength parameters were available. In most practical cases, however, cost considerations severely restrict the number of shear tests that are carried out, providing little justification for a more sophisticated shear strength model at this stage.

All rock blocks are assumed to be subject to gravitational loading. It is also possible to specify up to 20 additional force vectors and up to 20 additional acceleration vectors on each block to simulate the effects of rock bolt loads and seismic effects respectively. If the orientation of a given seismic acceleration is unknown it can be specified to act along the potential movement direction of the block, giving the worst possible case. It is, in addition, possible to specify a uniform fluid pressure acting on any plane in the block model. When applied to the main free face this fluid pressure simulates the effects of a uniform support pressure on the face; when applied to one or more of the discontinuity sets the effect is to simulate fluid pressure on discontinuities within the rock mass. In this latter case the fluid pressure can be specified to vary as a linear function of distance from the rock face. One of the more difficult aspects of preparing the input data for the block analysis is to establish realistic values for fluid pressures on each discontinuity set. The best approach is to measure actual water levels in a grid of piezometers adjacent to the face being studied. If this is not feasible, water pressures can be estimated from continuum, or discontinuum, hydrogeological models of the rock mass, subjected to appropriate boundary conditions.

3 BLOCK GENERATION

A fundamental feature of the input data for the program is the specification of the orientation, attitude and size of the main free face. The size is defined in terms of the coordinates of the corners of a square or rectangular generation area containing a smaller sample area within which block stability is analysed. By making the generation area sufficiently large compared with the sample area it is possible to minimise errors caused by a reduction in discontinuity frequency close to the boundaries of the generation area. A ratio in areas of 2:1 was found to be sufficient in most cases. The discontinuities are first generated as linear traces in the plane of the rock face and then extended back into the rock mass by applying the geometrical model outlined below. This simplification, which yields a dramatic saving in computational effort, is possible when analysing only tetrahedral blocks since any discontinuity involved in delimiting a block must intersect the rock face.

The first step in the block generation process is to create a two-dimensional random realisation of discontinuity traces, for up to 10 separate discontinuity sets, within the generation area. By analysing the input data for orientation and frequency it is possible to calculate the expected frequency for each discontinuity set along the boundaries of the sample area. Generation proceeds by allowing the mid-point of each trace to obey a two-dimensional Poisson process within the generation area, selecting random values of trace length and three-dimensional orientation from the appropriate parent distributions. Generation continues until all sets have achieved their expected sample area boundary frequency. This approach has the benefit of constraining the generation process for a particular face orientation to reflect measured frequency values. A hard copy of the realisation in the plane of the rock face can be plotted if required. Figure 1 shows a 20 by 20m sample area containing 1592

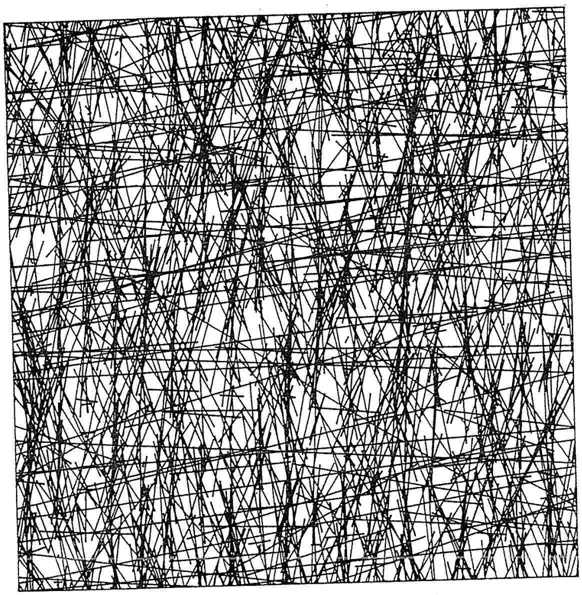

Figure 1 Random realisation of discontinuity traces

traces in a random realisation of eight discontinuity sets for the case study presented later in this paper.

The discontinuity traces are extended back into the rock mass by applying the following geometrical model. A discontinuity trace, of length ℓ, is assumed to be a chord of a circular discontinuity of radius r that has its centre a distance t from the rock face, this distance being measured at right angles to the trace in the plane of the discontinuity, as shown in Figure 2. By simple geometry

$$r = \sqrt{((\ell/2)^2+t^2)}$$

The offset t is assumed to be a random uniform variable in the range $-k\ell$ to $+k\ell$, negative values indicating that the discontinuity centre lies in free air. The parameter k was selected following recommendations by Lamas (1986) who showed that a value of $k=0.5\sqrt{3}$ would give a maximum discontinuity radius r equal to the trace length ℓ. Lamas noted that for a given trace length ℓ, and a uniform distribution of the offset t, the distribution of radii r would be of exponential shape, truncated at $\ell/2$ and censored at ℓ. This simple size/shape model has the following advantages:

(i) The original trace length distribution is preserved so no assumptions are made concerning the distribution of discontinuity areas.
(ii) A strong correlation between discontinuity trace length and radius is imposed.

It is believed that more sophisticated models of discontinuity geometry are not justified until more is known about discontinuity size and shape in real rock masses.

The discontinuity realisation, held in numerical form, is first interrogated to produce a list of all the intersection points. This list is then analysed to identify all groups of three discontinuity traces that together form closed triangles within the sample area. Each of these triangles is examined to determine:

(i) whether the discontinuity orientations and locations form a kinematically feasible tetrahedral block, and
(ii) whether the discontinuities are large enough to intersect within the rock mass and delimit the block.

4 STABILITY ANALYSIS

The geometrical properties of all blocks that satisfy both of the above geometrical criteria are passed to subroutines that determine translational and rotational stability. Gravitational, hydraulic, seismic and other forces acting on each block are analysed to determine the resultant force on the block and thereby deduce the category of block behaviour following the methods explained by Priest (1985). Four categories are recognised as follows:

Category 0: The block is kinematically stable, so that in a rigid rock mass it cannot move, whatever the magnitude of the resultant force.
Category 1: The block, if unstable will slide on a single discontinuity plane.
Category 2: The block, if unstable, will slide on two discontinuity planes along their line of intersection.
Category 3: The block is free to move in the direction of the resultant force.

Blocks in category 0 are assigned a factor of safety of 10.0, those in category 3 a factor of safety of zero. The normal and shear forces on the plane(s) of sliding for those blocks in categories 1 and 2 are input, together with the appropriate values of cohesion and friction, to the Coulomb shear strength model to determine the factor of safety against translational sliding, as explained in Chapter 8 of Priest (1985). For those blocks that have a factor of safety less than 1.0, the magnitude of a fictitious equilibrating force is computed. This equilibrating force is defined such that if applied against the direction of translation for the given mechanism, the block would be restored to a factor of safety of 1.0. The sum of the normal components of the equilibrating forces of all blocks provides a conservative measure of the required support pressure; this sum is here termed the support pressure index.

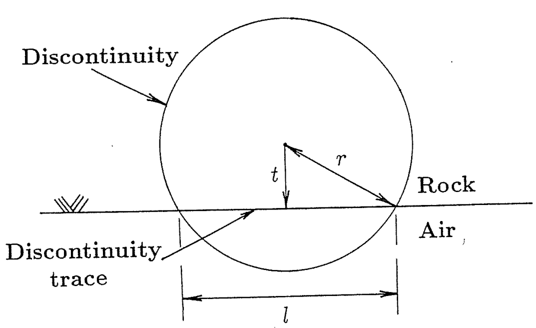

Figure 2 Model for discontinuity shape

A total of 12 rotational modes and 6 toppling modes of displacement are analysed for any tetrahedral block that is found to have a factor of safety greater than 1.0 for the translational mechanisms. Each of the rotational modes is a rigid body rotation about an axis constructed normal to one of the planes forming the tetrahedron, and located so that it passes through one of the vertices lying in that plane. There are four planes bounding each block and three vertices in each plane, giving 12 independent rotational modes. Each of the toppling modes is a rigid body rotation about an axis formed by the block corner joining a pair of vertices. There are 6 such block corners and therefore 6 independent toppling modes. Each rotational and toppling mode is analysed for both a positive and a negative sense of rotation, giving a total of 36

independent toppling and rotational mechanisms. Any given rotational or toppling mechanism is deemed to be unstable if the following two conditions are met:
(i) the mechanism is kinematically feasible, and
(ii) the mechanism leads to a nett loss in the potential energy of the block.
These two conditions are tested for a given mechanism by allowing it to generate a small angle of virtual rotation in the block. The new locations of the centroid of each face of the block are then inspected. If none of these centroids has moved into the adjacent rock the mechanism is deduced to be kinematically feasible. The second condition is tested by monitoring the centroid of the block; if this centroid moves with a positive component in the direction of the resultant force there is a nett loss in potential energy. The analysis is not sensitive to the magnitude of the applied virtual rotation; a small angle, say 1°, is usually adequate.

The results of the analysis of translational and rotational stability are presented in tabular form, listing the following data for any block whose volume exceeds some specified threshold value: volume, area of the face triangle, height measured normal to the rock face, factor of safety, category of translational behaviour, plane(s) of sliding, equilibrating force and number of unstable mechanisms of toppling and rotation. These data can be processed further, if required, to produce histograms of any of the parameters at any specified class interval. The program also incorporates an option to carry out a deterministic analysis of a single block whose geometry is specified by the orientation and location of four planes. This facility proved to be valuable for validating the kinematic and stability analysis subroutines.

All kinematically feasible tetrahedral blocks in the three-dimensional random realisation are identified and analysed. Because a given element of rock may lie within more than one block, there is a tendency to overestimate the volume of potentially unstable rock by a factor of between 2 and 5. To overcome this problem, a three-dimensional grid of elements 50 by 50 is set up to cover the sample area and extend back into the rock mass, as shown in Figure 3. Each element in the grid is initially assigned a factor of safety $F_e=20.0$ and a number of modes of rotational instability $N_e=0$. When the factor of safety F_b and the number of modes of rotational instability N_b of a given block have been computed, the block is superimposed on the grid at its correct location. If, for any element within the block, $F_e>F_b$ then F_e is assigned the value F_b. Similarly if, for any element, $N_e<N_b$ then N_e is assigned the value N_b. As each block is analysed, the values F_e and N_e for the associated elements are updated in this way to generate a three-dimensional map of rock stability. When all the blocks have been analysed the map is interrogated to produce histograms of the number of elements, and hence the volume of rock, in terms of factor of safety and number of modes of rotational instability. These histograms are then converted to cumulative form. Slices through the three-dimensional map can be taken, to illustrate the spatial distribution of instability at the face or within the rock mass. Assessment of rock face stability, by this analysis of instability within a grid element system, is illustrated in the following case study.

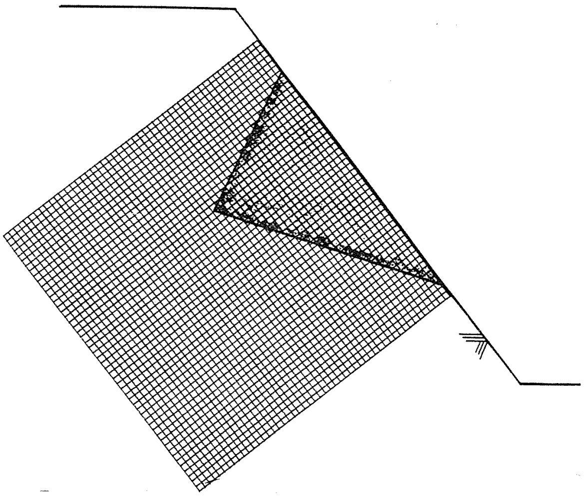

Figure 3 Grid element system

5 CASE STUDY

The case study is concerned with an assessment of slope stability in the hangingwall of the iron ore mine at Koolan Island, Western Australia, operated by BHP Minerals Division. The typical cross-section in Figure 4 shows that the hangingwall, on the southern side of the mine, is composed of Arbitration Cove Quartzite overlying Elgee Siltstone. The crest of this southern face is currently between 30 and 80m above sea level and is located, at its closest point, less than 100m north of the shoreline. Plans for the final phase of development at Koolan Island involve excavating to a maximum depth of approximately 82m below sea level, with an overall slope angle of up to 60° on the hangingwall, constructed in 12m and 24m high benches and 70° batter angles.

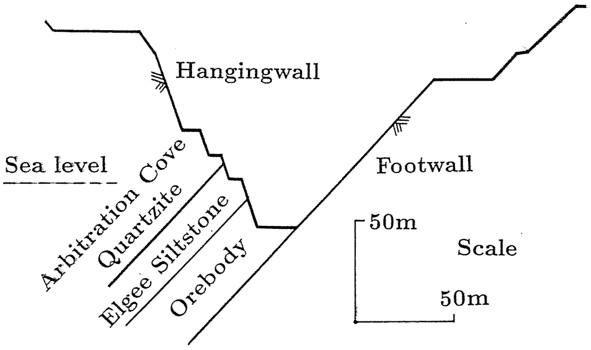

Figure 4 Cross-section through mine, Koolan Island

Although the Arbitration Cove Quartzite is materially very strong, with a uniaxial compressive strength exceeding 150MPa, it is heavily jointed and could present rigid block stability problems at faces subject to sea water pressure. Discontinuity data were collected from exposed faces in the quartzite by the senior author and BHP staff utilising scanline and photographic sampling techniques. The data were processed by the methods referred to in Section 2. Table 1 summarises the following data for the eight discontinuity sets identified: mean dip direction/dip amount (Orn.), Fisher's constant (FC), mean trace length (Len.), mean normal frequency (Freq.) and the Coulomb shear strength parameters c and ϕ determined from shear box tests on 300mm square samples. These data were

Table 1 Discontinuity characteristics

Set	Orn. deg.	FC	Len. m	Freq. m^{-1}	c kPa	ϕ deg.
1	113/79	262	3.7	1.72	15	30
2	100/59	99	3.7	3.03	15	30
3	027/27	84	2.4	1.67	20	35
4	036/55	106	1.9	1.35	20	35
5	006/65	69	2.4	1.96	20	35
6	345/68	100	2.2	0.27	20	35
7	309/70	41	4.1	4.55	15	30
8	215/49	91	100	1.89	10	28

input to the program BLOCK, together with details of the face geometry and water pressures. The rock face was taken to be non-overhanging with a dip direction of 030°; dip angles of 55, 60, 65 and 70° were modelled in a parameter study. A 30m square generation area containing a 20m square sample area was adopted thoughout. Water pressure gradients of 3, 5, 10 and 20 kPa/m were applied under gravitational loading, with no additional forces or acceleration vectors. The four face dip angles and four water pressure gradients gave a parameter study requiring 16 separate runs. The random realisation in Figure 1 shows 1592 discontinuity traces intersecting the 65° rock face; this contains 29184 intersections, 15149 face triangles and 2664 tetrahedral blocks. Each run required between 20 and 25 minutes CPU time on a VAX 11/785. Figure 5 shows a map of the 65° rock face indicating the areas of the face that are unstable under a water pressure gradient of 20kPa/m. Figure 6 contains graphs of the volume of rock per unit area of face that has a factor of safety less than F, for the four different water pressure gradients at the 65° face. Table 2 summarises the results of the complete parameter study, listing the volume of rock per unit area of face with a factor of safety less than 1.0 ($V_{1.0}$) and the support pressure index for each run.

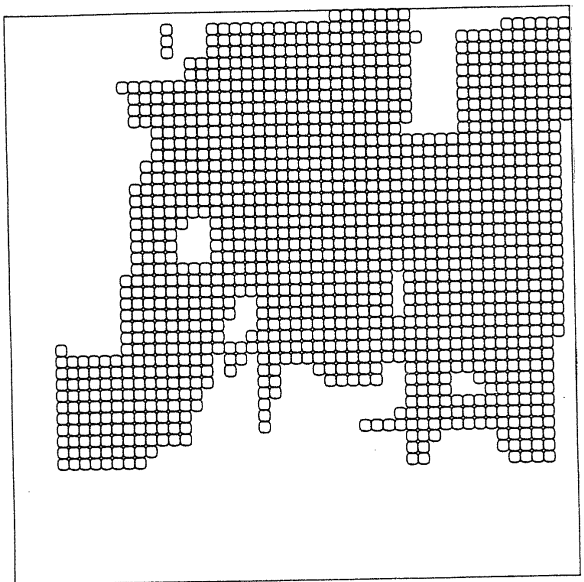

Figure 5 Map of unstable rock elements, 65° face

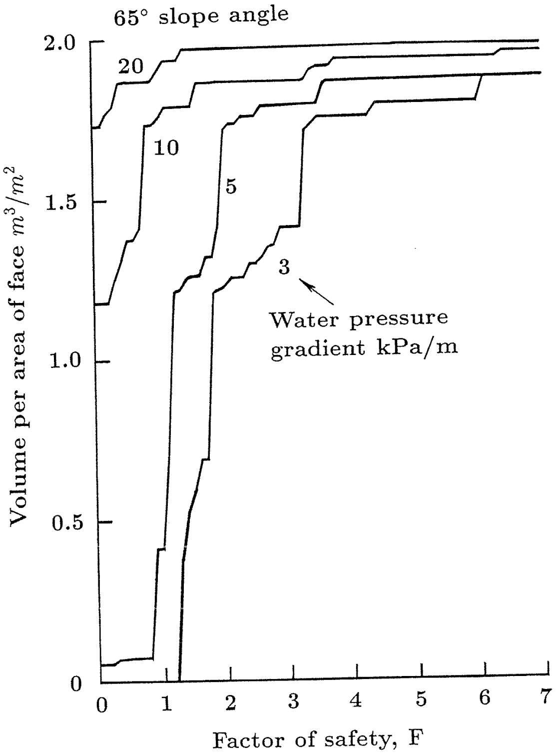

Figure 6 Rock volume versus factor of safety, 65° face

Table 2 Results of parameter study

Volume per area with $F \leq 1.0$ ($V_{1.0}$)
Support pressure index

Slope	55°	60°	65°	70°
3kPa/m	0.0	0.0	0.0	0.007
	0.0	0.0	0.020	0.253
5kPa/m	0.005	0.0	0.413	0.437
	0.250	0.117	13.08	29.63
10kPa/m	1.005	0.606	1.758	1.500
	294.4	319.2	548.3	500.6
20kPa/m	1.039	0.637	1.907	1.572
	1118.9	812.4	1563.8	1410.9

Figure 6 and Table 2 show the consistent influence of water pressure on face stability. Although Table 2 indicates a general tendency for the steeper slopes to be less stable, there are some anomalies; for example the 65° slope seems to be more stable than the 70° slope at water pressure gradients of 3 and 5 kPa/m but less stable at gradients of 10 and 20 kPa/m. To investigate this, 30 further runs were carried out at a slope angle of 65° and water pressure gradient of 5kPa, adopting a different random number generator seed for each run. The results of this series of runs are presented in Figure 7 as a histogram of $V_{1.0}$, at a class interval of 0.2m. The wide distribution of values illustrates the influence that random variability in discontinuity geometry can have on rock face stability. The high values of $V_{1.0}$ are caused by the chance occurrence of one or two large, unfavourably orientated discontinuities in the realisation, leading to the formation of large unstable tetrahedral blocks and high failure volumes. These random effects are not diminished

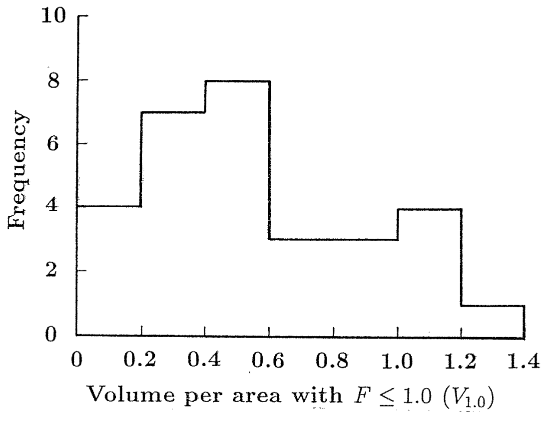

Figure 7 Histogram of unstable rock volumes

by taking a larger face area, and hence larger sample size, since although the number of blocks increases with the square of the width of the sample area, the largest block volume increases with the cube of this dimension; the chance occurence of just a few large unstable blocks will always dominate face stability. This overiding influence of major discontinuities on face stability is confirmed by observations in the mine, where unfavourably orientated, persistent discontinuities are seen to control block behaviour.

6 CONCLUSIONS

The statistical simulation of rigid block failure provides an effective method for representing the influence of discontinuity characteristics and face geometry on rock mass stability. A range of distribution types is used to model variability in the geometrical characteristics of up to 10 discontinuity sets. The adoption of a relatively simple tetrahedral block model makes it feasible to analyse the stability of each block in a three-dimensional realisation containing several thousand discontinuities. A brief case example illustrates how, for a given group of discontinuity characteristics, water pressure and face orientation control stability. When several different random realisations were generated for a single face, from the same discontinuity characteristics, a wide range of stability indices was observed. The low stability end of this range is caused by the chance occurrence of a small number of extensive, unfavourably orientated discontinuities forming large unstable tetrahedral blocks and high failure volumes.

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The program BLOCK is dedicated to the memory of Dave Champion.

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